Investigation of the influence of extra noises in seed beams on continuous-variable entanglement generation*

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The influence of the extra classical noises in seed beams on the entanglement between the signal and the idler modes of the output fields generated by a non-degenerate optical parametric amplifier operating at deamplification is investigated theoretically and experimentally. With the increase of the extra classical noises in the seed beams, the correlation degree of the output entangled optical fields, which is scaled by the quantum noise limit, decreases rapidly. The experimental results are in good agreement with the theoretical calculations.

Keywords: quantum entanglement, continuous variable, classical noise

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1. Introduction

Quantum communication and quantum information processing based on exploiting continuous variable (CV) entanglement of optical fields have been extensively investigated.[1−7] Einstein–Podolsky–Roson (EPR) entangled states with quantum correlations of amplitude and phase quadratures produced from optical parametric amplifiers (OPAs) have been used in most CV quantum information systems as necessary entanglement resources.[2−7] Utilizing EPR entanglement shared between sender and receiver various communication feats, such as determinately teleporting an unknown quantum state,[2] quantum dense coding of classical information transmission[4] and creating entanglement between two distant systems never directly interacting,[3] have been successfully performed. These feats cannot be realized with any classical communication schemes without the assistance of entanglement. The presence and the quality of entanglement are the most important factors for the success in quantum information implementation. Thus, verifying and quantifying entanglement are significant procedures in quantum information experiments and practical applications. The quantively theoretical criteria on two-party (EPR)[8,9] and multipartite CV entanglement[10−13] have been proposed. In Refs. [8] and [9], the author provided was a simple borderline to check experimentally the EPR entanglement of two macroscopic optical fields, that is, the quantum noise limit (QNL) of the total correlation variances of amplitude and phase quadratures (a pair of the canonical conjugate variables). When the variances of two commuting combinations of the variables, for example amplitude sum and phase difference or amplitude difference and phase sum, are smaller than those of the corresponding QNL determined by quantum noises of coherent state optical fields, the two optical fields are entangled, otherwise they are separable. It has been theoretically demonstrated that the non-separability criterion normalized to that of the QNL is a necessary and sufficient condition for a state of bright optical field being entangled.[14] Two independent single-mode squeezed states generated by degenerate OPAs (DOPAs) can be coupled on a beamsplitter to produce an entangled state with quantum correlations of both amplitude and phase quadratures.[2] More conveniently, a two-mode squeezed state generated by a nondegenerate OPA (NOPA) can be directly transformed into a pair of entangled optical beams with a beamsplitter.[4,15] The bright EPR beams with the quadrature amplitude anticorrelation and the quadrature phase correlation can be generated by NOPA operating at deamplifi-

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cation with injected seed beams.[3,4,16] In the case of NOPA deamplification, the phase difference between the pump light and the seed beam is $(2n+1)\pi$ (n is any integer). A seed laser beam can often be well approximated by a coherent state at higher analysis frequency, but there are some extra lower frequency classical noises in the laser beam usually due to the existence of the relaxation oscillation in the all-solid state lasers. In this paper, we analyse the influence of excess classical noises on the entanglement generation and measurement of EPR optical beams.

2. Theoretical analysis

The schematic diagram of the phase sensitive NOPA is shown in Fig. 1, two seed fields $a_{\rm i}$, $a_{\rm s}$ with the same frequency $\omega_{\rm i}=\omega_{\rm s}=\omega$ and orthogonal polarizations as well as a pump field a_0 with frequency $\omega_0=2\omega$ are injected into the NOPA with a second-order nonlinear optical crystal, simultaneously.

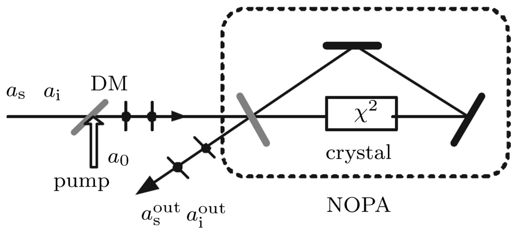

Fig. 1. Schematic diagram for the phase sensitive NOPA. DM, dichroic mirror of HR for pump and HT for seed beam.

For simplicity and without the loss of generality, we assume that the intensities of the injected signal and idler fields are identical. The pump field is considered as a classical light without depletion during the optical parametric process, i.e., $a_0 >> a_{\rm i}$, $a_{\rm s}$. In previous theoretical analyses, the injected seed beams were usually thought as idea coherent beams.[3,4,8−13,17] As is well known, coherent state optical fields are the closest quantum mechanical approximation of an idealized classical coherent light, which contain only quantum noises without any classical noises. A coherent state of light ($a_{\rm coh}$) is a minimum uncertainty state and the variances of its amplitude ($X_{\rm coh}=(a_{\rm coh}+a_{\rm coh}^*)/\sqrt{2}$) and phase ($Y_{\rm coh}=(a_{\rm coh}-a_{\rm coh}^*)/{\rm i}\sqrt{2}$) quadratures are equal, which are the QNLs of fluctuations of quadrature components and usually are normalized ($\langle\delta^2 X_{\rm coh}\rangle=\langle\delta^2 Y_{\rm coh}\rangle=1/2$).[18] The noises of any classical light are not able to be less than the QNLs because the quantum noises are always involved. The noise ($a_{\rm l}$) values of the signal laser beam and idler laser beam injected into NOPA usually each contain two parts, one is the unavoidable quantum fluctuation ($a_{\rm ql}$) and the other is the extra classical noise ($a_{\rm cl}$),

$$a_{\rm l} = a_{\rm ql}+a_{\rm cl}, \tag{1}$$

where $l={\rm s,i}$ stand for signal and idler respectively. Under the condition of perfect phase matching, the output signal ($a_{\rm s}^{\rm out}$) and idler ($a_{\rm i}^{\rm out}$) beams from the NOPA operated at deamplification are expressed as[17]

$$\begin{aligned} a_{\rm s}^{\rm out} &= a_{\rm qs}\cosh r + a_{\rm cs}\cosh r - a_{\rm qi}^*\sinh r - a_{\rm ci}^*\sinh r,\\ a_{\rm i}^{\rm out} &= a_{\rm qi}\cosh r + a_{\rm ci}\cosh r - a_{\rm qs}^*\sinh r - a_{\rm cs}^*\sinh r, \end{aligned} \tag{2}$$

where r is the correlation parameter which depends on the length and the effective nonlinear coefficient of the crystal inside the NOPA, the loss of optical cavity and the intensity of pump field. The value of the correlation factor r is taken to be in a range of $0<r<\infty$, $r=0$ means without any correlation between two output beams, while $r\to\infty$ corresponds to perfect correlation. Rewriting Eq.(2) with the amplitude quadrature $X_{\rm l}=(a_{\rm l}+a_{\rm l}^*)/\sqrt{2}$ and the phase quadrature $Y_{\rm l}=(a_{\rm l}-a_{\rm l}^*)/{\rm i}\sqrt{2}$, we obtain

$$\begin{aligned} X_{\rm s}^{\rm out} &= X_{\rm qs}\cosh r + X_{\rm cs}\cosh r\\ &\quad - X_{\rm qi}\sinh r - X_{\rm ci}\sinh r,\\ X_{\rm i}^{\rm out} &= X_{\rm qi}\cosh r + X_{\rm ci}\cosh r\\ &\quad - X_{\rm qs}\sinh r - X_{\rm cs}\sinh r,\\ Y_{\rm s}^{\rm out} &= Y_{\rm qs}\cosh r + Y_{\rm cs}\cosh r\\ &\quad + Y_{\rm qi}\sinh r + Y_{\rm ci}\sinh r,\\ Y_{\rm i}^{\rm out} &= Y_{\rm qi}\cosh r + Y_{\rm ci}\cosh r\\ &\quad + Y_{\rm qs}\sinh r + Y_{\rm cs}\sinh r, \end{aligned} \tag{3}$$

where $X_{\rm l}^{\rm out}$ and $Y_{\rm l}^{\rm out}$ denote the amplitude quadrature and the phase quadrature of the output fields ($a_{\rm l}^{\rm out}$), and $X_{\rm ql}$ and $Y_{\rm ql}$ ($X_{\rm cl}$, $Y_{\rm cl}$) represent the amplitude quadrature and phase quadrature of the quantum noises (classical noises) in the injected beams $a_{\rm ql}$ ($a_{\rm cl}$), respectively. From Eqs. (3) we can easily calculate the correlation variances of the amplitude quadrature and the phase quadrature between two output fields, and they are expressed as

$$\begin{aligned} &\langle\delta^2 X_+\rangle\\ &= \langle\delta^2(X_{\rm s}^{\rm out}+X_{\rm i}^{\rm out})\rangle\\ &= {\rm e}^{-2r}(\langle\delta^2 X_{\rm qs}\rangle+\langle\delta^2 X_{\rm cs}\rangle+\langle\delta^2 X_{\rm qi}\rangle+\langle\delta^2 X_{\rm ci}\rangle),\\ &\langle\delta^2 Y_-\rangle\\ &= \langle\delta^2(Y_{\rm s}^{\rm out}-Y_{\rm i}^{\rm out})\rangle \end{aligned}$$

$$= \mathrm{e}^{-2r}(\langle\delta^2 Y_{\mathrm{qs}}\rangle + \langle\delta^2 Y_{\mathrm{cs}}\rangle + \langle\delta^2 Y_{\mathrm{qi}}\rangle + \langle\delta^2 Y_{\mathrm{ci}}\rangle), \quad (4)$$

and

$$\begin{aligned}&\langle\delta^2 X_+\rangle + \langle\delta^2 Y_-\rangle \\ = \ & \mathrm{e}^{-2r}(\langle\delta^2 X_{\mathrm{qs}}\rangle + \langle\delta^2 X_{\mathrm{cs}}\rangle + \langle\delta^2 X_{\mathrm{qi}}\rangle + \langle\delta^2 X_{\mathrm{ci}}\rangle \\ &+ \langle\delta^2 Y_{\mathrm{qs}}\rangle + \langle\delta^2 Y_{\mathrm{cs}}\rangle + \langle\delta^2 Y_{\mathrm{qi}}\rangle + \langle\delta^2 Y_{\mathrm{ci}}\rangle). \quad (5)\end{aligned}$$

For the quantum noise, we take $\langle\delta^2 X_{\mathrm{ql}}\rangle = \langle\delta^2 Y_{\mathrm{ql}}\rangle = 1/2$ to be the normalized QNL.[18] For the extra coherent noise, we take $\langle\delta^2 X_{\mathrm{cl}}\rangle = \langle\delta^2 Y_{\mathrm{cl}}\rangle = c^2$, where c is a value depending on the magnitude of extra classical noise. If there is no excess classical noise in the injected seed beams ($c = 0$), Eq. (5) will become

$$\langle\delta^2 X_+\rangle + \langle\delta^2 Y_-\rangle = 2\,\mathrm{e}^{-2r}. \quad (6)$$

In the case of $r = 0$ without any correlation we have $\langle\delta^2 X_+\rangle + \langle\delta^2 Y_-\rangle = 2$, that is, the borderline to identify quantum entanglement proposed in Refs. [8] and [9]. When the combined correlation variance on the left-hand side is smaller than 2 the two optical fields are partially entangled. If $r \to \infty$ Eq. (6) trends to zero and the perfect entanglement is reached.

If there are some excess classical noises in the seed beams, the combined quadrature correlation variance equals

$$\langle\delta^2 X_+\rangle + \langle\delta^2 Y_-\rangle = 2\,\mathrm{e}^{-2r}(1 + 2c^2). \quad (7)$$

It means that the entanglement between the output beams will be influenced by the noises of input seed beams and the correlation degree of the output entangled optical fields decreases rapidly with extra classical noises increasing.

3. Experimental setup and results

The experimental setup is shown in Fig. 2. The pump field of 540 nm wavelength and the injected seed beams of 1080 nm wavelength are produced from a home-made continuous wave (CW) frequency-doubled and frequency-stabilized Nd:YAP/KTP laser (Nd:YAP, Nd-doped YAlO3 perovskite, KTP, potassium titanyl phosphate) (YuGuang Company F-IVB)[19] and then are divided with a mirror coated with high reflectivity (HR) for 1080 nm and high transmission (HT) for 540 nm. The 540 nm laser directly pumps the NOPA. The light beam of 1080 nm is sent to a mode cleaner for filtering the additional noises and improving the space mode matching efficiency with the NOPA. Then the light beam at 1080 nm is divided into two parts of orthogonal polarizations with a polarizing-beam-splitter (PBS1), i.e., the injected signal and idler beams. The signal (idler) beam passes through an amplitude modulator AM1(2) and a phase modulator PM1(2) which are connected with a noise signal generator (it is not drawn in the figure) for adding excess noises according to the requirement of the experiment. The half wave plates HWP1 and HWP2 are used to align the polarization directions of light beams for achieving the type-II phase matching in the nonlinear KTP crystal. The NOPA cavity consists of a 10-mm long type-II KTP crystal and a concave mirror with a 50 mm radius of curvature, which is mounted upon a piezoelectric transducer (PZT) to lock the cavity length. One end face of the crystal is coated with HR at 1080 nm and HT at 540 nm for being used as an input mirror of the NOPA, the other end face is coated with anti-reflection at both 1080 nm and 540 nm. The output fields are extracted from a concave mirror with HR for 540 nm and a transmission of 3.2% for 1080 nm. The output signal and the idler beams are separated by PBS3. A 50/50 beamsplitter (BS3), detectors D1 and D2 compose a detection system to measure the correlation variances. Using a PZT mounted on mirror M1, we can lock the relative phase between the pump field and the injected signal to $2n\pi$ or $(2n+1)\pi$ to enforce the NOPA operating in parametric amplification or deamplification conditions, respectively.[4,15] The entangled states with the quadrature-amplitude correlation and the quadrature-phase anticorrelation ($\langle\delta^2 X_-\rangle + \langle\delta^2 Y_+\rangle < 2$), and the entangled states with the quadrature-amplitude anticorrelation and the quadrature-phase correlation ($\langle\delta^2 X_+\rangle + \langle\delta^2 Y_-\rangle < 2$), can be produced from a NOPA operating at amplification and deamplification, respectively.[4,15,20] In the experiment, we locked the NOPA to operate at deamplification, so the correlation variances of the amplitude sum $\langle\delta^2 X_+\rangle$ and the phase difference $\langle\delta^2 Y_-\rangle$ can be simultaneously measured with a Bell-state direct detector consisting of BS3, D1 and D2.[16] The phase difference of two beams on BS3 was aligned to $\pi/2$ using a PZT mounted on mirror M2 to satisfy the requirement of the direct Bell-state measurement.[16] The detected photocurrents by D1 and D2 were combined by a positive (negative) power combiner (+/−) to obtain the amplitude sum (phase difference) photocurrents, of which the noise spectra were measured by a spectrum analyzer (SA). During the experiment, the powers of the pump laser and the injected signal beam were kept at 90 mW (below the oscillation threshold power of 180 mW) and 6.3 mW, respectively. In this condition the output entangled state with a power of 31 μW was obtained.

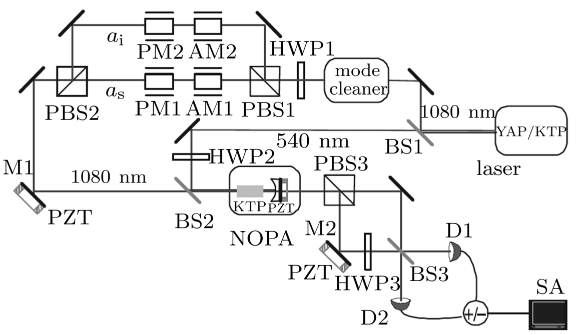

Fig. 2. Schematic of experimental setup: Nd:YAP/KTP, laser source; AM1-2, amplitude modulator; PM1-2, phase modulator; HWP1-3, $\lambda/2$ wave plate; PBS, polarizing beam splitter; BS1, dichroic mirror of HR for 1080 nm and HT for 540 nm; BS2, dichroic mirror of HR for 540 nm and HT for 1080 nm; BS3, 50/50 beam splitter; M1-2, high-reflection mirror; PZT, piezoelectric transducer; D1-2, ETX500 InGaAs photodiode detectors; +/−, positive/negative power combiner; SA, spectrum analyzer.

First, we used a laser beam in a coherent state as an injected signal of the NOPA and blocked the pump field to produce an output coherent state with the same power of the generated entangled states (31 μW), which would form the QNL for the measurement of variances.[16] For confirming the QNL we also checked the standard with a real white light.[21] Then with the existence of the pump laser we measured the correlation variances of the amplitude sum (Fig. 3) and the phase difference (Fig. 4) at 3 MHz, both of which are 3.1 ± 0.2 dB below the corresponding QNL. In Figs. 3 and 4 the trace i represents the measured QNL and trace ii denotes the measured correlation variances. To study the influence of the excess classical noises on the correlation variance, we added some classical noises on the amplitude quadratures and the phase quadratures of the injected signal and idler beams with AM1(2) and PM1(2) modulators, respectively. The measured results are shown in Fig. 5. Symbol ■ ($\star$) denotes the measured correlation variance of the amplitude (phase) quadrature of the entangled signal and idler output fields from the NOPA with a pump of 90 mW, which is also a function of the magnitude of the excess classical noise (c). The trace i denotes the QNL and the trace ii represents the theoretical fit curve according to Eq. (7) with $r = 0.36$ corresponding to the measured correlation of 3.1 dB below the QNL (see Figs. 3 and 4). The experimentally measured values and the theoretical curves from Eq. (7) are in very good agreement. It means that the simple theoretical model can fit the experimental phenomenon quite well. It is obvious that the correlation variance increases with the increase of the excess classical noise (c). When the magnitude of c is smaller than 0.72, the correlation variances of the amplitude quadrature and the phase quadrature, $\langle\delta^2(X_s^{out} + X_i^{out})\rangle$ and $\langle\delta^2(Y_s^{out} - Y_i^{out})\rangle$, are both below the QNL, and the entanglement criterion $\langle\delta^2(X_s^{out} + X_i^{out})\rangle + \langle\delta^2(Y_s^{out} - Y_i^{out})\rangle < 2$ is satisfied. In this case we can confirm that there is quantum entanglement between the output signal and idler optical fields from the NOPA. However, when $c > 0.72$, the correlation variances are larger than the QNL, where the quantum correlations are covered by the large excess classical noises.

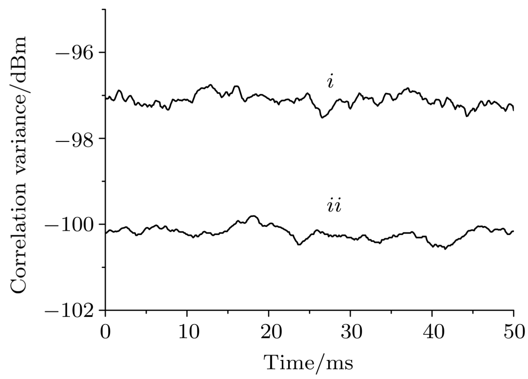

Fig. 3. Measured noise powers of the sum of amplitude quadratures of EPR beams at 3.0 MHz. Trace i, SNL; Trace ii, the correlation variance. The measurement parameters of SA are a resolution band width of 10 kHz and a video band width of 30 Hz.

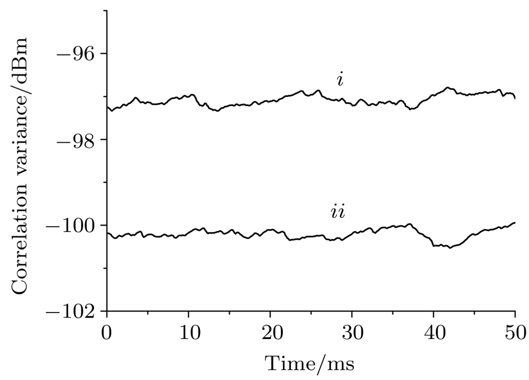

Fig. 4. Measured noise powers of the difference of phase quadratures of EPR beams at 3.0 MHz. Trace i, SNL; Trace ii, the correlation variance. The measurement parameters of SA are a resolution band width is 10 kHz and a video band width of 30 Hz.

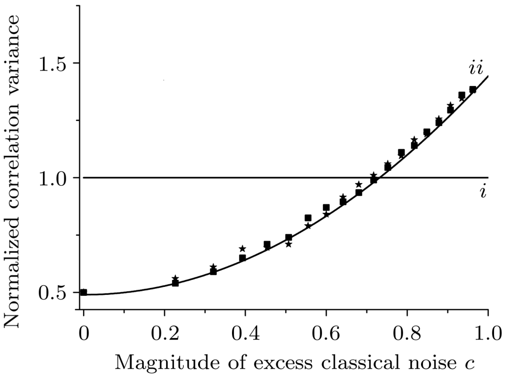

Fig. 5. Correlation variances versus magnitude of excess classical noise in input seed beams. Trace i denotes the QNL; trace ii represents the calculated function of the correlation variance vs the magnitude of the excess noise attached on the injected seed beams. Symbols ■ and ⋆ denote the measured amplitude and phase correlation variances.

4. Conclusion

The theoretical calculations and the experimental results show that the entanglement degree is influenced by the excess noise of input seed beam. When the magnitude of excess noise c is smaller than 0.72, the correlation variances of the amplitude quadrature and the phase quadrature are both below the QNL; when $c > 0.72$ in our system, the correlation variances are larger than the QNL, which means the quantum correlation between amplitude quadrature and phase quadrature of optical fields does not exist. We can see that the existence of the extra classical noises in the seed beam has a serious influence on the generation of CV EPR entangled optical fields, so we need to reduce these influences. Besides, it should be emphasized that the QNL needs to be scaled with a real coherent state of light without any excess classical noise in the homodyne detection system, otherwise the measured quantum entanglement will not be correct. Fortunately, the mode cleaners with high finesse have been used in such systems to filter the extra classical noises in the laser beams and the ideal coherent light beams have been available.

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