Quantum Dense Coding Exploiting a Bright Einstein-Podolsky-Rosen Beam

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(Received 18 July 2001; published 11 January 2002)

Highly efficient quantum dense coding for continuous variables has been experimentally accomplished by means of exploiting a bright Einstein-Podolsky-Rosen (EPR) beam with anticorrelation of amplitude quadratures and correlation of phase quadratures. Two bits of classical information are encoded on two quadratures of a half of a bright EPR beam, then are simultaneously decoded by the other half of the EPR beam with the sensitivities beyond that of the shot noise limit. A high degree of immunity to unauthorized eavesdropping has been experimentally demonstrated.

DOI: 10.1103/PhysRevLett.88.047904 PACS numbers: 03.67.Hk, 42.50.Dv

Quantum teleportation and quantum dense coding are two typical examples exploiting nonlocal quantum correlation of entangled states in quantum information to perform otherwise impossible tasks. Quantum teleportation is the disembodied transport of an unknown quantum state from one place to another [1]. Quantum dense coding provides a method by which two bits of classical information can be transmitted by sending one qubit of quantum information [2]. Discrete and continuous variable teleportations have been performed experimentally for single-photon polarization states [3–5] and the coherent state of electromagnetic field modes [6], respectively. The dense coding, originally proposed [2], successively discussed and experimentally demonstrated, is mostly concentrated within the setting of discrete quantum variables [7–9]. Later, the schemes realizing highly efficient dense coding for continuous variables are theoretically proposed, in that the two-mode squeezed-state entanglement is exhibited to achieve unconditional signal transmission [10–12]. So far, the experimental demonstration on the quantum dense coding for continuous variables has not to our knowledge been presented. There are two main motivations to realize experimentally the dense coding for continuous quantum variables. First, the information capacity always beats single-mode coherent-state communication and surpasses single-mode squeezed state communication if the average photon numbers $\overline{n} > 1$ [10]. Second, dense coding is highly immune to unauthorized eavesdropping, since the signal-to-noise ratios in information channels are very small due to the huge noise floors far above the shot noise limit (SNL) in the signal channel itself if the other half of an Einstein-Podolsky-Rosen (EPR) beam is not available to decode the signals. Furthermore, any attempt to extract information will reveal itself by a decrease in the signal-to-noise ratio and an increase of quantum fluctuation. Therefore the dense coding for continuous variables with enough robust quantum correlation has potential application for future quantum communication and the developing quantum information process [13,14].

An EPR beam with correlated amplitude quadratures and anticorrelated phase quadratures may be produced by a continuous nondegenerate optical parametric amplifier (NOPA) operating in the state of amplification. In that case, the variances of the difference of amplitude quadratures $\langle \delta(X_1 - X_2)^2 \rangle$ and the sum of phase quadratures $\langle \delta(Y_1 + Y_2)^2 \rangle$ are both smaller than the SNL defined by the vacuum fluctuation [15,16]. For a NOPA operating at deamplification the quantum correlations of quadrature components between the signal and idler modes have inverse signs, i.e., $\langle \delta(X_1 + X_2)^2 \rangle <$ SNL and $\langle \delta(Y_1 - Y_2)^2 \rangle <$ SNL [17]. It was theoretically proven in our previous publication [11] that the bright EPR beam having nonzero average intensity with the anticorrelation of amplitude quadratures $[\langle \delta(X_1 + X_2)^2 \rangle \longrightarrow 0]$ and the correlation of phase quadrature $[\langle \delta(Y_1 - Y_2)^2 \rangle \longrightarrow 0]$ can be used for realizing quantum teleportation and dense coding of continuous variables by means of the direct detection for photocurrents and two radio frequency (rf) power splitters. Based on the proposed scheme, we performed the first dense coding experiment for continuous variables, using simultaneous measurements of phase and amplitude signals with a sensitivity beyond that of the SNL.

The experimental diagram is shown in Fig. 1. The entangled EPR beam is generated from a NOPA consisting of an α-cut type II KTP ($KTiOPO_4$, potassium titanyl phosphate) crystal (10 mm long), the front face of which is coated to be used as the input coupler (the transmission >95% at 540 nm wavelength and ~0.5% at 1080 nm) and the other face of which is coated with the dual-band antireflection at both 540 and 1080 nm, as well as a concave mirror of 50 mm curvature radius, which is used as the output coupler of the EPR beam at 1080 nm (the transmission of 5% at 1080 nm and high reflectivity at 540 nm). The output coupler is mounted on a piezoelectric transducer to lock actively the cavity length on resonance with the injected seed wave at 1080 nm using the FM sideband technique. By fine-tuning the crystal temperature the birefringence between signal and idler waves in KTP is compensated and the simultaneous resonance in the cavity is reached. The process of adjusting temperature to meet double resonance can be monitored with an oscilloscope during scanning the length of cavity.

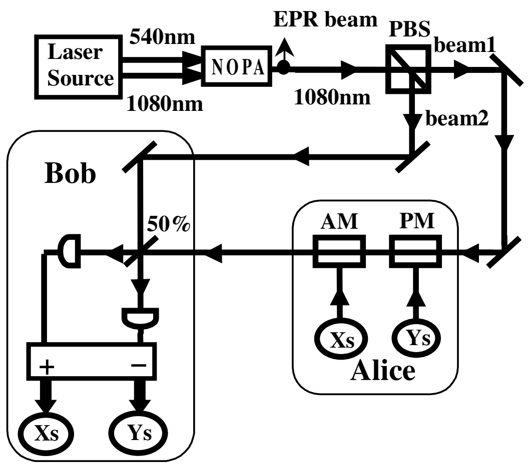

FIG. 1. Schematic of the experimental apparatus for dense coding for continuous variables. Two bits of classical information X_s and Y_s are encoded on the amplitude and phase quadratures of a half of an EPR beam (beam 1) at Alice, then are decoded by the other half of the EPR beam (beam 2) at Bob.

Once the double resonance is completed, the NOPA is locked on the frequency of the injected seed wave [16]. The measured finesse, the free spectral range, and the linewidth of the parametric oscillator are 110, 2.8 GHz, and 26 MHz, respectively. The pump source of NOPA is a homemade all-solid-state intracavity frequency-doubled and frequency-stabilized continuous wave ring Nd:YAP ($Nd:YAlO_3$, yttrium-aluminum-perovskite) laser. The output second-harmonic wave at 540 nm and the leaking fundamental wave at 1080 nm are used as the pump light and the seed wave, respectively. The laser-diode pumped all-solid-state laser and the semimonolithic F-P configuration of the parametric cavity make the system very stable, so that the frequency and phase of light waves can be efficiently locked during the dense coding experiment. Locking the relative phase between the pump laser and injected seed wave of NOPA to $(2n + 1)\pi$, where n is integer, to enforce the NOPA operating at deamplification, the entangled EPR beam with the quantum anticorrelation of amplitude quadratures and the correlation of phase quadratures was generated. The two halves of the EPR beam are just the signal and idler modes of the light field produced from type II parametric down conversion; thus they have orthogonal polarizations originally. The correlations measured by self-homodyne detectors [11,18] between the quadrature-phase amplitudes of the two halves of the EPR beam are shown in Fig. 2. Both variances of $\langle\delta(X_1 + X_2)^2\rangle$ [Fig. 2(a)] and $\langle\delta(Y_1 - Y_2)^2\rangle$ [Fig. 2(b)] measured directly are ~4 dB below that of the SNL (considering the electronics noise that is ~8 dB below the SNL, the actual fluctuation should be ~5.4 dB below that of the SNL). The product of the correspondent conditional variances of the EPR beam is $\langle\delta(X_1 + X_2)^2\rangle\langle\delta(Y_1 - Y_2)^2\rangle = 0.63$. The bright EPR beam of ~70 μW (the correspondent photon-number flow is about 3.8×10^{14} s^{-1}) was obtained at the following operation parameters of NOPA: the pump power 150 mW just below the power of the oscillation threshold of 175 mW and the polarization of that was along the b axis of the KTP crystal. The power of the injected seed wave was 10 mW before entering the input coupler of the cavity and that was polarized at 45° relative to the b axis.

The two halves (1 and 2) of the EPR entangled beam were distributed to the sender Alice and the receiver Bob, respectively. At Alice the classical amplitude and phase signals were modulated on one half of the EPR beam (beam 1), which led to a displacement signal sent via the quantum channel. Since each half of the EPR beam had huge noise individually, for perfect EPR entanglement $\langle\delta(X_{1(2)})^2\rangle \longrightarrow \infty$ and $\langle\delta(Y_{1(2)})^2\rangle \longrightarrow \infty$, the signal-to-noise ratios in the beam 1 for both amplitude and phase signals tended to zero, so no one other than Bob could gain any signal information from the modulated beam under ideal conditions. At Bob, the signals were decoded with the aid of the other half of the EPR beam (beam 2), which was combined with the modulated first

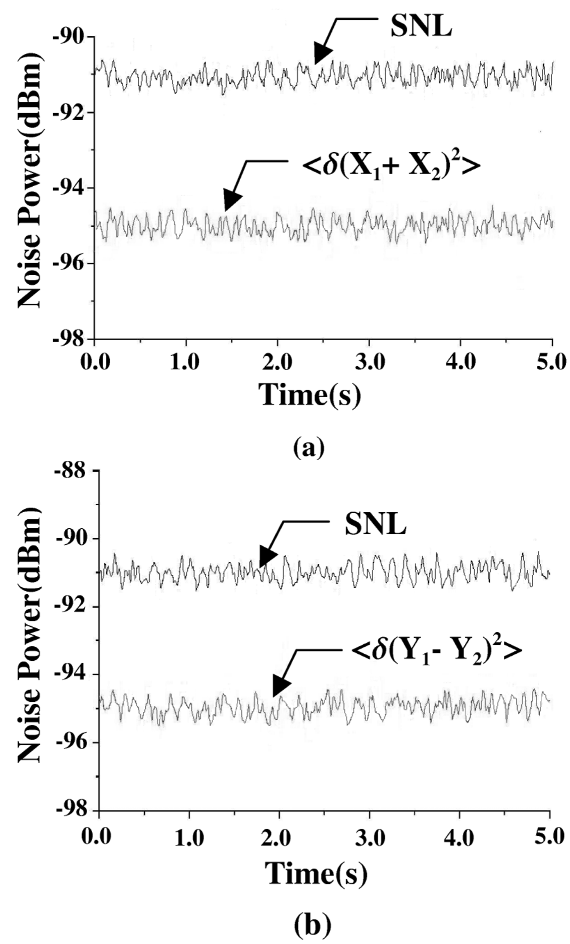

FIG. 2. Spectral densities of photocurrent fluctuations $\langle\delta(X_1 + X_2)^2\rangle$ (a) and $\langle\delta(Y_1 - Y_2)^2\rangle$ (b); SNL, the shot noise limit. Acquisition parameters: radio frequency (rf) $\Omega/2\pi = 2$ MHz, resolution bandwidth $\Delta\Omega/2\pi = 30$ KHz, video bandwidth 0.1 KHz; the electronics noise is ~8 dB below the SNL.

half of the EPR beam (beam 1) on a 50% beam splitter, in our experiment which consisted of two polarized beam splitters and a half-wave plate [19], and, before combination, a $\pi/2$ phase shift was imposed between them according to the requirement of direct measurement of the Bell state [11]. The bright outcome from two ports of the beam splitter were directly detected by a pair of photodiodes D_1 and D_2 (ETX500 InGaAs), and then each photocurrent of D_1 and D_2 was divided into two parts through the power splitters. The sum and difference of the divided photocurrent were nothing else but the transmitted amplitude and phase signals from Alice to Bob, which are expressed by [11]

$$i_+(\Omega) = \frac{1}{\sqrt{2}}\{[X_1(\Omega) + X_2(\Omega)] + X_s(\Omega)\}, \quad (1)$$

$$i_-(\Omega) = \frac{1}{\sqrt{2}}\{[Y_1(\Omega) - Y_2(\Omega)] + Y_s(\Omega)\}, \quad (2)$$

where $X_s(\Omega)$ and $Y_s(\Omega)$ are the modulated amplitude and phase signals on the first half of the EPR beam at the sender Alice. With the perfect EPR entangled beam $\langle\delta(X_1 + X_2)^2\rangle \longrightarrow 0$ and $\langle\delta(Y_1 - Y_2)^2\rangle \longrightarrow 0$, Eqs. (1) and (2) are simplified:

$$i_+(\Omega) = \frac{1}{\sqrt{2}}\{X_s(\Omega)\}, \quad (3)$$

$$i_-(\Omega) = \frac{1}{\sqrt{2}}\{Y_s(\Omega)\}. \quad (4)$$

It means that under ideal conditions, the signals $X_s(\Omega)$ and $Y_s(\Omega)$ encoded on the amplitude quadrature and phase quadrature of beam 1 are simultaneously retrieved at the receiver Bob without any error. In general, both encoded signals will be recovered with a sensitivity beyond that of the SNL when the beam 1 and the beam 2 are quantum entangled. Of course, the sum of the amplitude quadratures between the two halves of the EPR beam commutes with the difference of its phase quadrature; therefore the detected variances in them can be below that of the SNL simultaneously, and not violate the uncertainty principle [20]. Figure 3 shows the directly measured amplitude [Fig. 3(a)] and phase [Fig. 3(b)] signals at Bob, which are the signals of 2 MHz modulated on the first half of the EPR beam (beam 1) by the amplitude and phase modulators at Alice. It can be seen that the original signals are retrieved with the high signal-to-noise ratios of ~4 dB and ~3.6 dB beyond that of the SNL under the help of the other half of the EPR beam (beam 2) (accounting for the electronics noise of ~8 dB below the SNL, the actual value should be ~5.4 dB and ~4.8 dB, respectively). Compared with the previously completed sub-shot-noise-limit optical measurements and quantum nondemolition measurements for a signal, in which the squeezed state light have been applied [20–26], our experiments have achieved the simultaneous

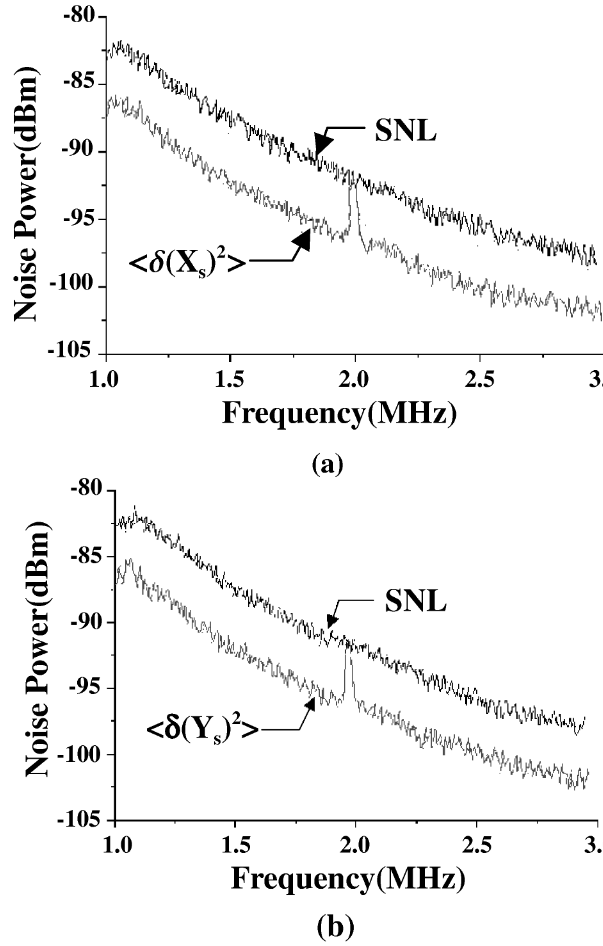

FIG. 3. Measured amplitude (a) and phase signal (b) at Bob, when EPR beam 1 is phase and amplitude modulated at 2 MHz at Alice; SNL, the shot noise limit. Acquisition parameters: measured frequency range 1.0–3.0 MHz, resolution bandwidth 30 KHz, video bandwidth 0.1 KHz; the electronics noise is ~8 dB below the SNL.

measurements of two signals modulated on the amplitude and phase quadratures, respectively, with the precision beyond that of the SNL by means of exploiting the EPR entanglement. Although when sending and modulating two coherent beams a factor of 2 in channel capacity may also be gained, the noise floors of recovered signals at the receiving station cannot be reduced below the SNL.

As was well discussed in Ref. [27], in which a signal is transmitted via two quantum channels of the EPR pair, the individual signal channel in our scheme has a high degree of immunity to unauthorized interception because of very low signal-to-noise ratios. Figure 4 demonstrates the security of the signal channel against eavesdropping, where the trace 4(a) is the fluctuation spectrum of the first half of the EPR beam 1 with the modulated signals measured individually at Bob while the other half of the EPR beam is not applied, which is ~4.4 dB above that of the corresponding SNL [trace 4(b)] (after the correction to the electronics noise floor, it should actually be ~5.4 dB above the SNL).

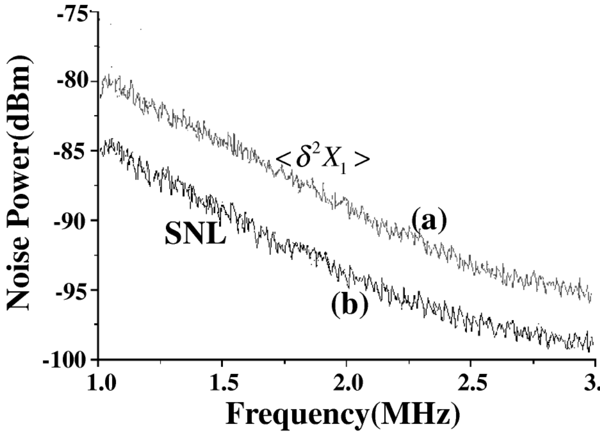

FIG. 4. Spectral density of photocurrent fluctuations of EPR beam 1 with the modulation signals [trace (a)], the modulated signals are submerged in the noise background. SNL, the shot noise limit [trace (b)]. Acquisition parameters: measured frequency range 1.0–3.0 MHz, resolution bandwidth 30 KHz, video bandwidth 0.1 KHz; the electronics noise is ~5.6 dB below the SNL.

It is obvious that no signal can be extracted since the signals are totally submerged in the large noise background.

In conclusion, the first dense coding for continuous variables has been experimentally realized by exploiting the bright EPR beam from NOPA operating at deamplification and the direct measurement of the Bell state. The amplitude and phase signals of 2 MHz modulated on the first half of the EPR beam at a sending station were simultaneously retrieved at a remote receiving station with high sensitivities which were ~4 dB and ~3.6 dB beyond that of the SNL due to using the quantum entanglement between the two halves of the EPR beam. To implement the coding of communication, we consider, for example, the specific encoding scheme of binary pulse code modulation, in which the data are independently encoded as two trains of 1 and 0 pulse signals at some rf frequency (2 MHz in our experiment) that are impressed on the amplitude and phase quadrature, respectively, of the first half of the EPR beam using electric-optical modulators. The EPR beam from the continuous NOPA used in our experiment is two-mode squeezed light. The higher the squeezing, the better the quantum correlation and the noisier the EPR beam halves [20,27]. It has been theoretically demonstrated [10,11] that the scheme of dense coding exploiting squeezed-state entanglement should allow unconditional signal transmission with high efficiency. On the other hand, the huge noise in the signal channel protects the signal against unauthorized eavesdropping; only the authorized receiver, who holds the other half of the EPR beam, can decode the transmitted signals with a certain signal-to-noise ratio defined by encoding. The detailed procedure about quantum cryptography of continuous variables using EPR correlation has been discussed in previous publications [27–29]. The mature technique for producing the EPR beam from NOPA and the simplicity of direct measurement make the presented scheme relatively straightforward and valuable to be applied to the developing quantum information science.

The work is supported by the National Nature Science Foundation (No. 69837010, No. 19974021, and No. 60178012).

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