Coherence Phenomena in the Phase-Sensitive Optical Parametric Amplification inside a Cavity

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We theoretically and experimentally demonstrate coherence phenomena in optical parametric amplification inside a cavity. The mode splitting in the transmission spectra of a phase-sensitive optical parametric amplifier is observed. Especially, we show that a very narrow dip and peak, which are the shape of a δ function, appear in the transmission profile. The origin of the coherence phenomenon in this system is the interference between the harmonic pump field and the subharmonic seed field in cooperation with dissipation of the cavity.

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Introduction.—Coherence and interference effects play very important roles in determining the optical properties of quantum systems. Electromagnetically induced transparency (EIT) [1] in quantum-mechanical atomic systems is a well understood and thoroughly studied subject. EIT has been utilized in a variety of applications, such as lasing without inversion [2], slow and stored light [3,4], enhanced nonlinear optics [5], and quantum computation and communication [6]. Relying on destructive quantum interference, EIT is a phenomenon where the absorption of a probe laser field resonant with an atomic transition is reduced or even eliminated by the application of a strongly driving laser to an adjacent transition. Since EIT results from destructive quantum interference, it has been recently recognized that similar coherence and interference effects also occur in classical systems, such as plasma [7], coupled optical resonators [8], and mechanical or electric oscillators [9]. In particular, the phenomenology of the EIT and dynamic Stark effect is studied theoretically in a dissipative system composed by two coupled oscillators under linear and parametric amplification using the quantum optics model in Ref. [10]. The classical analog of EIT is not only helpful to understand deeply the physical meaning of the EIT phenomenon but also offers a number of itself important applications, such as slow and stored light by coupled optical resonators [11].

In this Letter, we extend the model in Ref. [10] and present a new system—a phase-sensitive optical parametric amplifier (OPA) to demonstrate coherence effects theoretically and experimentally. We observe mode splitting in the transmission spectra of the OPA. Especially, we show that a very narrow dip and peak, which are the shape of a δ function, appear in the transmission profile. This phenomenon results from the interference between the harmonic pump field and the subharmonic seed field in OPA. The destructive and constructive interference correspond to an optical parametric deamplifier and amplifier, respectively, which are in cooperation with dissipation of the cavity. The absorptive and dispersive responses of an optical cavity for the probe field are changed by optical parametric interaction in the cavity. The phase-sensitive optical parametric amplifier presents a number of new characteristics of coherence effects.

Theoretical model.—Consider the interaction of two optical fields of frequencies ω and 2ω, denoted by subharmonic and harmonic wave (the pump), which are coupled by a second-order, type-I nonlinear crystal in an optical cavity as shown in Fig. 1. The cavity is assumed to be a standing-wave cavity and only resonant for the subharmonic field with dual-port of transmission $T_{\rm HR}$ and T_c, internal losses A, and length L (round-trip time $\tau = 2L/c$). We consider both the subharmonic seed beam $a^{\rm in}$ and harmonic pump beam $\beta^{\rm in}$, which are injected into the back port ($T_{\rm HR}$ mirror) of the cavity, where the relative phase between them is adjusted by a movable mirror outside the cavity. The $T_{\rm HR}$ mirror is a high reflectivity mirror at the subharmonic wavelength, yet has a high transmission coefficient at the harmonic wavelength, and the T_c mirror has a high reflectivity coefficient for the harmonic wave. The harmonic wave makes a double pass through the nonlinear medium. The equation of motion for the mean value of the subharmonic intracavity field can then be derived by the semiclassical method [12] as

$$\tau \frac{da}{dt} = -i\tau\Delta a - \gamma a + g\beta^{\rm in} a^* + \sqrt{2\gamma_{\rm in}} a^{\rm in}. \qquad (1)$$

The decay rate for internal losses is $\gamma_l = A/2$, and the damping associated with the coupling mirror and the back

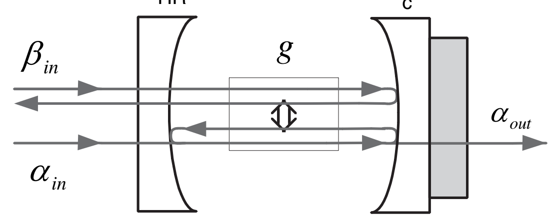

FIG. 1. Schematic of an optical parametric amplifier in a standing-wave cavity.

mirror is $\gamma_c = T_c/2$ and $\gamma_{\rm in} = T_{\rm HR}/2$, respectively. The total damping is denoted by $\gamma = \gamma_{\rm in} + \gamma_c + \gamma_l$. Δ is the detuning between the cavity-resonance frequency ω_c and the subharmonic field frequency ω. The strength of the interaction is characterized by the nonlinear coupling parameter g. Equation (1) is complemented with the boundary conditions $a^{\rm out} = \sqrt{2\gamma_c}a$ and $a^{\rm ref} = -a^{\rm in} + \sqrt{2\gamma_{\rm in}}a$ to create propagating beams, where $a^{\rm out}$ is the transmitted field from the coupling mirror T_c and $a^{\rm ref}$ is the reflected field from the back mirror $T_{\rm HR}$. The phase-sensitive optical parametric amplifier always operates below the threshold of optical parametric oscillation $\beta_{\rm th}^{\rm in} = \gamma/g$. Equation (1) ignores the third-order term [13] describing the conversion losses due to harmonic generation. For simplicity, we assume that the phase of the pump field is zero in any case; i.e., $\beta^{\rm in}$ is a real and positive value. The intracavity field a and the injected field $a^{\rm in}$ are expressed as $a = \alpha\exp(-i\phi)$ and $a^{\rm in} = A_{\rm in}\exp(-i\varphi)$, respectively. Here α and $A_{\rm in}$ are real values; ϕ and φ are the relative phase between the intracavity field and the pump field as well as between the seed field and the pump field, respectively. If the harmonic pump is turned off, the throughput for the nonimpedance matched subharmonic seed beam is given by $a^{\rm out}_{\rm no\ pump} = 2\sqrt{\gamma_c\gamma_{\rm in}}A_{\rm in}/(\gamma + i\tau\Delta)$. The subharmonic seed beam is subjected to either amplification or deamplification, depending on the chosen relative phase between the subharmonic field and the pump field.

Case 1.—Consider the transmitted intensity of the subharmonic seed beam as a function of the detuning Δ between the subharmonic field frequency and the cavity-resonance frequency and keep the pump field of frequency $\omega_p = 2\omega$ constant. Setting the derivative to zero ($d\alpha/dt = 0$) and separating the real and imaginary parts of Eq. (1), the steady state solutions of the amplitude and relative phase of the intracavity field are given by

$$\begin{aligned} -\gamma\alpha + g\beta^{\rm in}\alpha\cos 2\phi + \sqrt{2\gamma_{\rm in}}A_{\rm in}\cos(\phi - \varphi) = 0, \\ -\tau\Delta\alpha + g\beta^{\rm in}\alpha\sin 2\phi + \sqrt{2\gamma_{\rm in}}A_{\rm in}\sin(\phi - \varphi) = 0. \end{aligned} \tag{2}$$

When the amplitude and relative phase of the subharmonic seed beam are given, the transmitted intensity of the subharmonic beam is obtained from Eq. (2) and the boundary condition. Figure 2(a) shows a Lorentzian profile of the subharmonic transmission when the pump field is absent. This corresponds to the typical transmitted spectrum of the optical empty cavity. When the injected subharmonic field is out of phase ($\varphi = \pi/2$) with the pump field, the subharmonic transmission profile is shown in Figs. 2(b)–2(d) for different pump powers, in which there is a symmetric mode splitting. The transmitted power of the subharmonic beam is normalized to the power in the absence of the pump and zero detuning. The transmission spectra show that the dip becomes deeper and two peaks higher as the pump intensity increases. The origin of mode splitting in the transmission spectra of OPA is destructive interference in cooperation with dissipation of the cavity. If the subharmonic field resonates in the cavity perfectly, i.e., $\Delta = 0$, the subharmonic intracavity field and the pump field are exactly out of phase and will interfere destructively to produce the deamplification for the subharmonic field in the nonlinear crystal. Thus, a dip appears at the zero detuning of the transmission profile. If the subharmonic field is not quite resonant in the cavity perfectly, that is, the subharmonic field's frequency is not exactly an integer multiple of the free spectral range (but close enough to build up a standing wave), the phase difference between the subharmonic intracavity field and the pump field will not be exactly out of phase and will increase as the detuning increases. The subharmonic intracavity field will change from deamplification to amplification as the phase difference increases. Thus, we see that the transmission profile has two symmetric peaks at two detuning frequencies. When the phase of the injected subharmonic field is deviated from out of phase with the pump field, i.e., $\varphi = \pi/2 \pm \theta$, an asymmetric mode splitting in the subharmonic transmission profile is illustrated in Figs. 2(e) and 2(f), in which the dip is deviated from the zero detuning of the transmission profile and two peaks have different amplitude.

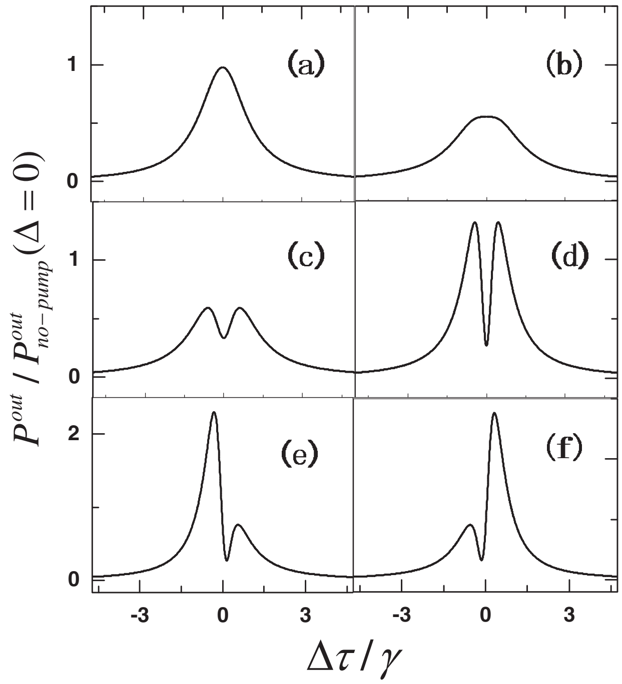

FIG. 2. The theoretical results for case 1: (a) without the pump field injection; (b) $\varphi = \pi/2$ and $\beta^{\rm in}/\beta^{\rm in}_{\rm th} = 0.33$; (c) $\varphi = \pi/2$ and $\beta^{\rm in}/\beta^{\rm in}_{\rm th} = 0.71$; (d) $\varphi = \pi/2$ and $\beta^{\rm in}/\beta^{\rm in}_{\rm th} = 0.9$; (e) $\varphi = \pi/2 - 0.07$ and $\beta^{\rm in}/\beta^{\rm in}_{\rm th} = 0.9$; (f) $\varphi = \pi/2 + 0.07$ and $\beta^{\rm in}/\beta^{\rm in}_{\rm th} = 0.9$.

Case 2.—Consider the subharmonic transmission profiles when the frequency of the pump field is fixed at $\omega_p = 2(\omega_c + \Omega)$. When scanning the frequency of the subharmonic seed beam, an idler field in the OPA cavity will be generated with the frequency $\omega_i = \omega_p - \omega$ due to energy conservation. The equation of motion of OPA becomes frequency-nondegenerate and is given by

$$\tau \frac{da}{dt} = -i\tau\Delta a - \gamma a + g\beta^{\rm in} a_i^* + \sqrt{2\gamma_{\rm in}} a^{\rm in},$$
$$\tau \frac{da_i}{dt} = -i\tau\Delta_i a_i - \gamma a_i + g\beta^{\rm in} a^*, \tag{3}$$

where a_i is the idler field in the OPA cavity. Δ_i is the detuning between the cavity-resonance frequency ω_c and the idler field frequency ω_i. Thus, the subharmonic transmission profile in this case is obtained from Eq. (3) for $\omega \neq \omega_i$ and Eq. (1) for $\omega = \omega_i$. When $\Omega = 0$, so $\Delta = -\Delta_i$, the stationary solution of the subharmonic and idle field is given by solving the mean-field equations of Eq. (3) and using the input-output formalisms. We obtain

$$A^{\rm out} = \frac{2\sqrt{\gamma_c \gamma_{\rm in}}}{i\tau\Delta + \gamma - \frac{(g\beta^{\rm in})^2}{i\tau\Delta + \gamma}} A^{\rm in},$$
$$A_i^{\rm out} = \frac{2\sqrt{\gamma_c \gamma_{\rm in}} g\beta^{\rm in}}{(-i\tau\Delta + \gamma)^2 - (g\beta^{\rm in})^2} A^{\rm in*}. \tag{4}$$

We will record the total output power including the subharmonic and idle field. The transmitted power of the subharmonic beam is given by

$$P_{\rm out}^{\rm nor} = \begin{cases} \left|\frac{\gamma}{i\tau\Delta + \gamma - \frac{(g\beta^{\rm in})^2}{i\tau\Delta + \gamma}}\right|^2 + \left|\frac{\gamma g\beta^{\rm in}}{(-i\tau\Delta + \gamma)^2 - (g\beta^{\rm in})^2}\right|^2 & \text{if } \omega \neq \omega_i, \\ \frac{\gamma^2}{(\gamma \pm g\beta^{\rm in})^2} & \text{if } \omega = \omega_i. \end{cases} \tag{5}$$

Here $\pm$ corresponds to the deamplifier and amplifier in frequency-degenerate OPA. Figures 3(a) and 3(b) show that the very narrow dip and peak, which are the shape of a δ function, appear in the transmission profile. This novel coherence phenomenon results in that the destructive and constructive interference are established only in the point of $\omega = \omega_i$ and completely destroyed in the other frequencies.

Experiment.—The experimental setup is shown schematically in Fig. 4. A diode-pumped intracavity frequency-doubled continuous-wave ring $Nd{:}YVO_4/KTP$ single-frequency green laser serves as the light source of the pump wave (the second-harmonic wave at 532 nm) and the seed wave (the fundamental wave at 1064 nm) for OPA. The green beam doubly passes the acousto-optic modulator (AOM) to shift the frequency 440 MHz. The infrared beam doubly passes AOM to shift the frequency around 220 MHz. We actively control the relative phase between the subharmonic and the pump field by adjusting the phase of the subharmonic beam with a mirror mounted upon a piezoelectric transducer (PZT). Both beams are combined together by a dichroic mirror and injected into the OPA cavity. OPA consists of a periodically poled $KTiOPO_4$ (PPKTP) crystal (12 mm long) and two external mirrors separated by 63 mm. Both end faces of the crystal are polished and coated with an antireflector for both wavelengths. The crystal is mounted in a copper block, whose temperature is actively controlled at the millidegrees kelvin level around the temperature for an optical parametric process (31.3 °C). The input coupler M1 is a 30 mm radius-of-curvature mirror with a power reflectivity 99.8% for 1064 nm in the concave and a total transmissivity 70% for 532 nm, which is mounted upon a PZT to adjust the cavity length. The output wave is extracted from M2, which is a 30 mm radius-of-curvature mirror with a total transmissivity 3.3% for 1064 nm and a reflectivity 99% for 532 nm in the concave. Because of the large transmission of the input coupler at 532 nm, the pump field only passes the cavity twice without resonation. The measured cavity finesse is 148 with the PPKTP crystal, which indicates the total cavity loss of 4.24%. Because of the high nonlinear coefficient of PPKTP, the measured threshold power is only 35 mW.

First, we fix the frequency of the subharmonic and the pump field with $\omega_p = 2\omega$ and scan cavity length, which corresponds to the condition of case 1. Figure 5 shows the experimental results: (a) without the pump field, (b) $\varphi = \pi/2$ and $\beta^{\rm in}/\beta^{\rm in}_{\rm th} = 0.33$, (c) $\varphi = \pi/2$ and $\beta^{\rm in}/\beta^{\rm in}_{\rm th} = 0.71$, (d) $\varphi = \pi/2$ and $\beta^{\rm in}/\beta^{\rm in}_{\rm th} = 0.9$,

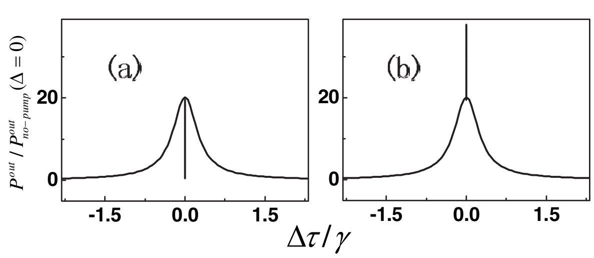

FIG. 3. The theoretical results for case 2: (a) $\varphi = \pi/2$ for deamplification; (b) $\varphi = 0$ for amplification.

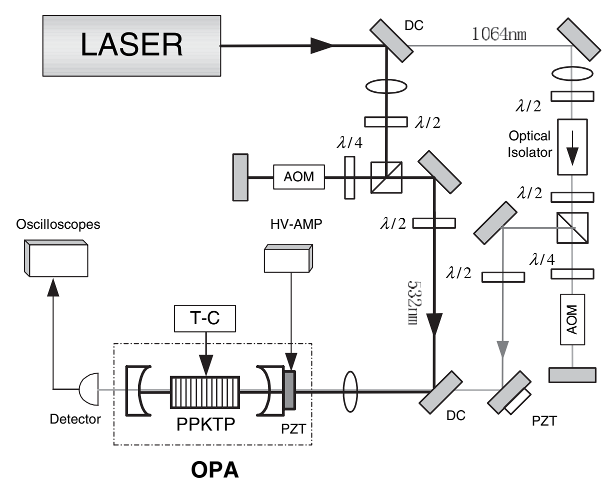

FIG. 4. Schematic of the experimental setup. DC: dichroic mirror; $\lambda/2$, half-wave plate; T-C, temperature controller, HV-AMP, high voltage amplifier.

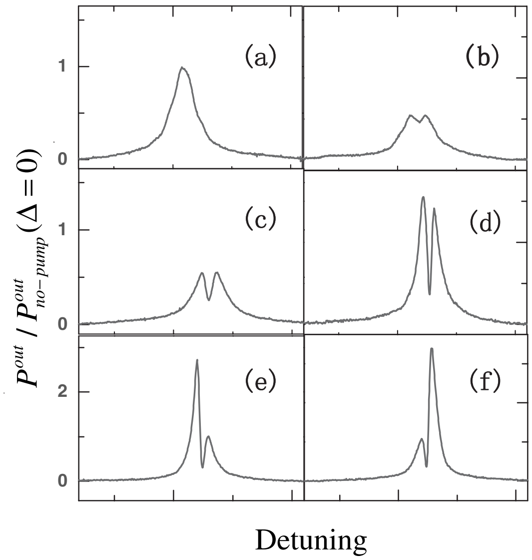

FIG. 5. The experimental results for case 1 corresponding to Fig. 2.

(e) $\varphi = \pi/2 - 0.07$ and $\beta^{\rm in}/\beta^{\rm in}_{\rm th} = 0.9$, (f) $\varphi = \pi/2 + 0.07$ and $\beta^{\rm in}/\beta^{\rm in}_{\rm th} = 0.9$. It can be seen that the experimental curves are in good agreement with the theoretical results shown in Fig. 2, which are obtained with the experimental parameters.

Then we fix the cavity length and frequency of the pump field and scan the frequency of the subharmonic field by the AOM, which corresponds to the condition of case 2. The output including the subharmonic and idle field is detected by a photodiode. There is a beat-note signal in the photocurrent with frequency proportional to the detuning. The very narrow dip and peak that appeared in a broad Lorentzian profile are observed experimentally as shown in Fig. 6. The insets in Fig. 6 show the enlarged narrow dip and peak by reducing the scanned range of frequency, which present the square shape. Because the measurement of a transmission profile is a dynamic process, the shape of a δ function for the narrow dip and peak in the theoretical model becomes a square shape in the experiment. The width of the square shape is $\sim$2 kHz, which is estimated from the voltage on the voltage-controlled oscillator of the AOM.

Conclusion.—We reported the theoretical and experimental results of coherence phenomena in the phase-sensitive optical parametric amplification inside a cavity. Splitting in the transmission spectra of OPA was observed. Mode splitting, as well known, occurs not only in a coupled quantum system but also in coupled optical resonators and in coupled mechanical and electronic oscillators. To the best of our knowledge, we are the first to observe mode splitting experimentally in the optical parametric process. This system will be important for practical optical and photonic applications, such as optical filters and delay lines, and closely relates to the coherent phenomenon of EIT predicted for quantum systems. OPA also has an important application as a squeezed light source. Our results may help us to investigate the quantum noise spectrum.

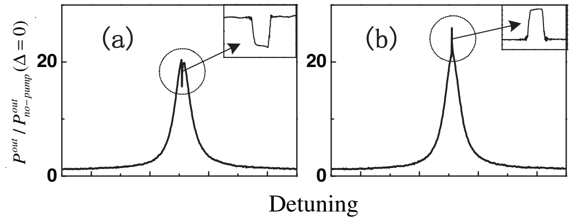

FIG. 6. The experimental results for case 2 corresponding to Fig. 3.

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