

Experimental Demonstration of Unconditional Entanglement Swapping for Continuous Variables

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The unconditional entanglement swapping for continuous variables is experimentally demonstrated. Two initial entangled states are produced from two nondegenerate optical parametric amplifiers operating at de-amplification. Through implementing the direct measurement of the Bell-state between two optical beams from each amplifier the remaining two optical beams, which have never directly interacted with each other, are entangled. The quantum correlation degrees of 1.23 and 1.12 dB below the shot noise limit for the amplitude and phase quadratures resulting from the entanglement swapping are measured straightly.

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It has been recognized that quantum entanglement is an important resource in quantum information and computation. Because of utilizing entanglement shared by sender and receiver together with local operations and classical communication, various feats of quantum communication, such as quantum teleportation [1–5] and quantum dense coding [6,7], have been experimentally demonstrated with both discrete and continuous quantum systems. Another novel and attractive task in quantum information is entanglement swapping, which means to entangle two quantum systems that have never directly interacted with each other. The entanglement swapping of discrete variables has already been achieved experimentally with single photons [8]. However, the post-selection is a standard intrinsic procedure in quantum communication systems of discrete variables because if no photon is detected, or the two photons simultaneously entering remote detectors are not in the same basis, the corresponding time slot has to be ignored and, hence, does not contribute to the raw data [9]. Later, the protocols of unconditional entanglement swapping for continuous variables (CVs) were theoretically proposed, in which the determinant squeezed-state entanglement of the continuous electromagnetic field was exploited; thus, the post-selection is not needed [10–12]. To the best of our knowledge, the entanglement swapping of CVs has not been experimentally accomplished so far. Thus it still is a real challenge to realize unconditional entanglement swapping without post-selection of "successful" events by photon detections. In this Letter, we will present the first experimental realization of CV entanglement swapping.

Figure 1 is the schematic of the experimental setup. The laser is a homemade continuous wave intracavity frequency-doubled and frequency stabilized Nd-doped YAlO₃ perovskite (Nd:YAP)/KTP (KTiOPO₄, potassium titanyl phosphate) ring laser consisting of five mirrors [13]. The second harmonic wave output at 0.54 μm and the fundamental wave output at 1.08 μm from the laser source

are used for the pump field and the injected signal of two nondegenerate optical parametric amplifiers (NOPAs), respectively. For obtaining a pair of symmetric Einstein-Podolsky-Rosen (EPR) entangled optical beams, the two NOPAs (NOPA1 and NOPA2) were constructed in identical configuration, both of which consist of an α -cut type-II KTP crystal and a concave mirror. The front face of KTP is coated to be used as the input coupler. The concave mirror (the output coupler of EPR beam) is mounted on a piezoelectric transducer (PZT) for locking actively the cavity length of NOPA on resonance with the injected signal at 1.08 μm . Through a parametric down conversion process of type II phase match, an EPR beam with anticorrelated amplitude quadratures and correlated phase quadratures may be produced from a NOPA operating in the state of de-amplification; that is, the pump field and the injected signal are out of phase [7]. The entangled two modes of

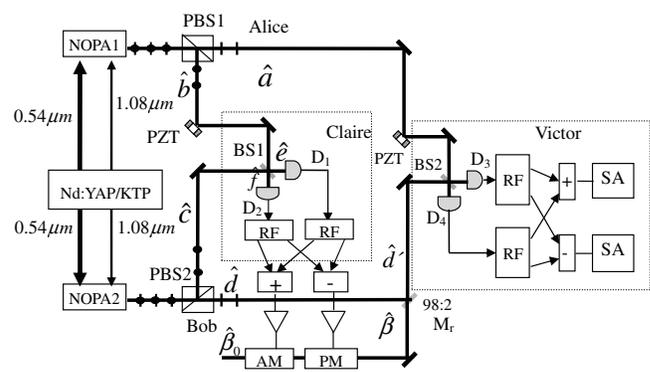


FIG. 1. Schematic of the experimental setup. Nd:YAP/KTP—laser source; NOPA—nondegenerate optical parametric amplification; PBS—polarizing optical beamsplitter; BS—50% optical beam splitter; RF—frequency splitter; +—positive power combiner; —negative power combiner; AM—amplitude modulator; PM—phase modulator; D_{1-4} —photodiode detector (ETX500 InGaAs); SA—spectrum analyzer; M_r —98:2 optical beam splitter

EPR beam are just the signal and idler modes produced from the process, which have identical frequency with the injected signal at $1.08 \mu\text{m}$ and the orthogonal polarization with each other [7]. In the case, the variances of the sum of amplitude quadratures and the difference of phase quadrature for the two entangled modes are both smaller than the shot noise limit (SNL) defined by the vacuum fluctuation. Because the same laser serves as the pump field and the injected signal source of two NOPAs, the classical coherence between a pair of EPR beams generating from two NOPAs is ensured.

The two entangled optical modes, \hat{a} , \hat{b} and \hat{c} , \hat{d} from NOPA1 and NOPA2 are distributed to Alice and Bob, respectively. Alice (Bob) divides mode \hat{a} and \hat{b} (\hat{c} and \hat{d}) in orthogonal polarization with polarizing-beam-splitter PBS1 (PBS2). Initially, Alice and Bob do not share an entangled state. However, we will see that Alice and Bob can establish the entanglement of mode \hat{a} and \hat{d} if they ask Claire for her assistance and send mode \hat{b} and \hat{c} to her. Claire performs a joint measurement of mode \hat{b} and \hat{c} by the direct detection system of Bell-state; that is, both the variances of the sum of amplitude quadratures $\langle \delta^2(\hat{X}_{\hat{b}} + \hat{X}_{\hat{c}}) \rangle$ and the difference of phase quadratures $\langle \delta^2(\hat{Y}_{\hat{b}} - \hat{Y}_{\hat{c}}) \rangle$ are simultaneously measured by using a self-homodyne detector with two radio frequency (rf) splitters and two (positive and negative) power combiners to produce the classical photocurrents \hat{i}_+ and \hat{i}_- [14]. Claire's detection of mode \hat{b} and \hat{c} projects mode \hat{a} and \hat{d} on an inseparable entangled state, the entanglement of which is not changed by any local operation on mode \hat{a} or \hat{d} as classical displacements [11,12]. However, the entanglement of mode \hat{a} and \hat{d} cannot be used or exhibited without information about Claire's measurement results [11]. For exhibiting the entanglement of mode \hat{a} and \hat{d} , we send the photocurrents \hat{i}_+ and \hat{i}_- detected by Claire to Bob, where Bob implements the amplitude modulation and phase modulation on a coherent state light $\hat{\beta}_0$ with \hat{i}_+ and \hat{i}_- by means of amplitude (AM) and phase (PM) modulator, respectively. The coherent light $\hat{\beta}_0$ is a part divided from the fundamental wave of the laser source; thus, it has the identical frequency with the EPR beams at $1.08 \mu\text{m}$. The modulated optical mode $\hat{\beta}$ becomes $\hat{\beta}$:

$$\hat{\beta} = \hat{\beta}_0 + g_+ \hat{i}_+ + i g_- \hat{i}_- \quad (1)$$

The parameters g_+ and g_- describe the amplitude and phase gain for the transformation from photocurrent to

output light field ($g_+ = g_- = g$ in the experiment for simplification). Then Bob combines mode \hat{d} and $\hat{\beta}$ at a mirror M_r of reflectivity $R = 98\%$. In this manner the mode \hat{d} is displaced to \hat{d}' :

$$\hat{d}' = \sqrt{R}(\xi_2 \hat{d} + \sqrt{1 - \xi_2^2} \hat{v}_{\hat{d}}) + \sqrt{1 - R}(\hat{\beta}_0 + g_+ \hat{i}_+ + i g_- \hat{i}_-); \quad (2)$$

ξ_2 and $\hat{v}_{\hat{d}}$ are the imperfect transmission efficiency and vacuum noise introduced by losses of mode \hat{d} . If in spite of mode \hat{a} , the process is an quantum teleportation of mode \hat{b} from Claire to Bob based on exploiting the entanglement between mode \hat{c} and \hat{d} [11]. Here, mode \hat{b} is the teleported input state and Claire corresponds to the sender (Alice) in normal teleportation systems [2]. The initial entangled state, mode \hat{c} and \hat{d} , serves as the quantum resource used for teleporting the quantum information of input state \hat{b} from Claire to Bob (the receiver). Claire's Bell-state detection on mode \hat{b} and \hat{c} collapses Bob's mode \hat{d} into a state conditioned on the measurement outputs (\hat{i}_+ , \hat{i}_-); that is, Claire's joint measurement on mode \hat{b} and \hat{c} teleports the quantum information of mode \hat{b} to mode \hat{d} by means of the quantum entanglement between mode \hat{c} and \hat{d} . Hence, after receiving this classical information from Claire, Bob is able to construct the teleported state \hat{b} via a simple phase-space displacement of the EPR field \hat{d} [2]. For avoiding the optical loss of mode \hat{d} , which will unavoidably reduce entanglement, in our experiment AM and PM transform the photocurrents (\hat{i}_+ , \hat{i}_-) into a complex field amplitude $\hat{\beta}$ first, which is then combined with the EPR beam \hat{d} at the mirror M_r to affect the displacement of \hat{d} to \hat{d}' . To verify that the entanglement swapping has been accomplished during the process, we measure the quantum correlations of the sum of amplitude quadratures and the difference of phase quadratures between mode \hat{a} and \hat{d}' with spectrum analyzers (SAs). If both the quantum fluctuations of the sum and difference photocurrents are less than the corresponding SNL, the mode \hat{a} and \hat{d}' are in an entangled state [12]. Through analogous calculation with Refs. [11,12], but taking into account the imperfect detection efficiency of the detectors ($\eta < 1$) and the imperfect transmission efficiency of the optical system ($\xi_{1-4} < 1$), we can obtain the noise power spectra of the sum and difference photocurrents. The calculated variances of the sum and the difference photocurrents are

$$\begin{aligned} \langle \delta^2 i_+^v \rangle &= \langle \delta^2 i_-^v \rangle \\ &= \frac{1}{4}(\eta \xi_3 - g_{\text{swap}} \eta \xi_4)^2 e^{2r_1} + \frac{1}{4}(\sqrt{R} \eta \xi_2 \xi_4 - g_{\text{swap}} \eta \xi_4)^2 e^{2r_2} + \frac{1}{4}(\eta \xi_3 + g_{\text{swap}} \eta \xi_4)^2 e^{-2r_1} + \frac{1}{4}(\sqrt{R} \eta \xi_2 \xi_4 \\ &\quad + g_{\text{swap}} \eta \xi_4)^2 e^{-2r_2} + 1 - \eta^2 + \frac{1}{2} \eta^2 (2 - \xi_3^2 - \xi_4^2) + \frac{1}{2} \eta^2 (1 - R \xi_2^2) \xi_4^2 + \frac{g_{\text{swap}}^2 (1 - \eta^2 \xi_1^2) \xi_4^2}{\xi_1^2}. \end{aligned} \quad (3)$$

$\xi_1, \xi_2, \xi_3,$ and ξ_4 are the transmission efficiency for mode \hat{b} (\hat{c}), \hat{a} , \hat{a} , and \hat{d}' . η is the detection efficiency of each detector; here we have assumed that the detection efficiency of all detectors (D1–D4) is equal. $g_{\text{swap}} = \frac{1}{\sqrt{2}}\sqrt{1-R}\eta\xi_1g$ is the normalized gain factor. r_1 and r_2 are the correlation parameters (also named as squeezing parameters) for two initial EPR beams from NOPA1 and NOPA2, respectively. The correlation parameter r is a system parameter depending on the strength and the time of parametric interaction

(nonlinear coefficient of crystal, pump intensity, finesse of cavity, and so on). Under given experimental conditions, the correlation parameter deciding entanglement degree is a constant value, thus we say, squeezed-state entanglement of CV is determinant.

Minimizing Eq. (3) we get the optimum gain factor for the maximum entanglement:

$$g_{\text{swap}}^{\text{opt}} = \frac{\eta^2[(e^{4r_1} - 1)e^{2r_2}\xi_3 + e^{2r_1}(e^{4r_2} - 1)\sqrt{R}\xi_2\xi_4]\xi_1^2}{\{4e^{2(r_1+r_2)} + \eta^2[e^{2r_1} + e^{2r_2} + e^{4r_1+2r_2} + e^{2r_1+4r_2} - 4e^{2(r_1+r_2)}]\xi_1^2\}\xi_4}. \quad (4)$$

Figure 2 is the calculated noise power of $\langle\delta^2 i_+^v\rangle = \langle\delta^2 i_-^v\rangle$ normalized to SNL as a function of the correlation parameters r_1 and r_2 in the numerical calculation $\xi_1^2 = 0.970$, $\xi_2^2 = 0.950$, $\xi_3^2 = 0.966$, $\xi_4^2 = 0.968$, $\eta^2 = 0.90$, and $R = 0.98$ are taken, which are the real parameters of our experimental system. The dark star designated in Fig. 2 corresponds to the correlation variance deserved with the r_1 and r_2 obtained in the experiment, which is 71.9% of the SNL (corresponding to 1.43 dB below the SNL).

In the experiments, at first, we locked both NOPA1 and NOPA2 to resonate with the injected signal of $1.08 \mu\text{m}$ from the Nd:YAP/KTP-laser and locked the relative phase between the pump light of $0.54 \mu\text{m}$ and the injected signal to $(2n+1)\pi$ (n is integers) for enforcing two NOPAs operating at de-amplification. In this case, the EPR beam of about $70 \mu\text{W}$ was obtained from each NOPA at the pump power about 150 mW just below the power of its oscillation threshold of about 175 mW and the injected signal power of 10 mW before entering the input coupler of

the NOPA cavity. With respect to the squeezed vacuum state with average intensity close to zero produced from an optical parametric amplifier without injected signal [2], we say, the obtained EPR beam is bright. The measured correlation degrees of amplitude sum and phase difference at the sideband mode of 2 mHz are $\langle\delta^2(\hat{X}_a + \hat{X}_b)\rangle = \langle\delta^2(\hat{Y}_a - \hat{Y}_b)\rangle = 4.10 \pm 0.20 \text{ dB}$ below the SNL for NOPA1 and $\langle\delta^2(\hat{X}_c + \hat{X}_d)\rangle = \langle\delta^2(\hat{Y}_c - \hat{Y}_d)\rangle = 4.30 \pm 0.17 \text{ dB}$ below the SNL for NOPA2. Considering the influence of electronic noise level, which is 11.3 dB below the SNL, the actual correlations of quadrature components of EPR light beams should be 4.9 dB for NOPA1 and 5.1 dB for NOPA2, respectively.

Substituting the actual correlation parameters of the two initial EPR beams (\hat{a}, \hat{b}) and (\hat{c}, \hat{d}), $r_1 = 0.564$ (4.9 dB) and $r_2 = 0.587$ (5.1 dB), into Eq. (4), we have $g_{\text{swap}}^{\text{opt}} = 0.74$. According to the optimum gain value the classical channels from Claire to Bob are carefully adjusted in a manner described in Ref. [5] to the optimum value of $g_{\text{swap}}^{\text{opt}} = 0.74 \pm 0.02$. Claire performs a combining Bell-state measurement of mode \hat{b} and \hat{c} and sends the measured photocurrents \hat{i}_+^c and \hat{i}_-^c to Bob to modulate a coherent state light $\hat{\beta}_0$. Locking the relative phase of mode \hat{d} and $\hat{\beta}$ on M_r to $2n\pi$, the displacement of mode \hat{d} to \hat{d}' in the reflective field is completed. The intensity of $\hat{\beta}_0$ is aligned to make the intensity of mode \hat{d}' equal to that of mode \hat{a} for satisfying the requirement of Bell-state detection at Victor [14]. For locking the relative phase between mode \hat{d} and $\hat{\beta}$ to $2n\pi$, a PZT is placed in the optical path of $\hat{\beta}$ (not shown in Fig. 1) and the locking technique of DC interference fringe is utilized [5].

Successively, Victor implements a direct Bell-state measurement on mode \hat{a} and \hat{d}' . The trace 4 in Fig. 3(a) and 3(b) are the measured correlation noise powers of the amplitude sum 3(a), $\langle\delta^2(\hat{X}_a + \hat{X}_{d'})\rangle$, and the phase difference 3(b), $\langle\delta^2(\hat{Y}_a - \hat{Y}_{d'})\rangle$, at the sideband mode of 2 MHz , respectively, both of which are below the corresponding SNL (trace 3). The anticorrelation of the amplitude quadratures and the correlation of the phase quadratures are 1.23 dB and 1.12 dB below the SNL, respectively, (after

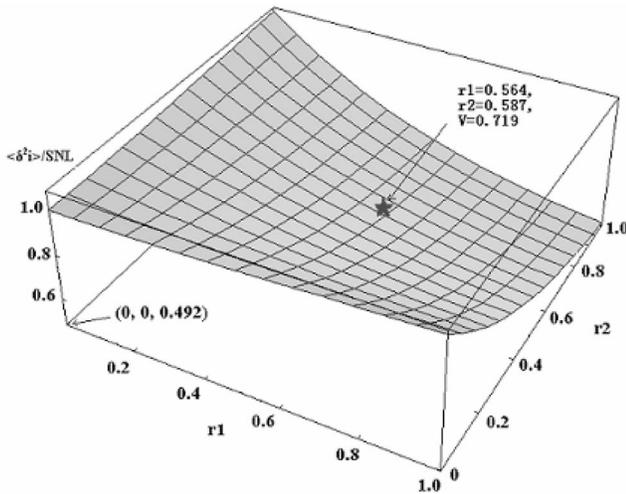


FIG. 2. The fluctuation variances of $\langle\delta^2 i_+^v\rangle = \langle\delta^2 i_-^v\rangle$ normalized to the SNL as a function of correlation parameters (r_1 and r_2) of the initial EPR beams. The dark star corresponds to the experimental values $r_1 = 0.564$ (4.9 dB), $r_2 = 0.587$ (5.1 dB), where $\langle\delta^2 i_+^v\rangle = \langle\delta^2 i_-^v\rangle = 0.719$ (SNL = 1).

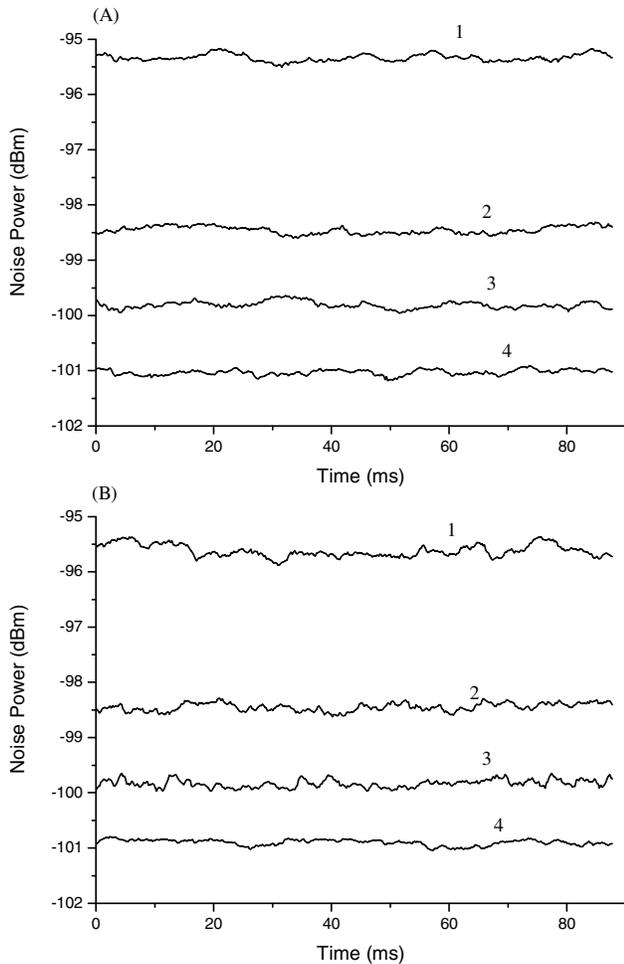


FIG. 3. The correlation noise powers resulting from entanglement swapping at 2 MHz as a function of time. (a) 1, The noise power of the amplitude sum without the classical information from Claire; 2, The noise power of amplitude of mode \hat{a} ; 3, SNL; 4, The correlation noise power of the amplitude sum with the classical information from Claire. (b) 1, The noise power of the phase difference without the classical information from Claire; 2, The noise power of amplitude of mode \hat{d}' ; 3, SNL; 4, The correlation noise power of the phase difference with the classical information from Claire. The measurement parameters of SA: Resolution band width is 10 kHz; Video band width is 30 Hz.

considering the influence of the electronics noise they should be 1.34 dB and 1.22 dB, respectively), which is reasonable agreement with the calculated value (1.43 dB). The initial entanglement of mode \hat{a} and \hat{b} is 4.9 dB ($r_1 = 0.564$), so the percentage of entanglement preserved after swapping with respect to the initial entanglement value is about 29%. The trace one in Figs. 3(a) and 3(b) are the noise power spectra of the amplitude sum and phase difference of mode \hat{a} and \hat{d}' when the classical channels of \hat{i}_+ and \hat{i}_- from Claire to Bob are blocked, which are much

higher than traces 4 and also the SNL (trace 3). It verifies obviously the conclusion of Ref. [11] that the entanglement of mode \hat{a} and \hat{d}' can not be exhibited and used without the assistance of Claire's measurement results. Even the amplitude noise of single mode \hat{a} [trace 2 in 3(a)] or mode \hat{d}' [trace 2 in 3(b)] is also higher than the correlation noise of two modes and the SNL. The results are agreeable with the characteristic of EPR entangled state light [15]. The measured results show that the entanglement between mode \hat{a} and \hat{d}' (\hat{d}), which have never interacted with each other, is truly established.

We achieved the unconditional entanglement swapping due to exploiting the determinant squeezed-state entanglement initially produced from two NOPAs pumped by the same laser. For a system with perfect detection and transmission efficiencies, the calculated degree of entanglement on the swapped pair of modes according to Eq. (3) should be 2.37 dB which is an upper boundary set by imperfectly initial entanglement degrees of EPR beams used for swapping (4.9 dB and 5.1 dB). The long-term intensity and frequency stability of laser source as well as good mechanic and thermal stabilities of NOPAs are the important requirements for demonstrating the experiments. This experiment realized the unconditional teleportation of CV entanglement. Therefore, this presented experimental protocol may have remarkable application potential in quantum communication and computation.

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