Deterministic Distribution of Multipartite Entanglement and Steering in a Quantum Network by Separable States

Meihong Wang,[1,2] Yu Xiang,[3,2,4] Haijun Kang,[1,2] Dongmei Han,[1,2] Yang Liu,[1,2] Qiongyi He,[3,2,4,*] Qihuang Gong,[3,2,4] Xiaolong Su,[1,2,†] and Kunchi Peng[1,2]

[1]*State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan 030006, China*

[2]*Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan, Shanxi 030006, China*

[3]*State Key Laboratory for Mesoscopic Physics, School of Physics, Frontiers Science Center for Nano-optoelectronics & Collaborative Innovation Center of Quantum Matter, Peking University, Beijing 100871, China*

[4]*Beijing Academy of Quantum Information Sciences, Beijing 100193, China*

(Received 31 July 2020; accepted 9 December 2020; published 30 December 2020)

As two valuable quantum resources, Einstein-Podolsky-Rosen entanglement and steering play important roles in quantum-enhanced communication protocols. Distributing such quantum resources among multiple remote users in a network is a crucial precondition underlying various quantum tasks. We experimentally demonstrate the deterministic distribution of two- and three-mode Gaussian entanglement and steering by transmitting separable states in a network consisting of a quantum server and multiple users. In our experiment, entangled states are not prepared solely by the quantum server, but are created among independent users during the distribution process. More specifically, the quantum server prepares separable squeezed states and applies classical displacements on them before spreading out, and users simply perform local beam-splitter operations and homodyne measurements after they receive separable states. We show that the distributed Gaussian entanglement and steerability are robust against channel loss. Furthermore, one-way Gaussian steering is achieved among users that is useful for further directional or highly asymmetric quantum information processing.

DOI: 10.1103/PhysRevLett.125.260506

Quantum entanglement is an important resource for quantum communication and computation [1]. Besides entanglement, Einstein-Podolsky-Rosen (EPR) steering has also been identified as a valuable resource for secure quantum information tasks [2–5]. The states exhibiting steering are a strict subset of the entangled states, and a strict superset of the Bell-nonlocal states [6]. Distinct from both inseparability and Bell nonlocality, the steerability of two directions between the entangled parties could be asymmetric [7,8] even it can only present in one direction [9], which has been successfully demonstrated in the pioneer works using continuous variable (CV) Gaussian states [9–13], discrete variable (DV) systems [14–17], and a hybrid CV-DV system [18]. Remarkably, EPR steering has been created recently in massive [19,20] and high-dimensional systems [21–25]. The concept of steering is important to quantum networks since it provides a way to verify entanglement, without the trustworthy requirement of the equipment at all nodes of the network. This has abundant applications to one-sided device-independent (1SDI) quantum key distribution [26–28], quantum secret sharing (QSS) [10,29], secure quantum teleportation [30,31], and subchannel discrimination [32,33].

At the current technology level, it is practical to establish a network consisting of a quantum server, which has the ability to prepare and manipulate quantum states, and two or more users who are merely able to perform local measurements on their states [Fig. 1(a)]. Consequently, how to distribute entanglement by the quantum server to make it shared among remote users becomes a crucial issue. The conventional method is to directly generate multipartite entangled states by a quantum server locally and then send to remote nodes. Alternatively, there are indirect ways to build entanglement among users, e.g., distributing entanglement by performing joint measurement (entanglement swapping) [34–36], or by transmitting separable states [37–44]. In the scheme of distributing entanglement via separable ancilla, instead of preparing entanglement directly by the quantum server, entanglement between two users is created by local operations, classical communication, and transmission of a separable ancillary mode. It has been shown that this indirect method has advantages for distribution of mixed Werner states with depolarizing and dephasing noise [42,45]. While significant progress has been reported in recent years [37–41], as well as first experiments implemented between two qubits [42] and between two Gaussian modes [43,44], the study of this efficient scheme is still in its infancy, and it is fair to say that our understanding of how powerful nonlocality can be provided by this method remains very limited so far.

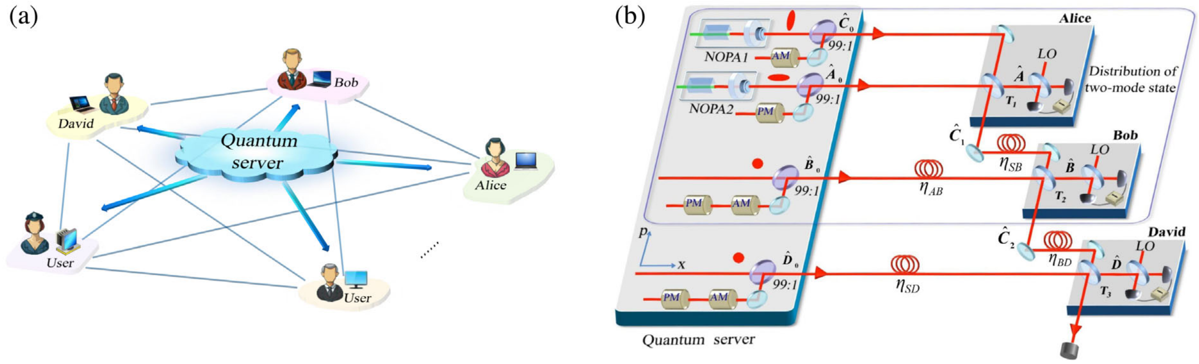

FIG. 1. Schematic of the distribution experiment. (a) Schematic of the quantum network. Quantum server produces quantum states and sends separable states to users. The quantum resource is shared by users after local operations. (b) Schematic of the experimental setup. Two squeezed states with −3 dB squeezing ($V_s = 0.50$) and +5.5 dB antisqueezing ($V_a = 3.55$) are produced by two nondegenerate optical parametric amplifiers (NOPA1 and NOPA2). Displacements for all modes are implemented by coupling modulated coherent beams with quantum states on 99:1 beam splitters. The correlated noise is added by amplitude (AM) and phase (PM) modulators, respectively. The distributed states are measured by balanced homodyne detectors for partial reconstruction of the covariance matrix. The lossy channel is simulated by a half-wave plate and a polarization beam splitter.

For instance, a generalized scheme was proposed to distribute Gaussian EPR steering by separable states [46], however, by reanalyzing data from those pioneer experiments [43,44], we find that none of them were able to demonstrate the shared EPR steering. As steerability is stronger than inseparability, in general it is harder to distribute steerability than inseparability. Moreover, towards a quantum network, it becomes an even more worthwhile objective to deeply explore the experimental feasibility of distributing multipartite entanglement and steering between more than two users with separable states. In addition, considering the practical channel loss, how to distribute as large as possible steerability at minimal cost is another important problem.

In this Letter, we experimentally demonstrate the deterministic distribution of Gaussian entanglement and steering with separable ancillary states both in two-user and multiuser scenarios. In the experiment, a quantum server prepares independent squeezed states and applies classical displacements on them, which makes initial states fully separable, and then distributes them to users; each user performs a local beam-splitter operation on the received states and transmits one output state of the beam splitter to the next user, where the classical displacements ensure the separability between the transmitted mode and the rest of the states in the network. Instead of providing a particular example to show the entanglement distribution via separable ancilla for two users [43,44], we rather experimentally implement the distribution of maximal steerability in general by optimizing the displacements according to the initial squeezing level, transmittance of the beam splitter, and transmission efficiencies in the channels. The distributed Gaussian entanglement and steerability are robust against channel loss. Furthermore, moving beyond two parties brings up richer steerability structures including one-way and one-to-multimode steering by mere transmission of separable ancillas, which could be used for providing unprecedented security for a future quantum internet [47,48].

We demonstrate the distribution of multipartite Gaussian entanglement and steering where entangled states are generated deterministically and information is encoded in the position or momentum quadratures of photonic harmonic oscillators [1]. In our experiment, two bright squeezed states are generated by two nondegenerate optical parametric amplifiers (NOPAs). Each of the NOPAs consists of a potassium titanyl phosphate (KTP) crystal and an output coupling mirror. The schematic of the experimental setup is illustrated in Fig. 1(b), and the details of the experiment can be found in Ref. [49]. The output states are measured in the time domain when the signals of the homodyne detectors are demodulated at a sideband frequency of 3 MHz with a bandwidth of 30 kHz. The demodulated signals are recorded simultaneously by a digital storage oscilloscope at the sampling rate of 500 KS/s.

In the experimental process, a series of correlated displacements (Gaussian noises) need to be optimized and added to realize this indirect distribution. Consequently, much effort is made to make sure the classical noises in same quadratures are canceled at the users' stations, that is, the added Gaussian noises must be synchronized. To do so, all of the displacements added on the amplitude and phase modulators are taken from two independent noise sources, respectively. With the increase of the number of users involved in the network, comes the requirement of even more effort to synchronize the added noises on all amplitude and phase modulators. In addition, more relative phases on the beam splitters need to be controlled precisely in the distribution of the three-mode state.

The process for distributing Gaussian entanglement and steering to three users, which is the smallest instance of a true quantum network, contains three steps. In the first step, the quantum server prepares a position (or amplitude quadrature) squeezed state $\hat{C}_{\rm in}$ and a momentum (or phase quadrature) squeezed state $\hat{A}_{\rm in}$ generated from two NOPAs, and two coherent (or vacuum) states $\hat{B}_{\rm in}$ and $\hat{D}_{\rm in}$ [49]. Then appropriate local classical displacements are applied to all modes according to the following relations:

$$\begin{aligned} \hat{x}_{C_0} &\to \hat{x}_{C_{\rm in}} + \mathcal{F}_C x_{\rm dis}, & \hat{p}_{A_0} &\to \hat{p}_{A_{\rm in}} + \mathcal{F}_A p_{\rm dis}, \\ \hat{x}_{B_0} &\to \hat{x}_{B_{\rm in}} + \mathcal{F}_B x_{\rm dis}, & \hat{p}_{B_0} &\to \hat{p}_{B_{\rm in}} - \mathcal{F}_B p_{\rm dis}, \\ \hat{x}_{D_0} &\to \hat{x}_{D_{\rm in}} + \mathcal{F}_D x_{\rm dis}, & \hat{p}_{D_0} &\to \hat{p}_{D_{\rm in}} - \mathcal{F}_D p_{\rm dis}, \end{aligned} \tag{1}$$

where $\hat{x}_j$ and $\hat{p}_j$ represent the position and momentum observables of the state corresponding to the subscript j, satisfying the canonical commutation relation $[\hat{x}_j, \hat{p}_j] = 2i$. The classical displacements are determined by $x_{\rm dis}$ and $p_{\rm dis}$ which obey Gaussian distribution with the same variance, and coefficients $\mathcal{F}_k$ $(k = A, B, C, D)$ corresponding to each mode. The coefficient $\mathcal{F}_k(T_i, \eta, V_{s,a})$ is a function of transmittance of beam-splitter T_i, transmission efficiency η in the channel, variances of squeezing V_s and antisqueezing V_a of the input squeezed states. Since $\hat{A}_{\rm in}$, $\hat{B}_{\rm in}$, $\hat{C}_{\rm in}$, $\hat{D}_{\rm in}$ are prepared independently and the added displacements are local operations and classical communication, the resulting states $\hat{A}_0$, $\hat{B}_0$, $\hat{C}_0$, $\hat{D}_0$ sent from the quantum server to users are fully separable.

In the second step, optical modes $\hat{A}_0$ and $\hat{C}_0$ are transmitted to Alice. We assume that Alice is close to the quantum server, i.e., $\eta_{SA} = 1$, while optical modes $\hat{B}_0$ and $\hat{D}_0$ are transmitted to Bob and David through lossy channels (the case for $\eta_{SA} \neq 1$ is discussed in Ref. [49]). In the two-user scenario, only optical mode $\hat{B}_0$ is transmitted to Bob, while David is not involved.

In the third step, all users perform beam-splitter operations on their received optical modes and measure the obtained states with homodyne detectors. Alice couples modes $\hat{A}_0$ and $\hat{C}_0$ on a balanced beam splitter with $T_1 = 1/2$, then keeps one output mode $\hat{A}$ and sends the other one $\hat{C}_1$ to Bob. The displacement operations on initial input modes ensure the separability across $\hat{C}_1|\hat{A}\hat{B}_0$ and $\hat{B}_0|\hat{A}\hat{C}_1$ splittings but entanglement between $\hat{A}$ and $\hat{B}_0\hat{C}_1$, which is essential for the present protocol. Bob couples the ancillary mode $\hat{C}_1$ and his mode $\hat{B}_0$ on the beam splitter T_2. Up to this stage, two-mode entanglement and steering between modes $\hat{A}$ and $\hat{B}$ (one of the output modes of Bob's beam splitter) are established. Meanwhile, the distributed steerability $\mathcal{G}^{A\to B}$ can be maximized by optimizing displacement coefficient $\mathcal{F}_B$, which was not uncovered by previous studies.

In the distribution for three users, the other output mode of Bob's beam splitter $\hat{C}_2$ is sequentially transmitted to David. A further challenge, apart from the requirement for separability across $\hat{C}_1|\hat{A}\hat{B}_0$ splitting, is that we need to carefully design the displacement on mode $\hat{D}_{\rm in}$ to keep the second ancillary mode $\hat{C}_2$ separable from all the users' modes $\hat{A}\hat{B}\hat{D}_0$. David couples the received mode $\hat{C}_2$ with his displaced mode $\hat{D}_0$ on the beam splitter T_3, and hence quantum entanglement and steering among three users, including modes $\hat{A}$, $\hat{B}$, and $\hat{D}$, can be built. Similarly, the Gaussian steerability $\mathcal{G}^{A\to BD}$ can be maximized by adjusting the displacement coefficient $\mathcal{F}_D$.

The distributed entangled states and the measurements both have Gaussian nature, thus, to detect Gaussian entanglement between subsystems N and M (each subsystem contains n and m modes, respectively) we adopt the positive partial transposition (PPT) criterion [55] which is necessary and sufficient when $n = 1$ and $m \geq 1$. The separable condition is that all symplectic eigenvalues of the covariance matrix after the partially transposition $\sigma_{NM}^{\top_N}$ are not smaller than 1 [49].

The steerability between two partitions $(N \to M)$ is quantified by the criterion from Ref. [56], where it was given by

$$\mathcal{G}^{N\to M}(\sigma_{NM}) = \max\left\{0, -\sum_{j:\bar{\nu}_j^{NM\backslash N}<1} \ln(\bar{\nu}_j^{NM\backslash N})\right\}. \tag{2}$$

Here, $\bar{\nu}_j^{NM\backslash N}$ $(j = 1, \ldots, m)$ denote the symplectic eigenvalues of the Schur complement $\bar{\sigma}_{NM\backslash N} = \mathcal{M} - \gamma^{\mathsf{T}}\mathcal{N}^{-1}\gamma$ of subsystem N, with diagonal blocks $\mathcal{N}$ and $\mathcal{M}$ corresponding to the reduced states of subsystems and the off-diagonal matrices γ and γ^{T} encoding the intermodal correlations between subsystems. A nonzero $\mathcal{G}^{N\to M} > 0$ denotes the presence of steering from N to M, and a higher value means stronger steerability. The steerability in the opposite direction $\mathcal{G}^{M\to N}$ can be obtained by swapping the roles of $\mathcal{N}$ and $\mathcal{M}$. The covariance matrices of generated states after each step are detailed in Ref. [49].

In this scheme, the crucial idea is that the ancillary modes $(\hat{C}_1, \hat{C}_2)$ in the channels are separable from the other modes. The conditions for separability depend on the parameters $\mathcal{F}_k(T_i, \eta, V_{s,a})$, $x_{\rm dis}$, and $p_{\rm dis}$. Without losing generality, we fix the variances of $x_{\rm dis}$ and $p_{\rm dis}$ to 1.50, $T_1 = 1/2$, $\mathcal{F}_A = \mathcal{F}_C = 1$, then the condition for separability across $\hat{C}_1|\hat{A}\hat{B}_0$ splitting in the two-user scenario only depends on the parameter $\mathcal{F}_B$, and that for the separability across $\hat{C}_2|\hat{A}\hat{B}\hat{D}_0$ splitting in the three-user scenario depends on the parameters $\mathcal{F}_B$ and $\mathcal{F}_D$. Additionally, on the basis of satisfying the above separable conditions, we optimize $\mathcal{F}_k$ to achieve the highest distributed steerabilities for each desired distribution direction.

To evaluate the performance of the present entanglement and steering distribution network, we investigate the effect of channel loss in our experiment since the transmission

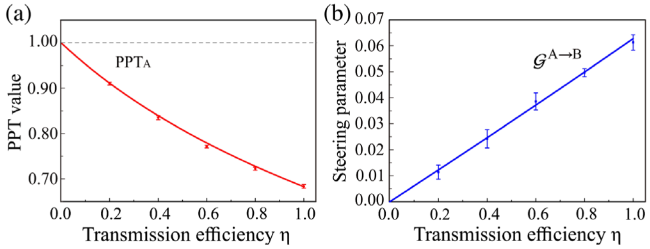

FIG. 2. Experimental results for two users. (a) The minimum symplectic eigenvalues PPT_A with respect to $\hat{A}|\hat{B}$ splitting is always smaller than 1. (b) The steerability $\mathcal{G}^{A\to B}$ is obtained and robust against loss in channels. Error bars represent one standard deviation and are obtained based on the statistics of measured noise variances.

distance of the quantum state is limited by inevitable loss in a practical quantum network. In the case of two users, in order to achieve the highest Gaussian steerability $\mathcal{G}^{A\to B}$, the optimized displacement coefficient on mode $\hat{B}_{\mathrm{in}}$ is set to $\mathcal{F}_B = \sqrt{2\eta_{AB}(1-T_2)}V_a/[\sqrt{\eta_{SB}T_2}(V_a+V_s)]$, where T_2 is the transmittance of Bob's beam splitter, and η_{AB} and η_{SB} are the transmission efficiencies for the channels from Alice to Bob and from quantum server to Bob, respectively. Thus, the maximal distributed steerability $\mathcal{G}^{A\to B}$ is given by

$$\mathcal{G}^{A\to B} = \ln\left[\frac{V_a+V_s}{(1-\eta_{AB}+\eta_{AB}T_2)(V_a+V_s)+2\eta_{AB}(1-T_2)V_sV_a}\right]. \tag{3}$$

We experimentally fix $T_2 = 1/2$ and set $\eta_{SB} = \eta_{AB} = \eta$, then the largest distributed steerability $\mathcal{G}^{A\to B}$ is

$$\mathcal{G}^{A\to B} = \ln\left[\frac{2(V_a+V_s)}{(2-\eta)(V_a+V_s)+2\eta V_sV_a}\right] \tag{4}$$

with $\mathcal{F}_B \approx 1.24$. When all channels are ideal, i.e., $\eta = 1$, we measure the covariance matrix $\sigma_{AB_0C_1}$ and verify the conditions for separability across $\hat{C}_1|\hat{A}\hat{B}_0$ splitting and $\hat{B}_0|\hat{A}\hat{C}_1$ splitting according to their minimum PPT values $1.264 > 1$ and $1.182 > 1$, respectively, while $\hat{A}$ is entangled with group of $\hat{B}_0\hat{C}_1$ due to its minimum PPT value $0.701 < 1$, under the above optimized displacement [49]. Note that when modes $\hat{C}_1$, $\hat{B}_0$ are transmitted in lossy channels, i.e., $\eta < 1$, the requirement for the separable conditions will be relaxed. As shown in Fig. 2, the distributed entanglement between Alice and Bob and one-way Gaussian steerability from Alice to Bob $\mathcal{G}^{A\to B}$ always exist when $\eta > 0$, which means this indirect distribution protocol is robust against channel loss.

After the successful distribution between two users, we extend this protocol to a three-user case. Figure 3 shows that the distributed three-mode entanglement and steerability are also robust against loss in quantum channels. As an example, the transmission efficiencies from quantum server to Bob, quantum server to David, Alice to Bob, and Bob to David are assumed to be the same. To achieve the maximum steerability $\mathcal{G}^{A\to BD}$, we optimize the displacements for modes $\hat{B}_{\mathrm{in}}$ and $\hat{D}_{\mathrm{in}}$ by $\mathcal{F}_B \approx 1.24$ and $\mathcal{F}_D = 2\sqrt{\eta}V_a/(V_a+V_s)$ with $T_2 = T_3 = 1/2$. Meanwhile, an additional condition for separability across $\hat{C}_2|\hat{A}\,\hat{B}\,\hat{D}_0$ splitting needs to be satisfied. Hence, we experimentally reconstruct the covariance matrix $\sigma_{\mathrm{ABC_2D_0}}$ [49], then verify that the minimum PPT value for splitting across $\hat{C}_2|\hat{A}\,\hat{B}\,\hat{D}_0$ is $1.177 > 1$ when $\eta = 1$. Similarly, when modes $\hat{C}_2$, $\hat{D}_0$ are transmitted in lossy channels, the separable condition required by splitting across $\hat{C}_2|\hat{A}\,\hat{B}\,\hat{D}_0$ is more easily satisfied.

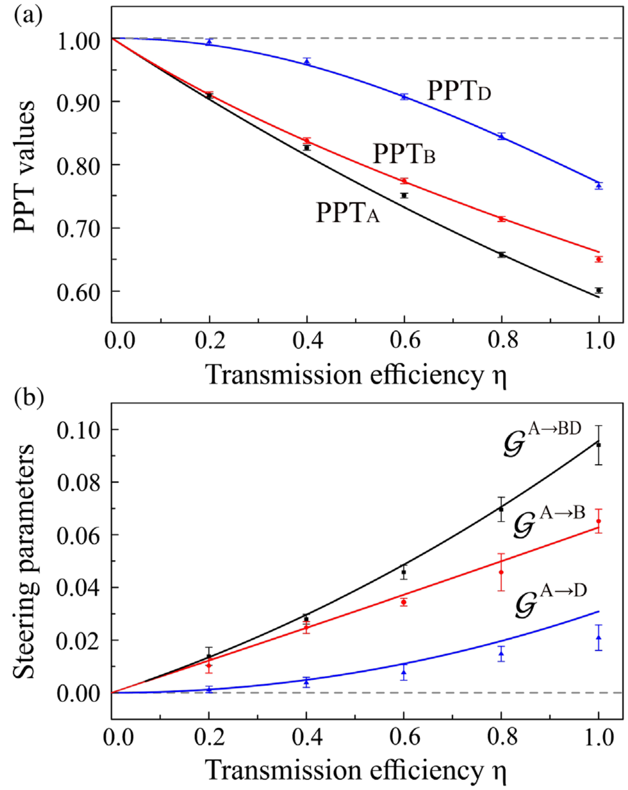

FIG. 3. Experimental results for three users. (a) All of the minimum symplectic eigenvalues PPT_A (black), PPT_B (red), and PPT_D (blue) with respect to $\hat{A}|\hat{B}\,\hat{D}$, $\hat{B}|\hat{A}\,\hat{D}$, and $\hat{D}|\hat{A}\,\hat{B}$ splittings are always smaller than 1. (b) The steerabilities $\mathcal{G}^{A\to BD}$, $\mathcal{G}^{A\to B}$, and $\mathcal{G}^{A\to D}$ are obtained and robust against channel losses. Error bars represent one standard deviation and are obtained based on the statistics of measured noise variances.

It is clearly shown in Fig. 3(a) that three-mode entanglement is shared among Alice, Bob, and David after the distribution. Different from entanglement, only the one-way steerabilities $\mathcal{G}^{A\to BD} > 0$, $\mathcal{G}^{A\to B} > 0$ and $\mathcal{G}^{A\to D} > 0$ are achieved, and the collective steerability ($\mathcal{G}^{A\to BD}$) is always higher than the individual steerabilities ($\mathcal{G}^{A\to B}$ and $\mathcal{G}^{A\to D}$), as shown in Fig. 3(b). We also note that the steering from Bob to David does not exist in any case (i.e., $\mathcal{G}^{B\to D} = 0$). This result can be understood as a consequence of the monogamy relation proposed in Ref. [57] where two independent parties cannot steer a third party simultaneously under Gaussian measurements. Thus, $\mathcal{G}^{A\to D} > 0$ prohibits the possibility of $\mathcal{G}^{B\to D} > 0$.

Note that the present experimental results show the ability to distribute the Gaussian steerability from Alice to other users (including the individual user and the group of them) by transmitting separable modes. This is because squeezed states are transmitted to Alice firstly, and then separable modes are transmitted from Alice to other users sequentially, i.e., it has a sequential property in such a distribution scheme. It can also be understood in the following way: the final distributed steerability comes from the mixture of two initial squeezed states at Alice's station by a balanced beam splitter, Alice holds half of the information of the whole state, while Bob, David, and the ancillary mode together hold the other half, which makes it much harder for Bob himself, and even together with David, to steer Alice. This means that with the current parameters our experiment presents a highly asymmetric network with directional steerability from Alice to other users.

After successful distribution, the quantum resources shared among distant users are widely available for real-world applications to networked quantum information tasks. For instance, the hierarchical structure presented in this network, where Alice acts as a superior who can always steer (pilot) any of the subordinate users ($\mathcal{G}^{A\to B,D,BD} > 0$), can be applied to implement secure directional quantum key distribution and quantum teleportation from Alice to Bob (David, or their group).

Furthermore, by adjusting the displacements and the transmittances of beam splitters, our protocol can also distribute on-demand quantum resources for specific quantum information tasks. For example, the steerabilities $\mathcal{G}^{BD\to A} > 0$ and $\mathcal{G}^{B\to A} = \mathcal{G}^{D\to A} = 0$ are required for 1SDI QSS, where the dealer Alice sends a secret and players (Bob and David) are able to decode the information only with their collaboration [29]. To distribute such a resource via separable ancillas, we need to adjust the displacement coefficients $\mathcal{F}_B = 0.92$ and $\mathcal{F}_D = 1.70$ with initial −10 dB squeezing and +11 dB antisqueezing, such that the steerability $\mathcal{G}^{BD\to A}$ can be distributed for $0.80 < \eta \leq 1$ where $\mathcal{G}^{B\to A} = \mathcal{G}^{D\to A} = 0$. This means that 1SDI QSS can be implemented in the range of 4.90 km with a fiber loss of 0.2 dB/km [49].

In summary, we present deterministic distribution of multipartite quantum resources by combining quantum channels and classical communications in a network consisting of a quantum server and multiple users. We demonstrate that it is feasible to distribute not only Gaussian entanglement but also EPR steering among two and three users via separable ancillas. Moreover, the maximum steerability allowed by the present network structure is distributed by optimizing the experimental parameters. The distributed entanglement and steerability are robust against channel losses, which further confirms the significance and practical feasibility of the presented method. This work provides a distinct approach for distributing precious multipartite quantum resources and takes a step forward in studying potential applications of this kind of protocols in a quantum network.

This work was financially supported by National Natural Science Foundation of China (Grants No. 11834010, No. 61675007, No. 11975026, No. 62005149, and No. 12004011), National Key R&D Program of China (Grants No. 2016YFA0301402, No. 2018YFB1107205 and No. 2019YFA0308702). X. S. thanks the program of Youth Sanjin Scholar, and the Fund for Shanxi "1331 Project" Key Subjects Construction. Q. H. acknowledges the Beijing Natural Science Foundation (Z190005) and the Key R&D Program of Guangdong Province (Grant No. 2018B030329001).

M. W. and Y. X. contributed equally to this work.

*qiongyihe@pku.edu.cn
†suxl@sxu.edu.cn

[1] C. Weedbrook, S. Pirandola, R. García-Patrón, N. J. Cerf, T. C. Ralph, J. H. Shapiro, and S. Lloyd, Gaussian quantum information, Rev. Mod. Phys. **84**, 621 (2012).
[2] R. Uola, A. C. S. Costa, H. C. Nguyen, and O. Gühne, Quantum steering, Rev. Mod. Phys. **92**, 015001 (2020).
[3] D. Cavalcanti and P. Skrzypczyk, Quantum steering: A review with focus on semidefinite programming, Rep. Prog. Phys. **80**, 024001 (2017).
[4] M. D. Reid, P. D. Drummond, W. P. Bowen, E. G. Cavalcanti, P. K. Lam, H. A. Bachor, U. L. Andersen, and G. Leuchs, Colloquium: The Einstein-Podolsky-Rosen paradox: From concepts to applications, Rev. Mod. Phys. **81**, 1727 (2009).
[5] R. Gallego and L. Aolita, Resource Theory of Steering, Phys. Rev. X **5**, 041008 (2015).
[6] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Steering, Entanglement, Nonlocality, and the Einstein-Podolsky-Rosen Paradox, Phys. Rev. Lett. **98**, 140402 (2007).
[7] E. G. Cavalcanti, S. J. Jones, H. M. Wiseman, and M. D. Reid, Experimental criteria for steering and the Einstein-Podolsky-Rosen paradox, Phys. Rev. A **80**, 032112 (2009).
[8] A. Rutkowski, A. Buraczewski, P. Horodecki, and M. Stobińska, Quantum Steering Inequality with Tolerance for Measurement-Setting Errors: Experimentally Feasible Signature of Unbounded Violation, Phys. Rev. Lett. **118**, 020402 (2017).
[9] V. Händchen, T. Eberle, S. Steinlechner, A. Samblowski, T. Franz, R. F. Werner, and R. Schnabel, Observation of one-way Einstein-Podolsky-Rosen steering, Nat. Photonics **6**, 596 (2012).
[10] S. Armstrong, M. Wang, R. Y. Teh, Q. Gong, Q. He, J. Janousek, H. A. Bachor, M. D. Reid, and P. K. Lam,

Multipartite Einstein-Podolsky-Rosen steering and genuine tripartite entanglement with optical networks, Nat. Phys. **11**, 167 (2015).

[11] X. Deng, Y. Xiang, C. Tian, G. Adesso, Q. He, Q. Gong, X. Su, C. Xie, and K. Peng, Demonstration of Monogamy Relations for Einstein-Podolsky-Rosen Steering in Gaussian Cluster States, Phys. Rev. Lett. **118**, 230501 (2017).

[12] Z. Qin, X. Deng, C. Tian, M. Wang, X. Su, C. Xie, and K. Peng, Manipulating the direction of Einstein-Podolsky-Rosen steering, Phys. Rev. A **95**, 052114 (2017).

[13] Y. Cai, Y. Xiang, Y. Liu, Q. He, and N. Treps, Versatile multipartite Einstein-Podolsky-Rosen steering via a quantum frequency comb, Phys. Rev. Research **2**, 032046(R) (2020).

[14] D. Cavalcanti, P. Skrzypczyk, G. H. Aguilar, R. V. Nery, P. H. Souto Ribeiro, and S. P. Walborn, Detection of entanglement in asymmetric quantum networks and multipartite quantum steering, Nat. Commun. **6**, 7941 (2015).

[15] S. Wollmann, N. Walk, A. J. Bennet, H. M. Wiseman, and G. J. Pryde, Observation of Genuine One-Way Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **116**, 160403 (2016).

[16] K. Sun, X.-J. Ye, J.-S. Xu, X.-Y. Xu, J.-S. Tang, Y.-C. Wu, J.-L. Chen, C.-F. Li, and G.-C. Guo, Experimental Quantification of Asymmetric Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **116**, 160404 (2016).

[17] Y. Xiao, X.-J. Ye, K. Sun, J.-S. Xu, C.-F. Li, and G.-C. Guo, Demonstration of Multisetting One-Way Einstein-Podolsky-Rosen Steering in Two-Qubit Systems, Phys. Rev. Lett. **118**, 140404 (2017).

[18] A. Cavaillès, H. Le Jeannic, J. Raskop, G. Guccione, D. Markham, E. Diamanti, M. D. Shaw, V. B. Verma, S. W. Nam, and J. Laurat, Demonstration of Einstein-Podolsky-Rosen Steering Using Hybrid Continuous- and Discrete-Variable Entanglement of Light, Phys. Rev. Lett. **121**, 170403 (2018).

[19] M. Fadel, T. Zibold, B. Décamps, and P. Treutlein, Spatial entanglement patterns and Einstein-Podolsky-Rosen steering in Bose-Einstein condensates, Science **360**, 409 (2018).

[20] P. Kunkel, M. Prüfer, H. Strobel, D. Linnemann, A. Frölian, T. Gasenzer, M. Gärttner, and M. K. Oberthaler, Spatially distributed multipartite entanglement enables EPR steering of atomic clouds, Science **360**, 413 (2018).

[21] C.-M. Li, K. Chen, Y.-N. Chen, Q. Zhang, Y.-A. Chen, and J.-W. Pan, Genuine High-Order Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **115**, 010402 (2015).

[22] C.-M. Li, H.-P. Lo, L.-Y. Chen, and A. Yabushita, Experimental verification of multidimensional quantum steering, Opt. Commun. **410**, 956 (2018).

[23] Q. Zeng, B. Wang, P. Li, and X. Zhang, Experimental High-Dimensional Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **120**, 030401 (2018).

[24] J. Wang, S. Paesani, Y. Ding, R. Santagati, P. Skrzypczyk, A. Salavrakos, J. Tura, R. Augusiak, L. Mancinska, D. Bacco *et al.*, Multidimensional quantum entanglement with large-scale integrated optics, Science **360**, 285 (2018).

[25] Y. Guo, S. Cheng, X. Hu, B.-H. Liu, E.-M. Huang, Y.-F. Huang, C.-F. Li, G.-C. Guo, and E. G. Cavalcanti, Experimental Measurement-Device-Independent Quantum Steering and Randomness Generation Beyond Qubits, Phys. Rev. Lett. **123**, 170402 (2019).

[26] C. Branciard, E. G. Cavalcanti, S. P. Walborn, V. Scarani, and H. M. Wiseman, One-sided device-independent quantum key distribution: Security, feasibility, and the connection with steering, Phys. Rev. A **85**, 010301(R) (2012).

[27] T. Gehring, V. Händchen, J. Duhme, F. Furrer, T. Franz, C. Pacher, R. F. Werner, and R. Schnabel, Implementation of continuous-variable quantum key distribution with composable and one-sided-device-independent security against coherent attacks, Nat. Commun. **6**, 8795 (2015).

[28] N. Walk, S. Hosseini, J. Geng, O. Thearle, J. Y. Haw, S. Armstrong, S. M. Assad, J. Janousek, T. C. Ralph, T. Symul, H. M. Wiseman, and P. K. Lam, Experimental demonstration of Gaussian protocols for one-sided device-independent quantum key distribution, Optica **3**, 634 (2016).

[29] Y. Xiang, I. Kogias, G. Adesso, and Q. He, Multipartite Gaussian steering: Monogamy constraints and quantum cryptography applications, Phys. Rev. A **95**, 010101(R) (2017).

[30] Q. He, L. Rosales-Zárate, G. Adesso, and M. D. Reid, Secure Continuous Variable Teleportation and Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **115**, 180502 (2015).

[31] C.-Y. Chiu, N. Lambert, T.-L. Liao, F. Nori, and C.-M. Li, No-cloning of quantum steering, npj Quantum Inf. **2**, 16020 (2016).

[32] M. Piani and J. Watrous, Necessary and Sufficient Quantum Information Characterization of Einstein-Podolsky-Rosen Steering, Phys. Rev. Lett. **114**, 060404 (2015).

[33] S.-L. Chen, C. Budroni, Y.-C. Liang, and Y.-N. Chen, Natural Framework for Device-Independent Quantification of Quantum Steerability, Measurement Incompatibility, and Self-Testing, Phys. Rev. Lett. **116**, 240401 (2016).

[34] X. Su, C. Tian, X. Deng, Q. Li, C. Xie, and K. Peng, Quantum Entanglement Swapping Between Two Multipartite Entangled States, Phys. Rev. Lett. **117**, 240503 (2016).

[35] T. Kraft, S. Designolle, C. Ritz, N. Brunner, O. Gühne, and M. Huber, Quantum entanglement in the triangle network, arXiv:2002.03970.

[36] M. Navascués, E. Wolfe, D. Rosset, and A. Pozas-Kerstjens, Genuine network multipartite entanglement, Phys. Rev. Lett. **125**, 240505 (2020).

[37] T. S. Cubitt, F. Verstraete, W. Dür, and J. I. Cirac, Separable States can be Used to Distribute Entanglement, Phys. Rev. Lett. **91**, 037902 (2003).

[38] L. Mišta, Jr. and N. Korolkova, Distribution of continuous-variable entanglement by separable Gaussian states, Phys. Rev. A **77**, 050302(R) (2008).

[39] L. Mišta, Jr. and N. Korolkova, Improving continuous-variable entanglement distribution by separable states, Phys. Rev. A **80**, 032310 (2009).

[40] T. K. Chuan, J. Maillard, K. Modi, T. Paterek, M. Paternostro, and M. Piani, Quantum Discord Bounds the Amount of Distributed Entanglement, Phys. Rev. Lett. **109**, 070501 (2012).

[41] A. Streltsov, H. Kampermann, and D. Bruß, Quantum Cost for Sending Entanglement, Phys. Rev. Lett. **108**, 250501 (2012).

[42] A. Fedrizzi, M. Zuppardo, G. G. Gillett, M. A. Broome, M. P. Almeida, M. Paternostro, A. G. White, and T. Paterek,

Experimental Distribution of Entanglement with Separable Carriers, Phys. Rev. Lett. **111**, 230504 (2013).

[43] C. E. Vollmer, D. Schulze, T. Eberle, V. Händchen, J. Fiurášek, and R. Schnabel, Experimental Entanglement Distribution by Separable States, Phys. Rev. Lett. **111**, 230505 (2013).

[44] C. Peuntinger, V. Chille, L. Mšta, Jr., N. Korolkova, M. Förtsch, J. Korger, C. Marquardt, and G. Leuchs, Distributing Entanglement with Separable States, Phys. Rev. Lett. **111**, 230506 (2013).

[45] M. Zuppardo, T. Krisnanda, T. Paterek, S. Bandyopadhyay, A. Banerjee, P. Deb, S. Halder, K. Modi, and M. Paternostro, Excessive distribution of quantum entanglement, Phys. Rev. A **93**, 012305 (2016).

[46] Y. Xiang, X. Su, L. Mišta, Jr., G. Adesso, and Q. He, Multipartite Einstein-Podolsky-Rosen steering sharing with separable states, Phys. Rev. A **99**, 010104(R) (2019).

[47] H. J. Kimble, The quantum internet, Nature (London) **453**, 1023 (2008).

[48] S. Wehner, D. Elkous, and R. Hanson, Quantum internet: A vision for the road ahead, Science **362**, eaam9288 (2018).

[49] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.125.260506 for details of the experiment, the criterion of Gaussian entanglement, the distribution in lossy channels, verification of separable conditions, and the effect of loss between Alice and server. The Supplemental Material contains additional Refs. [50–54].

[50] X. Su, S. Hao, X. Deng, L. Ma, M. Wang, X. Jia, C. Xie, and K. Peng, Gate sequence for continuous variable one-way quantum computation, Nat. Commun. **4**, 2828 (2013).

[51] Y. Zhang, H. Wang, X. Li, J. Jing, C. Xie, and K. Peng, Experimental generation of bright two-mode quadrature squeezed light from a narrow-band nondegenerate optical parametric amplifier, Phys. Rev. A **62**, 023813 (2000).

[52] X. Su, Y. Zhao, S. Hao, X. Jia, C. Xie, and K. Peng, Experimental preparation of eight-partite cluster state for photonic qumodes, Opt. Lett. **37**, 5178 (2012).

[53] R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, Laser phase and frequency stabilization using an optical resonator, Appl. Phys. B **31**, 97 (1983).

[54] K. Huang, H. Le Jeannic, J. Ruaudel, O. Morin, and J. Laurat, Microcontroller-based locking in optics experiments, Rev. Sci. Instrum. **85**, 123112 (2014).

[55] R. Simon, Peres-Horodecki Separability Criterion for Continuous Variable Systems, Phys. Rev. Lett. **84**, 2726 (2000).

[56] I. Kogias, A. R. Lee, S. Ragy, and G. Adesso, Quantification of Gaussian Quantum Steering, Phys. Rev. Lett. **114**, 060403 (2015).

[57] M. D. Reid, Monogamy inequalities for the Einstein-Podolsky-Rosen paradox and quantum steering, Phys. Rev. A **88**, 062108 (2013).