Science in China Series G: Physics, Mechanics & Astronomy

Generation of GHZ-like and cluster-like quadripartite entangled states for continuous variable using a set of quadrature squeezed states

SU XiaoLong, JIA XiaoJun, XIE ChangDe & PENG KunChi†

State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-electronics, Shanxi University, Taiyuan 030006, China

We designed the experimental generation system of the optical GHZ-like and cluster-like quadripartite entangled states for continuous variables. We theoretically demonstrated that the two different types of quadripartite entangled states can be obtained by the linearly optical transformation of four amplitude-quadrature and phase-quadrature squeezed states produced from a pair of nondegenerate optical parametric amplifiers under appropriate phase relations. The criteria for full inseparability of quadripartite cluster-like state were deduced, and the dependency of the quadripartite entanglement on the initial squeezing degree, the transmission efficiencies of the system and the detection efficiency of homodyne detection were numerically calculated.

continuous variable quadripartite entanglement, GHZ-like state, cluster-like state

1 Introduction

The quantum entangled states involving more than two subsystems were named multipartite entangled states, in which the entanglement was shared by more than two parties. Generally, the wave function of the multipartite entangled states cannot be written to the direct product of the subsystem wave functions, so the quantum states are fully inseparable. There are different kinds of multipartite entangled states which have different constructions and variant physical features. Thus the wave function and the criterion of the full inseparability characterizing these multipartite entangled states are also not the same. The most typical and extensively applied two kinds of multipartite entangled states are GHZ state and Cluster state, the wave functions of which are denoted in the following expressions, respectively[1]:

Received December 11, 2006; accepted April 9, 2007
doi: 10.1007/s11433-008-0004-y
†Corresponding author (email: kcpeng@sxu.edu.cn)
Supported by the National Natural Science Foundation of China (Grant Nos. 10674088 and 60608012) and Program for Changjiang Scholars and Innovative Research Team in University (Grant No. IRT0516)

$$\left|\mathrm{GHZ}\right\rangle_N = 2^{-1/2}(\left|0\right\rangle_1\left|0\right\rangle_2 \ldots \left|0\right\rangle_N + \left|1\right\rangle_1\left|1\right\rangle_2 \ldots \left|1\right\rangle_N), \quad (1)$$

$$\left|C\right\rangle_N = 2^{-N/2} \bigotimes_{a=1}^{N} (\left|0\right\rangle_a \sigma_z^{(a+1)} + \left|1\right\rangle_a), \quad (2)$$

where N is the number of the subsystems involved in the multipartite entangled states, a stands for a single subsystem, $\bigotimes_{a=1}^{N}$ means that the operation of the direct product runs over all lattices sites ("nodes") occupied by the subsystems in the cluster state, $\sigma_z^{(a+1)}$ is the Pauli matrix of the "node" at (a+1) site and $\sigma_z^{(N+1)} \equiv 1$. Due to different constructions the entanglement persistency of cluster state, such as ~N/2 subsystems, has to be measured to disentangle an N-partite cluster state, however for an N-partite GHZ state the disentanglement can be achieved only by measuring a subsystem.

Multipartite entanglement is the basic resource for realizing quantum information network and quantum computation. Five photon Greenberger-Horne-Zeilinger (GHZ) polarization entangled states[2], four photon cluster entangled states[3] and one-way quantum computation based on four photon cluster entangled states[3] have been experimentally demonstrated. In recent years the investigation of continuous variable (CV) quantum information with amplitude-quadratures and phase-quadratures of electromagnetic fields has attracted extensive interest, however, the experimental study in this realm is still slower than the investigation with the discrete variable. CV tripartite GHZ entangled states have been generated by several groups and been applied in controlled dense coding quantum communication[4], quantum teleportation network[5] and quantum secret sharing[6], etc. The generation of CV multipartite entangled states of more than three subsystems has not been reported so far.

CV GHZ-like state of optical field is a simultaneous eigenstate of the sum of total amplitude (phase)-quadrature and the difference of relative phase (amplitude)-quadrature of N subsystems. When the variances of the sum and difference of these quadratures are below the corresponding standard quantum limit, the state is GHZ-like entangled state[4−6]. Besides GHZ-like states, there is another type of CV multipartite entangled state, which is cluster-like state[7]. This type of multipartite entangled state has the "chain structure" in one-dimension, and can be acquired by squeezed optical field and quantum nondemolition (QND) coupling between next two modes[7]. When $N = 2$ and $N = 3$, the cluster-like state is the same with GHZ-like state, however they are different when $N >$ 3. It has been recently proved that universal quantum computation can be achieved with CV cluster-like states as long as a non-Gaussian measurement can be performed[8].

In recent years, we have accomplished CV quantum dense coding[9], controlled dense coding quantum communication[4] and quantum entanglement swapping[10] utilizing the EPR entangled optical fields generated from nondegenerate optical parametric amplifier (NOPA) operating at deamplification. We consider that using NOPA as an initial source of squeezed and entangled states the experimental setup can be simplified efficiently, so it is beneficial for practical applications. For the necessities of developing quantum communication network and quantum computation, we designed the generation system of CV quadripartite entangled states using NOPAs as the basic devices. We have demonstrated theoretically that the quadripartite GHZ-like states and the cluster-like states could be generated by means of the QND coupling of four quadrature-squeezed states of light from a pair of NOPAs with certain phase relations. The dependencies of the quadripartite entanglement on the initial squeezing degree, the transmission efficiencies of the system and the detection efficiency of homodyne detectors were calculated and the criteria of the quantum full inseparability for quadripartite cluster-like state were deduced. The boundaries of the physical parameters which satisfy the full inseparability criteria of quadripartite GHZ-like and cluster-like states were numerically calculated. The results provide the valuable theoretical reference for the designs of the experimental systems.

2 Generation principle of quadripartite GHZ-like and cluster-like entangled states

2.1 The squeezing of the output fields from NOPA

The amplitude and phase quadratures of the quantized optical field a are defined as

$$X_a = \frac{1}{2}(a + a^\dagger),$$
$$Y_a = -\frac{i}{2}(a - a^\dagger), \quad (3)$$

where a and $a^\dagger$ are the annihilation and creation operators of the optical field. When the optical field is in a coherent state or vacuum state, the variances of the quadratures are $V(X_a) = V(Y_a) = 1/4$.

The amplitude ($X_{a,b}$) and phase ($Y_{a,b}$) quadratures of the signal and idler modes (a and b) from a NOPA consisting of a type-II nonlinear crystal and operating at deamplification (the pump light and the injected signal light are out of phase) are expressed respectively by[11]

$$X_a = X_a^{(0)} \cosh r - X_b^{(0)} \sinh r,$$
$$Y_a = Y_a^{(0)} \cosh r + Y_b^{(0)} \sinh r,$$
$$X_b = X_b^{(0)} \cosh r - X_a^{(0)} \sinh r, \quad (4)$$
$$Y_b = Y_b^{(0)} \cosh r + Y_a^{(0)} \sinh r,$$

where r is the squeezing parameter (or correlation parameter), which depends on the strength and the time of parametric interaction in NOPA. $X_i^{(0)}$ and $Y_i^{(0)}$ ($i = a, b$) are the amplitude and phase quadratures of the injected signal light (coherent state). The correlation variances of the output fields are $V(X_a + X_b) = V(Y_a - Y_b) = \mathrm{e}^{-2r}/2$. So the output fields, a and b, are a pair of Einstein-Podolsky-Rosen (EPR) entangled light beams with the anticorrelation of the amplitude quadratures and the correlations of the phase quadratures. $r = 0$ corresponds to no correlation, while $r \to \infty$ corresponds to ideal entanglement.

The polarization directions of the output fields from the NOPA are shown in Figure 1. The polarization directions of modes a and b are at 0° and 90° respectively, which correspond to the polarization directions of signal and idler optical modes in the type-II nonlinear crystal. The coupling modes c and d of modes a and b at the directions of ±45° are

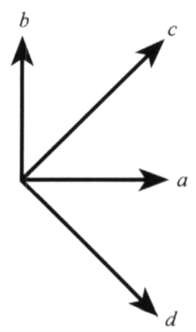

Figure 1 The polarization direction of the output fields from NOPA.

$$c = \frac{1}{\sqrt{2}}(a+b),$$
$$d = \frac{1}{\sqrt{2}}(a-b). \tag{5}$$

The amplitude and phase quadratures of coupling modes c and d are, respectively,

$$\begin{aligned} X_c &= \frac{1}{\sqrt{2}}(X_a + X_b) = \mathrm{e}^{-r} X_c^{(0)}, \\ Y_c &= \frac{1}{\sqrt{2}}(Y_a + Y_b) = \mathrm{e}^{+r} Y_c^{(0)}, \\ X_d &= \frac{1}{\sqrt{2}}(X_a - X_b) = \mathrm{e}^{+r} X_d^{(0)}, \\ Y_d &= \frac{1}{\sqrt{2}}(Y_a - Y_b) = \mathrm{e}^{-r} Y_d^{(0)}. \end{aligned} \tag{6}$$

It is obvious that modes c and d are the amplitude-quadrature and the phase-quadrature squeezed state of light, respectively[12].

2.2 Generation of quadripartite GHZ-like state

The generation system of quadripartite GHZ-like and cluster-like states is shown in Figure 2. The four quadrature squeezed optical fields (denoted by a_1, a_2, a_3 and a_4) from a pair of NOPAs (NOPA1 and NOPA2) are coupled with the optical beamsplitters. Suppose a_1 and a_4 are phase-quadrature squeezed lights, and a_2 and a_3 are amplitude-quadrature squeezed lights. Their amplitude and phase quadratures are expressed as follows, respectively:

$$\begin{aligned} X_{a_1} &= \mathrm{e}^{+r} X_{a_1}^{(0)}, \qquad Y_{a_1} = \mathrm{e}^{-r} Y_{a_1}^{(0)}, \\ X_{a_2} &= \mathrm{e}^{-r} X_{a_2}^{(0)}, \qquad Y_{a_2} = \mathrm{e}^{+r} Y_{a_2}^{(0)}, \\ X_{a_3} &= \mathrm{e}^{-r} X_{a_3}^{(0)}, \qquad Y_{a_3} = \mathrm{e}^{+r} Y_{a_3}^{(0)}, \\ X_{a_4} &= \mathrm{e}^{+r} X_{a_4}^{(0)}, \qquad Y_{a_4} = \mathrm{e}^{-r} Y_{a_4}^{(0)}, \end{aligned} \tag{7}$$

where we have considered that the squeezing degrees of the two NOPAs are the same, i.e. an

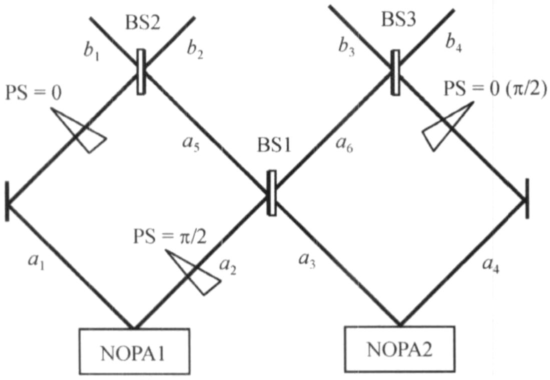

Figure 2 Generation system of quadripartite entangled states. NOPA: nondegenerate optical parametric amplifier; BS: 50% beam splitter; PS: phase shift.

identical squeezing parameter r is used. The variances of the amplitude and phase quadratures of the injected coherent field, $X_{a_i}^{(0)}$ and $Y_{a_i}^{(0)}$, are taken to be the shot noise limit (SNL), $V(X_{a_i}^{(0)}) = V(Y_{a_i}^{(0)}) = 1/4$.

For the generation of quadripartite GHZ-like state we couple the two amplitude squeezed lights a_2 and a_3 on a 50% beam splitter (BS1), with π/2 phase shift. The amplitude and phase quadratures of two output fields (modes a_5 and a_6) are, respectively,

$$\begin{aligned}
X_{a_5} &= \frac{1}{\sqrt{2}}(\sqrt{\xi_2}X_{a_2} + \sqrt{1-\xi_2}X_{v_2} - \sqrt{\xi_2}Y_{a_3} - \sqrt{1-\xi_2}Y_{v_3}),\\
Y_{a_5} &= \frac{1}{\sqrt{2}}(\sqrt{\xi_2}Y_{a_2} + \sqrt{1-\xi_2}Y_{v_2} + \sqrt{\xi_2}X_{a_3} + \sqrt{1-\xi_2}X_{v_3}),\\
X_{a_6} &= \frac{1}{\sqrt{2}}(\sqrt{\xi_2}X_{a_2} + \sqrt{1-\xi_2}X_{v_2} + \sqrt{\xi_2}Y_{a_3} + \sqrt{1-\xi_2}Y_{v_3}),\\
Y_{a_6} &= \frac{1}{\sqrt{2}}(\sqrt{\xi_2}Y_{a_2} + \sqrt{1-\xi_2}Y_{v_2} - \sqrt{\xi_2}X_{a_3} - \sqrt{1-\xi_2}X_{v_3}),
\end{aligned} \tag{8}$$

where ξ_2 is the transmission efficiencies of a_2 and a_3, X_{v_i} (Y_{v_i}) denotes the amplitude (phase) quadrature of the vacuum field introduced by losses. Then, coupling the optical fields a_5 and a_1 on a 50% beam splitter (BS2) with zero phase shift, and coupling the optical fields a_6 and a_4 on the other 50% beam splitter (BS3) with zero phase shift also, the amplitude and phase quadratures of the resultant output modes b_1, b_2, b_3 and b_4 are, respectively,

$$\begin{aligned}
X_{b_1}^G &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}X_{a_5} + \sqrt{1-\xi_3}X_{v_5} - \sqrt{\xi_1}X_{a_1} - \sqrt{1-\xi_1}X_{v_1}) + \sqrt{1-\eta}X_{v_7},\\
Y_{b_1}^G &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}Y_{a_5} + \sqrt{1-\xi_3}Y_{v_5} - \sqrt{\xi_1}Y_{a_1} - \sqrt{1-\xi_1}Y_{v_1}) + \sqrt{1-\eta}Y_{v_7},\\
X_{b_2}^G &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}X_{a_5} + \sqrt{1-\xi_3}X_{v_5} + \sqrt{\xi_1}X_{a_1} + \sqrt{1-\xi_1}X_{v_1}) + \sqrt{1-\eta}X_{v_8},\\
Y_{b_2}^G &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}Y_{a_5} + \sqrt{1-\xi_3}Y_{v_5} + \sqrt{\xi_1}Y_{a_1} + \sqrt{1-\xi_1}Y_{v_1}) + \sqrt{1-\eta}Y_{v_8},\\
X_{b_3}^G &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}X_{a_6} + \sqrt{1-\xi_3}X_{v_6} + \sqrt{\xi_4}X_{a_4} + \sqrt{1-\xi_4}X_{v_4}) + \sqrt{1-\eta}X_{v_9},\\
Y_{b_3}^G &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}Y_{a_6} + \sqrt{1-\xi_3}Y_{v_6} + \sqrt{\xi_4}Y_{a_4} + \sqrt{1-\xi_4}Y_{v_4}) + \sqrt{1-\eta}Y_{v_9},\\
X_{b_4}^G &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}X_{a_6} + \sqrt{1-\xi_3}X_{v_6} - \sqrt{\xi_4}X_{a_4} - \sqrt{1-\xi_4}X_{v_4}) + \sqrt{1-\eta}X_{v_{10}},\\
Y_{b_4}^G &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}Y_{a_6} + \sqrt{1-\xi_3}Y_{v_6} - \sqrt{\xi_4}Y_{a_4} - \sqrt{1-\xi_4}Y_{v_4}) + \sqrt{1-\eta}Y_{v_{10}},
\end{aligned} \tag{9}$$

where the superscript G denotes GHZ-like state, ξ_3 is the transmission efficiency of modes a_5 and a_6, ξ_1 and ξ_4 are the transmission efficiencies of modes a_1 and a_4 respectively, and η is the detection efficiency of the detectors. Substituting eqs. (7) and (8) into (9), and calculating the sum of the amplitude quadratures and the difference of the relative phase quadratures, we have

$$X_{b_1}^G + X_{b_2}^G + X_{b_3}^G + X_{b_4}^G = 2\sqrt{\eta\xi_2\xi_3}\,\mathrm{e}^{-r}X_{a_2}^{(0)} + 2\sqrt{\eta(1-\xi_2)\xi_3}\,X_{\nu_2} + \sqrt{2\eta(1-\xi_3)}(X_{\nu_5}+X_{\nu_6}) + \sqrt{1-\eta}(X_{\nu_7}+X_{\nu_8}+X_{\nu_9}+X_{\nu_{10}}), \qquad (10a)$$

$$Y_{b_1}^G - Y_{b_2}^G = -\sqrt{2\eta\xi_1}\,\mathrm{e}^{-r}Y_{a_1}^{(0)} - \sqrt{2\eta(1-\xi_1)}Y_{\nu_1} + \sqrt{1-\eta}(Y_{\nu_7}-Y_{\nu_8}), \qquad (10b)$$

$$Y_{b_2}^G - Y_{b_3}^G = \sqrt{\eta\xi_2\xi_3}\,\mathrm{e}^{-r}X_{a_3}^{(0)} + \sqrt{\eta(1-\xi_2)\xi_3}\,X_{\nu_3} + \frac{\sqrt{\eta\xi_1}}{\sqrt{2}}\mathrm{e}^{-r}Y_{a_1}^{(0)} + \frac{\sqrt{\eta(1-\xi_1)}}{\sqrt{2}}Y_{\nu_1} - \frac{\sqrt{\eta\xi_4}}{\sqrt{2}}\mathrm{e}^{-r}Y_{a_4}^{(0)} - \frac{\sqrt{\eta(1-\xi_4)}}{\sqrt{2}}Y_{\nu_4} + \frac{\sqrt{\eta(1-\xi_3)}}{\sqrt{2}}(Y_{\nu_5}-Y_{\nu_6}) + \sqrt{1-\eta}(Y_{\nu_8}-Y_{\nu_9}), \qquad (10c)$$

$$Y_{b_3}^G - Y_{b_4}^G = \sqrt{2\eta\xi_4}\,\mathrm{e}^{-r}Y_{a_4}^{(0)} + \sqrt{2\eta(1-\xi_4)}Y_{\nu_4} + \sqrt{1-\eta}(Y_{\nu_9}-Y_{\nu_{10}}), \qquad (10d)$$

$$Y_{b_1}^G - Y_{b_3}^G = \sqrt{\eta\xi_2\xi_3}\,\mathrm{e}^{-r}X_{a_3}^{(0)} + \sqrt{\eta(1-\xi_2)\xi_3}\,X_{\nu_3} - \frac{\sqrt{\eta\xi_1}}{\sqrt{2}}\mathrm{e}^{-r}Y_{a_1}^{(0)} - \frac{\sqrt{\eta(1-\xi_1)}}{\sqrt{2}}Y_{\nu_1} - \frac{\sqrt{\eta\xi_4}}{\sqrt{2}}\mathrm{e}^{-r}Y_{a_4}^{(0)} - \frac{\sqrt{\eta(1-\xi_4)}}{\sqrt{2}}Y_{\nu_4} + \frac{\sqrt{\eta(1-\xi_3)}}{\sqrt{2}}(Y_{\nu_5}-Y_{\nu_6}) + \sqrt{1-\eta}(Y_{\nu_8}-Y_{\nu_9}), \qquad (10e)$$

$$Y_{b_1}^G - Y_{b_4}^G = \sqrt{\eta\xi_2\xi_3}\,\mathrm{e}^{-r}X_{a_3}^{(0)} + \sqrt{\eta(1-\xi_2)\xi_3}\,X_{\nu_3} - \frac{\sqrt{\eta\xi_1}}{\sqrt{2}}\mathrm{e}^{-r}Y_{a_1}^{(0)} - \frac{\sqrt{\eta(1-\xi_1)}}{\sqrt{2}}Y_{\nu_1} + \frac{\sqrt{\eta\xi_4}}{\sqrt{2}}\mathrm{e}^{-r}Y_{a_4}^{(0)} + \frac{\sqrt{\eta(1-\xi_4)}}{\sqrt{2}}Y_{\nu_4} + \frac{\sqrt{\eta(1-\xi_3)}}{\sqrt{2}}(Y_{\nu_5}-Y_{\nu_6}) + \sqrt{1-\eta}(Y_{\nu_8}-Y_{\nu_9}), \qquad (10f)$$

$$Y_{b_2}^G - Y_{b_4}^G = \sqrt{\eta\xi_2\xi_3}\,\mathrm{e}^{-r}X_{a_3}^{(0)} + \sqrt{\eta(1-\xi_2)\xi_3}\,X_{\nu_3} + \frac{\sqrt{\eta\xi_1}}{\sqrt{2}}\mathrm{e}^{-r}Y_{a_1}^{(0)} + \frac{\sqrt{\eta(1-\xi_1)}}{\sqrt{2}}Y_{\nu_1} + \frac{\sqrt{\eta\xi_4}}{\sqrt{2}}\mathrm{e}^{-r}Y_{a_4}^{(0)} + \frac{\sqrt{\eta(1-\xi_4)}}{\sqrt{2}}Y_{\nu_4} + \frac{\sqrt{\eta(1-\xi_3)}}{\sqrt{2}}(Y_{\nu_5}-Y_{\nu_6}) + \sqrt{1-\eta}(Y_{\nu_8}-Y_{\nu_9}). \qquad (10g)$$

For quadripartite GHZ-like state, we can see from eq. (10a) that the noise of the sum of the amplitude quadratures depends on the noise of the amplitude quadrature of the amplitude squeezed light a_2, and the noises of the phase difference in (10b) and (10d) depend on the noise of the phase quadrature of phase squeezed light a_1 and a_4 respectively, while the noises of phase differences in eqs. (10c), (10e), (10f) and (10g) depend on the noises of both squeezed amplitude and phase quadratures. So the correlation variances may be below the corresponding SNL and satisfy the full inseparability criteria of quadrature entanglement for certain values of the squeezing parameter. In the ideal case ($r\to\infty,\ \xi\to 1,\ \eta\to 1$), the quadripartite GHZ-like state is a simultaneous eigenstate of $X_{b_1}^G + X_{b_2}^G + X_{b_3}^G + X_{b_4}^G \to 0$ and $Y_{b_i}^G - Y_{b_j}^G \to 0$ ($i, j = 1-4,\ i\neq j$).

2.3 Generation of quadripartite cluster-like state

In the system shown in Figure 2, the quadripartite cluster-like state can be generated also, once we control the phase shift between a_6 and a_4 to π/2, and keep the rest unchanging. In this case the amplitude and phase quadratures of the output fields are given by

$$
\begin{aligned}
X_{b_1}^C &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}X_{a_5} + \sqrt{1-\xi_3}X_{\nu_5} - \sqrt{\xi_1}X_{a_1} - \sqrt{1-\xi_1}X_{\nu_1}) + \sqrt{1-\eta}X_{\nu_7}, \\
Y_{b_1}^C &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}Y_{a_5} + \sqrt{1-\xi_3}Y_{\nu_5} - \sqrt{\xi_1}Y_{a_1} - \sqrt{1-\xi_1}Y_{\nu_1}) + \sqrt{1-\eta}Y_{\nu_7}, \\
X_{b_2}^C &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}X_{a_5} + \sqrt{1-\xi_3}X_{\nu_5} + \sqrt{\xi_1}X_{a_1} + \sqrt{1-\xi_1}X_{\nu_1}) + \sqrt{1-\eta}X_{\nu_8}, \\
Y_{b_2}^C &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}Y_{a_5} + \sqrt{1-\xi_3}Y_{\nu_5} + \sqrt{\xi_1}Y_{a_1} + \sqrt{1-\xi_1}Y_{\nu_1}) + \sqrt{1-\eta}Y_{\nu_8}, \\
X_{b_3}^C &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}X_{a_6} + \sqrt{1-\xi_3}X_{\nu_6} - \sqrt{\xi_4}Y_{a_4} - \sqrt{1-\xi_4}Y_{\nu_4}) + \sqrt{1-\eta}X_{\nu_9}, \\
Y_{b_3}^C &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}Y_{a_6} + \sqrt{1-\xi_3}Y_{\nu_6} + \sqrt{\xi_4}X_{a_4} + \sqrt{1-\xi_4}X_{\nu_4}) + \sqrt{1-\eta}X_{\nu_9}, \\
X_{b_4}^C &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}X_{a_6} + \sqrt{1-\xi_3}X_{\nu_6} + \sqrt{\xi_4}Y_{a_4} + \sqrt{1-\xi_4}Y_{\nu_4}) + \sqrt{1-\eta}Y_{\nu_{10}}, \\
Y_{b_4}^C &= \frac{\sqrt{\eta}}{\sqrt{2}}(\sqrt{\xi_3}Y_{a_6} + \sqrt{1-\xi_3}Y_{\nu_6} - \sqrt{\xi_4}X_{a_4} - \sqrt{1-\xi_4}X_{\nu_4}) + \sqrt{1-\eta}Y_{\nu_{10}},
\end{aligned}
\tag{11}
$$

where the superscript C denotes cluster-like state. Substituting eqs. (7) and (8) into eq. (11), and calculating the correlations of the amplitude and phase quadratures between the subsystems of cluster-like state, we have,

$$
\begin{aligned}
X_{b_1}^C + X_{b_2}^C + 2X_{b_3}^C &= 2\sqrt{\eta\xi_2\xi_3}\mathrm{e}^{-r}X_{a_2}^{(0)} - \sqrt{2\eta\xi_4}\mathrm{e}^{-r}Y_{a_4}^{(0)} + 2\sqrt{\eta(1-\xi_2)\xi_3}X_{\nu_2} \\
&\quad + \sqrt{2\eta(1-\xi_3)}(X_{\nu_5} + X_{\nu_6}) - \sqrt{2\eta(1-\xi_4)}Y_{\nu_4} + \sqrt{1-\eta}(X_{\nu_7} + X_{\nu_8} + 2X_{\nu_9}),
\end{aligned}
\tag{12a}
$$

$$
X_{b_3}^C - X_{b_4}^C = -\sqrt{2\eta\xi_4}\mathrm{e}^{-r}Y_{a_4}^{(0)} - \sqrt{2\eta(1-\xi_4)}Y_{\nu_4} + \sqrt{1-\eta}(X_{\nu_9} - X_{\nu_{10}}),
\tag{12b}
$$

$$
Y_{b_1}^C - Y_{b_2}^C = -\sqrt{2\eta\xi_1}\mathrm{e}^{-r}Y_{a_1}^{(0)} - \sqrt{2\eta(1-\xi_1)}Y_{\nu_1} + \sqrt{1-\eta}(Y_{\nu_7} - Y_{\nu_8}),
\tag{12c}
$$

$$
\begin{aligned}
-2Y_{b_2}^C + Y_{b_3}^C + Y_{b_4}^C &= -2\sqrt{\eta\xi_2\xi_3}\mathrm{e}^{-r}X_{a_3}^{(0)} - \sqrt{2\eta\xi_1}\mathrm{e}^{-r}Y_{a_1}^{(0)} - 2\sqrt{\eta(1-\xi_2)\xi_3}X_{\nu_3} \\
&\quad + \sqrt{2\eta(1-\xi_3)}(Y_{\nu_6} - Y_{\nu_5}) - \sqrt{2\eta(1-\xi_1)}Y_{\nu_1} + \sqrt{1-\eta}(-2Y_{\nu_8} + Y_{\nu_9} + Y_{\nu_{10}}),
\end{aligned}
\tag{12d}
$$

where (12b) and (12c) only depend on the squeezed quadrature of phase squeezed lights a_1 and a_4, respectively, while (12a) and (12d) depend on squeezed quadratures of both modes a_2, a_4 and a_1, a_3, respectively. In the ideal case ($r \to \infty$, $\xi \to 1$, $\eta \to 1$), the quadripartite cluster-like state is a simultaneous eigenstate with perfect amplitude correlations $X_{b_1}^C + X_{b_2}^C + 2X_{b_3}^C \to 0$, $X_{b_3}^C - X_{b_4}^C \to 0$ and perfect phase correlations $Y_{b_1}^C - Y_{b_2}^C \to 0$, $-2Y_{b_2}^C + Y_{b_3}^C + Y_{b_4}^C \to 0$.

3 The inseparability criteria of quadripartite entanglement

When the state vector of a quantum system comprising two or more than two subsystems cannot be written into the direct product of the state vectors of respective subsystem in any quantum mechanical picture, there is the quantum inseparability among these subsystems. The observation to any one subsystem must affect other subsystems even if they are separated far away spatially. The non-local quantum correlations are called the quantum entanglement. The inseparability criteria of CV quadripartite GHZ-like entanglement have been deduced in ref. [13]. The modes b_1, b_2, b_3 and b_4 are in quantum inseparable quadripartite GHZ-like state if they simultaneously violate any two of the following three inequalities:

$$V(Y_{b_1}^G - Y_{b_2}^G) + V(X_{b_1}^G + X_{b_2}^G + g_3 X_{b_3}^G + g_4 X_{b_4}^G) \geq 1, \tag{13a}$$

$$V(Y_{b_2}^G - Y_{b_3}^G) + V(g_1 X_{b_1}^G + X_{b_2}^G + X_{b_3}^G + g_4 X_{b_4}^G) \geq 1, \tag{13b}$$

$$V(Y_{b_3}^G - Y_{b_4}^G) + V(g_1 X_{b_1}^G + g_2 X_{b_2}^G + X_{b_3}^G + X_{b_4}^G) \geq 1, \tag{13c}$$

where g_i is the gain factor. For ideal squeezing ($r \to \infty$), $g_i = 1$, while for nonideal squeezing we need to change g_i to minimize the left hand of eq. (13), and the corresponding gain factor is called the optimal gain factor g_i^{opt}.

The criteria for the inseparability of the quadripartite cluster-like state have not been reported yet. We deduce the inseparability criteria of the CV quadripartite cluster-like entanglement according to the methods used in ref. [13]. At first we consider the following linear combinations:

$$\begin{aligned} \hat{u} &= h_1 \hat{X}_1 + h_2 \hat{X}_2 + \cdots + h_N \hat{X}_N, \\ \hat{v} &= g_1 \hat{Y}_1 + g_2 \hat{Y}_2 + \cdots + g_N \hat{Y}_N. \end{aligned} \tag{14}$$

The necessary condition for separability and partial separability of N partite is

$$V(\hat{u})_\rho + V(\hat{v})_\rho \geq f(h_1, h_2, \cdots, h_N, g_1, g_2, \cdots, g_N), \tag{15}$$

where $f(h_1, h_2, \cdots, h_N, g_1, g_2, \cdots, g_N)$ is the function of parameter $h_1, h_2, \cdots, h_N$ and $g_1, g_2, \cdots, g_N$. For any arbitrary partially separable form, the total density operator can be written as

$$\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,k_r,\cdots m} \otimes \hat{\rho}_{i,k_s,\cdots n} \tag{16}$$

with a distinct pair of "separable modes" (m, n), and the other modes $k_r \neq k_s$, where the weight $\eta_i \geq 0$ and $\sum_i \eta_i = 1$. van Loock et al. have proved that the condition for testing separability or partial separability of N partite is

$$V(\hat{u})_\rho + V(\hat{v})_\rho \geq \frac{1}{2}\left(\left| h_m g_m + \sum_r h_{k_r} g_{k_r} \right| + \left| h_n g_n + \sum_s h_{k_s} g_{k_s} \right| \right). \tag{17}$$

The states are entangled if the above condition is violated. The bound is $\sum_j \left| h_j g_j \right| / 2$ ($j = 1, \cdots, N$) for fully separable state, which is larger than the bound of the partial separable state. So if the necessary condition of partially separable state is violated, the necessary condition of fully separable state must be violated. For the quadripartite entanglement we consider four modes (k, l, m, n), which is the arbitrary combination of (1, 2, 3, 4). The following three inequalities are used for judging the CV quadripartite cluster-like entanglement:

$$V(X_{b_1}^C + X_{b_2}^C + g_3 X_{b_3}^C) + V(Y_{b_1}^C - Y_{b_2}^C) \geq 1, \tag{18a}$$

$$V(X_{b_3}^C - X_{b_4}^C) + V(-g_2 Y_{b_2}^C + Y_{b_3}^C + Y_{b_4}^C) \geq 1, \tag{18b}$$

$$V(g_1 X_{b_1}^C + X_{b_2}^C + 2X_{b_3}^C) + V(-2Y_{b_2}^C + Y_{b_3}^C + g_4 Y_{b_4}^C) \geq 2. \tag{18c}$$

For inequality (18a), we have $h_1 = h_2 = g_1 = -g_2 = 1$, $h_3 = g$, $g_3 = h_4 = g_4 = 0$. For the arbitrary combination of (k, l, m, n), if $(k, l, m, n) = (2, 3, 4, 1)$ and $(k, l, m, n) = (1, 3, 4, 2)$, the bound of all fluctuations is 1, then inequality (18a) can be used as the necessary condition of the partially separable states $\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,134} \otimes \hat{\rho}_{i,2}$, $\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,234} \otimes \hat{\rho}_{i,1}$, $\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,14} \otimes \hat{\rho}_{i,23}$ and $\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,13} \otimes \hat{\rho}_{i,24}$. But for $(k, l, m, n) = (1, 2, 4, 3)$ and $(k, l, m, n) = (1, 2, 3, 4)$, the bound of all fluctuations is 0, so the inequality (18a) cannot be used as the necessary condition of partially separable states $\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,124} \otimes \hat{\rho}_{i,3}$, $\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,123} \otimes \hat{\rho}_{i,4}$ and $\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,12} \otimes \hat{\rho}_{i,34}$. In the same way, we can test the type of partially separable states corresponding to inequalities (18b) and (18c). We can see that three inequalities in eq. (18) are the necessary condition for separable and partially separable quadripartite state:

$$\begin{aligned}
&\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,123} \otimes \hat{\rho}_{i,4} \quad \Rightarrow \text{(18b)},\\
&\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,124} \otimes \hat{\rho}_{i,3} \quad \Rightarrow \text{(18b), (18c)},\\
&\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,134} \otimes \hat{\rho}_{i,2} \quad \Rightarrow \text{(18a), (18c)},\\
&\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,234} \otimes \hat{\rho}_{i,1} \quad \Rightarrow \text{(18a)},
\end{aligned} \tag{19}$$

and

$$\begin{aligned}
&\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,12} \otimes \hat{\rho}_{i,34} \quad \Rightarrow \text{(18c)},\\
&\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,13} \otimes \hat{\rho}_{i,24} \quad \Rightarrow \text{(18a), (18b), (18c)},\\
&\hat{\rho} = \sum_i \eta_i \hat{\rho}_{i,14} \otimes \hat{\rho}_{i,23} \quad \Rightarrow \text{(18a), (18b)},
\end{aligned} \tag{20}$$

where the fully separable state is contained in the above several partially separable states. So the three inequalities in eq. (18) are sufficient to test all partially and fully separable quadripartite states. If the quadripartite state is cluster-like entanglement, the three inequalities in eq. (18) will be violated simultaneously.

4 Numerical calculation and results

According to the criterion of quadripartite GHZ-like entangled state and eq. (9), we can calculate the variances when we choose the same gain factor,

$$\begin{aligned}
&V(X_{b_1}^G + X_{b_2}^G + gX_{b_3}^G + gX_{b_4}^G) = \frac{1}{4}\eta\xi_2\xi_3(1+g)^2 \mathrm{e}^{-2r} + \frac{1}{4}\eta\xi_2\xi_3(1-g)^2 \mathrm{e}^{2r} + \frac{1}{2}(1-\eta\xi_2\xi_3)(1+g^2),\\
&V(gX_{b_1}^G + X_{b_2}^G + X_{b_3}^G + gX_{b_4}^G) = \frac{1}{4}\eta\xi_2\xi_3(1+g)^2 \mathrm{e}^{-2r} + \frac{1}{8}\eta(\xi_1+\xi_4)(1-g)^2 \mathrm{e}^{2r}\\
&\qquad + \frac{1}{2} + \frac{g^2}{2} - \frac{1}{4}\eta\xi_2\xi_3(1+g)^2 - \frac{1}{8}\eta(\xi_1+\xi_4)(1-g)^2,\\
&V(gX_{b_1}^G + gX_{b_2}^G + X_{b_3}^G + X_{b_4}^G) = \frac{1}{4}\eta\xi_2\xi_3(1+g)^2 \mathrm{e}^{-2r} + \frac{1}{4}\eta\xi_2\xi_3(1-g)^2 \mathrm{e}^{2r} + \frac{1}{2}(1-\eta\xi_2\xi_3)(1+g^2),\\
&V(Y_{b_1}^G - Y_{b_2}^G) = \frac{1}{2}\eta\xi_1 \mathrm{e}^{-2r} + \frac{1}{2}(1-\eta\xi_1),
\end{aligned} \tag{21}$$

$$V(Y_{b_2}^G - Y_{b_3}^G) = (\frac{1}{4}\eta\xi_2\xi_3 + \frac{1}{8}\eta\xi_1 + \frac{1}{8}\eta\xi_4)\mathrm{e}^{-2r} + \frac{1}{2} - \frac{1}{4}\eta\xi_2\xi_3 - \frac{1}{8}\eta\xi_1 - \frac{1}{8}\eta\xi_4,$$

$$V(Y_{b_3}^G - Y_{b_4}^G) = \frac{1}{2}\eta\xi_4\mathrm{e}^{-2r} + \frac{1}{2}(1-\eta\xi_4).$$

We can see that the two expressions of $V(X_{b_1} + X_{b_2} + gX_{b_3} + gX_{b_4})$ and $V(gX_{b_1} + gX_{b_2} + X_{b_3} + X_{b_4})$ are the same, and a little bit different from the expression of $V(gX_{b_1} + X_{b_2} + X_{b_3} + gX_{b_4})$, which leads to a little difference of the optimal gain factor. For simplicity and without loss of generality, the same gain factor is used in the calculation. Minimizing the left hand of the first inequality in eq. (13), the optimal gain factor of quadripartite GHZ-like entanglement is obtained:

$$g_{\mathrm{opt}}^G = \frac{\eta\xi_2\xi_3(\mathrm{e}^{4r}-1)}{\eta\xi_2\xi_3(\mathrm{e}^{2r}-1)^2 + 2\mathrm{e}^{2r}}. \tag{22}$$

According to the criteria of quadripartite cluster-like state in eqs. (18) and (12), we calculated the variances for judging the quadripartite cluster-like entanglement,

$$\begin{aligned}
V(X_{b_1}^C + X_{b_2}^C + g_3X_{b_3}^C) &= \left(\frac{1}{4}\eta\xi_2\xi_3\left(1+\frac{g_3}{2}\right)^2 + \frac{g_3{}^2}{8}\eta\xi_4\right)\mathrm{e}^{-2r} + \frac{1}{4}\eta\xi_2\xi_3\left(1-\frac{g_3}{2}\right)^2\mathrm{e}^{2r} \\
&\quad + \frac{1}{2} + \frac{g_3{}^2}{4} - \left(\frac{1}{2}+\frac{g_3{}^2}{8}\right)\eta\xi_2\xi_3 - \frac{g_3{}^2}{8}\eta\xi_4, \\
V(X_{b_3}^C - X_{b_4}^C) &= \frac{1}{2}\eta\xi_4\mathrm{e}^{-2r} + \frac{1}{2}(1-\eta\xi_4), \\
V(Y_{b_1}^C - Y_{b_2}^C) &= \frac{1}{2}\eta\xi_1\mathrm{e}^{-2r} + \frac{1}{2}(1-\eta\xi_1), \\
V(-g_2Y_{b_2}^C + Y_{b_3}^C + Y_{b_4}^C) &= \left[\frac{1}{4}\eta\xi_2\xi_3\left(1+\frac{g_2}{2}\right)^2 + \frac{g_2{}^2}{8}\eta\xi_1\right]\mathrm{e}^{-2r} + \frac{1}{4}\eta\xi_2\xi_3\left(1-\frac{g_2}{2}\right)^2\mathrm{e}^{2r} \\
&\quad + \frac{1}{2} + \frac{g_2{}^2}{4} - \left(\frac{1}{2}+\frac{g_2{}^2}{8}\right)\eta\xi_2\xi_3 - \frac{g_2{}^2}{8}\eta\xi_1, \\
V(g_1X_{b_1}^C + X_{b_2}^C + 2X_{b_3}^C) &= \left[\frac{1}{4}\eta\xi_2\xi_3\left(\frac{3}{2}+\frac{g_1}{2}\right)^2 + \frac{g_1{}^2}{2}\eta\xi_4\right]\mathrm{e}^{-2r} \\
&\quad + \left[\frac{1}{4}\eta\xi_2\xi_3\left(\frac{1}{2}-\frac{g_1}{2}\right)^2 + \frac{(1-g_1)^2}{8}\eta\xi_1\right]\mathrm{e}^{2r} \\
&\quad + \frac{5}{4} + \frac{g_1{}^2}{4} - \frac{(1-g_1)^2}{8}\eta\xi_1 - \frac{1}{4}\eta\xi_2\xi_3\left[\left(\frac{3}{2}+\frac{g_1}{2}\right)^2 + \left(\frac{1}{2}-\frac{g_1}{2}\right)^2\right] - \frac{1}{2}\eta\xi_4, \\
V(-2Y_{b_2}^C + Y_{b_3}^C + g_4Y_{b_4}^C) &= \left[\frac{1}{4}\eta\xi_2\xi_3\left(\frac{3}{2}+\frac{g_4}{2}\right)^2 + \frac{g_4{}^2}{2}\eta\xi_1\right]\mathrm{e}^{-2r} \\
&\quad + \left[\frac{1}{4}\eta\xi_2\xi_3\left(\frac{g_4}{2}-\frac{1}{2}\right)^2 + \frac{(1-g_4)^2}{8}\eta\xi_4\right]\mathrm{e}^{2r} \\
&\quad + \frac{5}{4} + \frac{g_4{}^2}{4} - \frac{(1-g_4)^2}{8}\eta\xi_4 - \frac{1}{4}\eta\xi_2\xi_3\left[\left(\frac{3}{2}+\frac{g_4}{2}\right)^2 + \left(\frac{1}{2}-\frac{g_4}{2}\right)^2\right] - \frac{1}{2}\eta\xi_1.
\end{aligned} \tag{23}$$

The optimal gain factors of the quadripartite cluster-like entanglement are calculated from eqs. (18) and (23). Supposing $\xi_1 = \xi_4$, we get the optimal gain factors $g^C_{2\text{opt}} = g^C_{3\text{opt}}$, $g^C_{1\text{opt}} = g^C_{4\text{opt}}$, which are expressed by

$$g^C_{2\text{opt}} = g^C_{3\text{opt}} = \frac{2\eta\xi_2\xi_3(\mathrm{e}^{4r}-1)}{4\mathrm{e}^{2r} + \eta\xi_2\xi_3(\mathrm{e}^{2r}-1)^2 - 2\eta\xi_4(\mathrm{e}^{2r}-1)}, \tag{24}$$

$$g^C_{1\text{opt}} = g^C_{4\text{opt}} = \frac{\eta[\xi_1\mathrm{e}^{2r} + \xi_2\xi_3(\mathrm{e}^{2r}+3) + \xi_4\mathrm{e}^{2r}](\mathrm{e}^{2r}-1)}{\eta\xi_1(\mathrm{e}^{2r}-1)\mathrm{e}^{2r} + \eta\xi_2\xi_3(\mathrm{e}^{2r}-1)^2 + [4+\eta\xi_4(\mathrm{e}^{2r}-1)]\mathrm{e}^{2r}}. \tag{25}$$

The left hands of the three inequalities in eq. (13) have totally analogous dependency on the squeezing parameter r. In the following, we numerically calculate the function of the correlation variance combination versus r in the inequality of eq. (13a) which are shown in Figure 3. Curve a in Figure 3 corresponds to the ideal case with $\xi_{1-4} \to 1$, $\eta \to 1$ and $g = 1$. We can see that when $r > 0.203$ (corresponding to the initial squeezing degree above 1.76 dB), the correlation variances in the left hands of eq. (13) are below the normalized SNL, i.e. the four modes generated is in a quadripartite GHZ-like entangled state. Curve b in Figure 3 is drawn with $\xi_{1-4} \to 1$, $\eta \to 1$ and $g = g^G_{\text{opt}}$ (the optimal gain). In this case, all variances are below the SNL for any squeezing of $r > 0$. Curve c in Figure 3 denotes the imperfect case with $\xi_1 = \xi_4 = 0.9$, $\xi_2 = \xi_3 = 0.95$, $\eta = 0.9$ and $g = g^G_{\text{opt}}$, the variances of which are always higher than the ideal curve b. For larger squeezing of $r > 0.8$, the correlation variances in curves a and b tend to identical values. However, for smaller squeezing, the effect of the gain factor is significant. If taking $g = 1$ (curve a), the correlation variances are larger than the SNL when $r < \sim 0.2$, but if taking $g = g^G_{\text{opt}}$ (curve b), the multipartite GHZ-like entanglement always exists for $r > 0$.

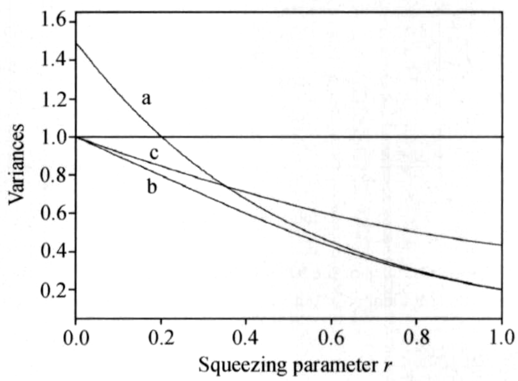

Figure 3 The functions of the correlation variances of the left hands in inequality (13a) of quadripartite GHZ-like entanglement criteria versus the squeezing parameter. a, The ideal case with $g = 1$; b, the ideal case with the optimal gain factor; c, the nonideal case with the optimal gain factor.

The expressions of the left hands of the inequalities (18a) and (18b) are analogous, so the dependencies of the correlation variances on the squeezing parameter r must be similar also. However the inequality (18c) is different from (18a) and (18b), so we numerically calculate the dependencies of (18a) and (18c) which are drawn in Figures 4 and 5, respectively. The functions of the correlation variances on the left hands of inequality (18a) versus the squeezing parameter are shown in

Figure 4. Curve a in Figure 4 shows that the correlation variances of inequality (18a) are below the normalized SNL when $r > 0.347$ (corresponding to the initial squeezing degree above 3 dB) with $\xi_{1-4} \to 1$, $\eta \to 1$ and $g = 1$, i.e. the inequality (18a) is violated. Curves b and c correspond to $g = g_{2\text{opt}}^{C}$ but ideal ($\xi_{1-4} \to 1$, $\eta \to 1$) and nonideal ($\xi_1 = \xi_4 = 0.9$, $\xi_2 = \xi_3 = 0.95$, $\eta = 0.9$) cases. The similar conclusions to the above discussion for Figure 3 can be obtained.

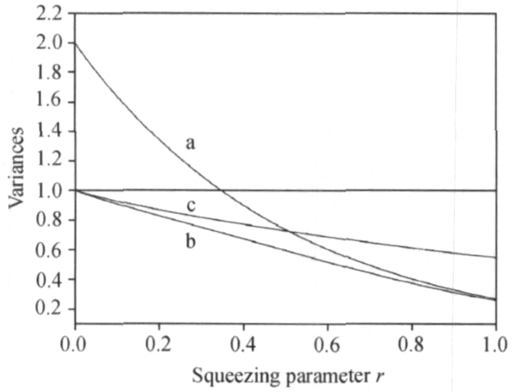

Figure 4 The functions of the correlation variances of the left hands in inequality (18a) of quadripartite cluster-like entanglement criteria versus the squeezing parameter. a, The ideal case with $g = 1$; b, the ideal case with the optimal gain factor; c, the nonideal case with optimal gain factor.

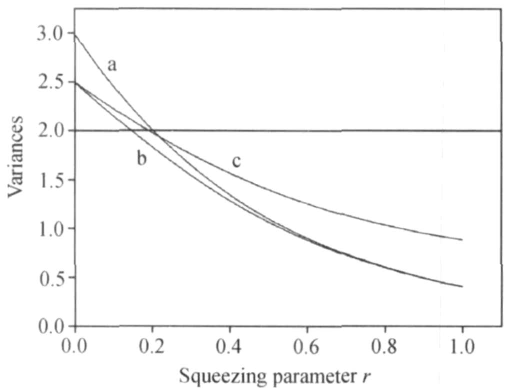

Figure 5 The functions of the correlation variances of the left hands in inequality (18c) of quadripartite cluster-like entanglement criteria versus the squeezing parameter. a, The ideal case with $g = 1$; b, the ideal case with the optimal gain factor; c, the nonideal case with the optimal gain factor.

The functions of the correlation variances on the left hand of inequality (18c) versus the squeezing parameter are shown in Figure 5. The curve a in Figure 5 shows that the correlation variances of inequality (18c) are below the normalized SNL (value 2) when $r > 0.203$ (corresponding to the initial squeezing degree above 1.76 dB), $\xi_{1-4} \to 1$, $\eta \to 1$ and $g = 1$, i.e. the inequality (18c) is violated. Curve b in Figure 5 denotes that inequality (18c) is violated when $r > 0.147$ (corresponding to the initial squeezing degree above 1.28 dB), $\xi_{1-4} \to 1$, and $\eta \to 1$ if taking $g = g_{1\text{opt}}^{C}$. Curve c in Figure 5 corresponds to the nonideal case with the optimal gain factor, where $\xi_1 = \xi_4 = 0.9$, $\xi_2 = \xi_3 = 0.95$, $\eta = 0.9$. Inequality (18c) is violated when $r > 0.192$ (corresponding to the initial squeezing degree above 1.28 dB). Comparing Figures 3—5, we can draw a conclusion that the quadripartite GHZ-like entangled state can be generated when the squeezing parameter is nonzero, while the quadripartite cluster-like entangled state can be generated only when the initial squeezing degree is above a certain level, even for the case taking $g = g_{1\mathrm{opt}}^{C}$ with ideal transmission and detection efficiencies. So the experimental conditions for generating the quadripartite cluster-like state are stricter than that needed for GHZ-like state.

5 Conclusion

We designed the generation systems of CV quadripartite GHZ-like and cluster-like entangled states in which the quadrature squeezed states from NOPAs and linear optical transforms were utilized. The dependencies of the quadripartite entanglement on the initial squeezing degree, the transmission efficiencies of the system and the detection efficiency of homodyne detection were calculated based on the inseparable criteria for CV quadripartite entanglement. The calculated results provide the theoretical reference for the designs of experimental systems.

1 Briegel H J, Raussendorf R. Persistent entanglement in arrays of interacting particles. Phys Rev Lett, 2001, 86: 910—913
2 Zhao Z, Chen Y A, Zhang A N, et al. Experimental demonstration of five-photon entanglement and open-destination teleportation. Nature, 2004, 430: 54
3 Walther P, Resch K J, Rudolph T, et al. Experimental one-way quantum computing. Nature, 2005, 434: 169—176
4 Jing J T, Zhang J, Yan Y, et al. Experimental demonstration of tripartite entanglement and controlled dense coding for continuous variables. Phys Rev Lett, 2003, 90: 167903
5 Yonezawa H, Aoki T, Furusawa A. Demonstration of a quantum teleportation network for continuous variables. Nature, 2004, 431: 430—433
6 Lance A M, Symul T, Bowen W P, et al. Tripartite quantum state sharing. Phys Rev Lett, 2004, 92: 177903
7 Zhang J, Brauntein S L. Continuous-variable Gaussian analog of cluster states. Phys Rev A, 2006, 73: 032318
8 Menicucci N C, van Loock P, Gu M, et al. Universal quantum computation with continuous-variable cluster states. quant-ph, 2006, 0605198
9 Li X Y, Pan Q, Jing J T, et al. Quantum dense coding exploiting an Einstein-Podolsky-Rosen beam. Phys Rev Lett, 2002, 88: 047904
10 Jia X J, Su X L, Pan Q, et al. Experimental demonstration of unconditional entanglement swapping for continuous variables. Phys Rev Lett, 2004, 93: 250503
11 Reid M D. Demonstration of Einstein-Podolsky-Rosen paradox using nondegenerate parametric amplification. Phys Rev A, 1989, 40: 913—923
12 Zhang Y, Wang H, Li X Y, et al. Experimental generation of bright two-mode quadrature squeezed light from a narrow-band nondegenerate optical parametric amplifier. Phys Rev A, 2000, 62: 023813
13 van Loock P, Furusawa A. Detecting genuine continuous-variable entanglement. Phys Rev A, 2003, 67: 052315