Measurement of intensity difference squeezing via non-degenerate four-wave mixing process in an atomic vapor

Yu Xu-Dong, Meng Zeng-Ming, and Zhang Jing

State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan 030006, China

(Received 31 December 2012; revised manuscript received 23 February 2013)

We report the measurement of the intensity difference squeezing via the non-degenerate four-wave mixing (FWM) process in a rubidium atomic vapor medium. Two pairs of balanced detection systems are employed to measure the probe and the conjugate beams, respectively. It is convenient to get the quantum shot noise limit, the squeezed and the amplified noise power spectra. We also investigate the influence of the input extra quadrature amplitude noise of the probe beam. The influence of the extra noise can be minimized and the squeezing can be optimized under the proper parameter condition. We measure the $-3.7$-dB intensity difference squeezing when the probe beam has a $3$-dB extra quadrature amplitude noise. This result is slightly smaller than $-4.1$ dB when the ideal coherent light (no extra noise) for the probe beam is used.

Keywords: four-wave mixing process, intensity difference squeezing, self-balanced detection

PACS: 42.50.-p, 42.50.Dv, 42.65.–k

DOI: 10.1088/1674-1056/22/9/094204

1. Introduction

Squeezed and entangled states of optical fields are important resources for quantum information processing, particularly with continuous variables. The generation of the squeezed state based on the four-wave mixing (FWM) process was proposed three decades ago and was used to demonstrate the squeezed state of the light for the first time experimentally. Recently, it attracts many attentions because a non-degenerate FWM process in atomic vapor can generate a high degree of quantum correlation between bright beams. This method shows a significant advantage compared with the optical parametric oscillator consisted of nonlinear crystal and a cavity, since it is very simple and does not require an optical cavity to enhance the nonlinearity. Several interesting works were performed, such as entangled images, the slow light, and the delay of EPR entanglement.

In this paper, we study the non-degenerate four-wave mixing process in a rubidium atomic vapor medium theoretically and experimentally. The main results are summarized as follows: i) An ideal FWM can be regarded as a linear phase insensitive amplifier. Ideally, the system is described by a single parameter, the parametric gain $G$. The input probe beam (power $P_0$) is amplified to produce an output probe beam with power $GP_0$, while the output conjugate beam (seeded only by vacuum) has power $(G-1)P_0$. The noise on the intensity difference between the probe and conjugate beams is $1/(2G-1)$.

Here, we show that it is not an optimized measurement scheme to obtain the maximum squeezing. A classically electric gain $g$ is introduced on the photocurrent of one of arms (the probe or conjugate beams). The maximum squeezing is obtained with $1/(2G-1+2\sqrt{G(G-1)})$ by optimizing electric gain. ii) In the experiments, the bright probe and conjugate beams are detected by two photodiodes, respectively. In order to obtain the shot noise level (SNL) (also called the standard quantum limit), two coherent laser beams need to be sent to two photodiodes, whose power are required to be equivalent to the probe and conjugate beams respectively. Thus, the SNL must often be calibrated when the powers of the probe and conjugate beams are changed. In this paper, the intensity difference squeezing is measured by the two pairs of self-balanced detection systems, which are employed to measure the bright probe and the conjugate beams, respectively. It is convenient to measure the SNL, intensity difference squeezing, and intensity sum (anti-squeezing component). iii) We study the influence of the extra noise of the seed probe beam on the intensity difference squeezing. By balancing the two losses and the gain in the FWM and adjusting the classical gain, we can minimize the influence of the extra noise and maximize the squeezing.

2. Theoretical analysis

2.1. Optimized measurement scheme

The quantum states we consider in this paper are described with the electromagnetic field annihilation operator \(\hat{a} = (\hat{X} + i\hat{Y})/2\), which is expressed in terms of the quadrature amplitude $\hat{X}$ and phase $\hat{Y}$ with the canonical commutation relations.

\[ \{\hat{X}, \hat{Y}\} = 0, \quad \{\hat{X}, \hat{X}^{\dagger}\} = \{\hat{Y}, \hat{Y}^{\dagger}\} = 1, \quad \{\hat{X}^{\dagger}, \hat{Y}^{\dagger}\} = 0.\]
The FWM process can be regarded as a nondegenerate optical parametric amplifier, so the evolution equations can be written in the form\[17,18\]

\[
\begin{align*}
\hat{a}_{\text{out}} &= \sqrt{G} \hat{a}_{\text{in}} + \sqrt{G - 1} \hat{b}_{\text{in}}^\dagger, \\
\hat{b}_{\text{out}} &= \sqrt{G - 1} \hat{a}_{\text{in}}^\dagger + \sqrt{G} \hat{b}_{\text{in}},
\end{align*}
\]  
(1)

where \(\hat{a}_{\text{in}}(\hat{a}_{\text{in}}^\dagger)\) and \(\hat{b}_{\text{in}}(\hat{b}_{\text{in}}^\dagger)\) are the annihilation (creation) operators of the signal and idler input fields, \(\hat{a}_{\text{out}}\) and \(\hat{b}_{\text{out}}\) are the annihilation operators of the output fields, \(G = \cosh^2(r)\) is the gain of FWM process, \(r\) is the squeezing parameter. The operators of the bright field can be expressed as \(\hat{a} = \alpha + \delta \hat{a}\), where \(\alpha\) is the mean value of the field and \(\delta \hat{a}\) is the quantum fluctuation. The corresponding fluctuations of the quadrature amplitude and the quadrature phase are \(\delta \hat{X} = \delta \hat{a}(\Omega) + \delta \hat{a}^\dagger(\Omega)\) and \(\delta \hat{Y} = -i[\delta \hat{a}(\Omega) - \delta \hat{a}^\dagger(\Omega)]\) according to the Fourier transformation \(\delta \hat{a}(\Omega) = (1/\sqrt{2\pi}) \int \delta \hat{a}(t) e^{-i\Omega t} dt\), where \(\Omega\) is the analysis frequency of the sidebands. Here, the signal input field is the bright field with \(\alpha_{\text{in}}\) and the idler input field is the vacuum field with \(b_{\text{in}} = 0\) and \(V(\delta \hat{X}_{\text{in}}) = V(\delta \hat{Y}_{\text{in}}) = 1\).

First, we present the theoretical analysis of the measurement scheme. The two output fields (the probe beam and the conjugate beam) are measured by two pairs of the self-balanced detection systems. The difference and the sum of the photocurrent fluctuation of each pair of the self-balanced detection system can be written as

\[
\begin{align*}
\hat{c}_{\text{in}} &= a \delta \hat{Y}_{\text{in}}, \\
\hat{d}_{\text{in}} &= b \delta \hat{X}_{\text{in}},
\end{align*}
\]

(3)

where \(\delta \hat{Y}_{\text{in}}\) and \(\delta \hat{X}_{\text{in}}\) are the quadrature phase operators of the input vacuum fields \(V(\delta \hat{Y}_{\text{in}}) = V(\delta \hat{X}_{\text{in}}) = 1\) from the empty ports of the 50/50 beam splitters of the detection systems, \(\delta \hat{Y}_{\text{out}}\) and \(\delta \hat{X}_{\text{out}}\) are the quadrature amplitude operators of the output fields, \(a = \sqrt{G} \alpha_{\text{in}}\) and \(b = \sqrt{G - 1} \alpha_{\text{in}}\) are the mean value of the output fields of the FWM. A classically electric gain \(g\) is introduced on the photocurrent \(\hat{c}_{\text{in}}\), which can be adjusted precisely in the experiment. Hence, the normalized variances of the differences and the sums of \(\hat{c}_{\text{in}}\) and \(\hat{d}_{\text{in}}\) are

\[
\begin{align*}
V(\hat{c}_{\text{in}} - g\hat{d}_{\text{in}}) &= V(\hat{c}_{\text{in}} + \hat{d}_{\text{in}}), \\
&= \frac{a^2}{a^2 + (gb)^2} V(\delta \hat{Y}_{\text{in}}) + \frac{(gb)^2}{a^2 + (gb)^2} V(\delta \hat{X}_{\text{in}}) = 1, \\
&\text{(5)}
\end{align*}
\]

\[
\begin{align*}
V(\hat{c}_{\text{in}} + \hat{d}_{\text{in}}) &= V(\hat{c}_{\text{in}} - g\hat{d}_{\text{in}}) + \frac{g^2}{a^2 + (gb)^2} V(\delta \hat{X}_{\text{out}}) + \frac{gb}{a^2 + (gb)^2} V(\delta \hat{Y}_{\text{out}}), \\
&\text{(6)}
\]

\[
\begin{align*}
&= \frac{[G - g(G - 1)]^2}{G + (G - 1)g^2} V(\delta \hat{X}_{\text{in}}) + \frac{G(G - 1)(1 + g)^2}{G + (G - 1)g^2} V(\delta \hat{Y}_{\text{in}}) + \frac{g^2}{G + (G - 1)g^2} V(\delta \hat{X}_{\text{out}}) + \frac{2gb}{G + (G - 1)g^2} V(\delta \hat{Y}_{\text{out}}),
\end{align*}
\]

\[
\begin{align*}
&= \frac{[G + g(G - 1)]^2}{G + (G - 1)g^2} V(\delta \hat{X}_{\text{in}}) + \frac{G(G - 1)(1 - g)^2}{G + (G - 1)g^2} V(\delta \hat{Y}_{\text{in}}) + \frac{g^2}{G + (G - 1)g^2} V(\delta \hat{X}_{\text{out}}) + \frac{2gb}{G + (G - 1)g^2} V(\delta \hat{Y}_{\text{out}}).
\end{align*}
\]

\[
\text{(7)}
\]

The variance of the difference or the sum of \(\hat{c}_{\text{in}}\) and \(\hat{d}_{\text{in}}\) (Eq. (5)) corresponds to the SNL. The variance of the difference between \(\hat{c}_{\text{in}}\) and \(\hat{d}_{\text{in}}\) (Eq. (6)) corresponds to the squeezed intensity difference noise power spectrum. The sum of \(\hat{c}_{\text{in}}\) and \(\hat{d}_{\text{in}}\) (Eq. (7)) corresponds to the intensity sum noise spectrum.

Now, we consider that the signal input field is the bright coherent light \((V(\hat{X}_{\text{in}}) = V(\hat{Y}_{\text{in}}) = 1)\). When \(g = \sqrt{G/(G - 1)}\), the intensity difference squeezing (Eq. (6)) becomes

\[
V\left(\frac{1}{\sqrt{2}} (\delta \hat{X}_{\text{out}} - \delta \hat{Y}_{\text{out}})\right) = 2G - 1 - 2\sqrt{G/(G - 1)} = e^{-2r},
\]

and the anti-squeezing component (Eq. (7)) becomes

\[
V\left(\frac{1}{\sqrt{2}} (\delta \hat{X}_{\text{out}} + \delta \hat{Y}_{\text{out}})\right) = 2G - 1 + 2\sqrt{G/(G - 1)} = e^{2r}.
\]

When \(g = 1\), equation (6) becomes

\[
V(\hat{c}_{\text{in}} - \hat{d}_{\text{in}}) = \frac{1}{2G - 1},
\]

and equation (7) becomes

\[
V(\hat{c}_{\text{in}} + \hat{d}_{\text{in}}) = 2G - 1 + \frac{2G(G - 1)}{2G - 1}.
\]

We plot the noise figure with \(g = 1\) and \(g = \sqrt{G/(G - 1)}\), as shown in Fig. 1. One can see that the classical gain \(g = \sqrt{G/(G - 1)}\) may maximize the squeezing in the measurement dramatically.

![Fig. 1.](color online) Theoretical calculation of the noise figure via the gain of the FWM when the classically electric gain is \(g = 1\) (blank lines \(a\) and \(c\)) and \(g = \sqrt{G/(G - 1)}\) (red lines \(b\) and \(d\)) respectively. The orange solid line is the SNL defined as \(-10\log V(\hat{c}_{\text{in}} + \hat{d}_{\text{in}})\). The dotted lines correspond to the noise figure of the intensity sum (anti-squeezing component), the solid lines correspond to the noise figure of the intensity difference (squeezing component).
2.2. Influence of the extra noise

Generally, the extra noise is very harmful to the squeezing and quantum entanglement. Therefore, we consider the influence of the extra noise \( N_c \) of the input signal field \( V(\hat{X}_{in}) = 1 + N_c \). The squeezing will be reduced (see Eq. (6)) if \( N_c > 0 \). We can obtain the optimal gain \( g \) from Eq. (6), by which the maximum squeezing can be detected

\[
g_{\text{opt}} = N_c + \frac{\sqrt{N_c^2 + 4G(2 + N_c)^2(G) - 1}}{2(2 + N_c)(G - 1)}. \tag{12}
\]

As we can see, the optimal gain \( g_{\text{opt}} \) becomes \( \sqrt{G}/(G - 1) \) when \( N_c = 0 \). Figure 2 presents the result of minimizing the influence of the extra noise and maximizing the squeezing. The intensity difference fluctuation with \( g = 1 \) is higher than the SNL when the gain \( G < 2.5 \) because of the influence of the extra noise \( N_c = 3 \). However, the intensity difference fluctuation with the optimal electric gain always keeps squeezing for \( G > 1 \).

![Figure 2](image)

Fig. 2. (color online) The squeezing with the extra noise \( N_c = 3 \). The black dotted line \( a \) corresponds to \( g = 1 \), the black solid line \( b \) corresponds to Eq. (12), the red solid line \( c \) is the SNL. The red dotted line \( d \) corresponds to the ideal condition \( (N_c = 0 \) and \( g = \sqrt{G}/(G - 1) \)).

Now, we consider another way to reduce the influence of the extra noise and obtain the maximal squeezing in the detection. The FWM process in the atomic vapor accompanies the large losses. Recently, McCorick and Jasperse developed an analytic distributed gain/loss model to describe the competition of the mixing and the absorption. Here, we will follow this model and just consider the gain and absorption as the separated processes. The output signal and idler fields of the FWM pass a lossy medium respectively

\[
\hat{c} = \sqrt{\eta_a} \hat{a}_{out} + \sqrt{1 - \eta_a} \hat{b}^+_d, \tag{13}
\]
\[
\hat{d} = \sqrt{\eta_b} \hat{b}_{out} + \sqrt{1 - \eta_b} \hat{b}^+_d, \tag{14}
\]

where \( \eta_a \) and \( \eta_b \) are the intensity transmission coefficients of the two fields in the absorption stages, \( \hat{b}^+_d \) and \( \hat{b}^+_d \) are the creation operators of the introduced vacuum fields in the absorption medium. In experiment, the different absorption coefficients for the signal and idler fields can be controlled by setting the detuning of the pump light. Equations (6) and (7) can be written as

\[
V(\hat{c}_{i+} - \hat{d}_{i+}) = \frac{1}{f_1} \left[ (\eta_c - \eta_b)(G - 1) \right] \sqrt{\left( V(\hat{c}_{i+}) - \hat{d}_{i+} \right) \left( V(\hat{d}_{i+}) - \hat{c}_{i+} \right)}
\]
\[
\eta_c \eta_b \left( 1 + \eta_c \eta_b \right) \left( 1 - \eta_c \eta_b \right) \left( 1 + \eta_c \eta_b \right) \left( 1 - \eta_c \eta_b \right).
\]

where \( f_1 = \sqrt{(4G - 1)/(G - 1)} \).

![Figure 3](image)

Fig. 3. (color online) Theoretical calculation of the noise figure when the losses are considered. The dotted lines correspond to the noise figure of the intensity sum, the solid orange line \( d \) is SNL, the other solid lines correspond to the noise figure of the intensity difference. Green lines \( a \) and \( e \) correspond to \( N_c = 3 \) and \( \eta_a = \eta_b = 1 \), black lines \( b \) and \( f \) correspond to \( N_c = 3 \), \( \eta_a = 0.9 \) when equation (20) is satisfied; red lines \( c \) and \( g \) correspond to \( N_c = 0 \), \( \eta_b = 0.9 \) when equation (17) is satisfied.
If $N_e > 0$, the condition of the maximum squeezing becomes

$$\eta_a = \left[\sqrt{4 + \frac{6}{(N_e + 2)G - 2}} - 1\right] \frac{G - 1}{G} \eta_b. \quad (20)$$

Therefore, equations (18) and (19) can be expressed as

$$V(i_{a,-} - i_{b,-}) = \frac{1}{f_2} \left\{ \left[\sqrt{4 + \frac{6}{(N_e + 2)G - 2}} - 2\right] \times (G - 1)^2 (1 + N_e) \eta_b^2 \right. \right.$$

$$+ \left[ (G - 1) \sqrt{4 + \frac{6}{(N_e + 2)G - 2}} - 2G + 1 \right] \times \frac{G - 1}{G} \eta_b$$

$$- \left[ \sqrt{4 + \frac{6}{(N_e + 2)G - 2}} - 1 \right]^2 \eta_b^2 \right\}$$

$$\times \frac{(G - 1)^2}{G} \eta_b^2 + (G - 1) \eta_b (1 - \eta_b) \}, \quad (21)$$

$$V(i_{a,+} + i_{d,+}) = \frac{1}{f_2} \left\{ \left[\sqrt{4 + \frac{6}{(N_e + 2)G - 2}} - 2\right] \times (G - 1)^2 (1 + N_e) \eta_b^2 \right. \right.$$  

$$+ \left[ (G - 1) \sqrt{4 + \frac{6}{(N_e + 2)G - 2}} + 1 \right] \times \frac{G - 1}{G} \eta_b$$

$$- \left[ \sqrt{4 + \frac{6}{(N_e + 2)G - 2}} - 1 \right]^2 \times (G - 1) \eta_b$$

$$- \left[ \sqrt{4 + \frac{6}{(N_e + 2)G - 2}} - 1 \right]^2 \eta_b^2 \right\}$$

$$\times \frac{(G - 1)^2}{G} \eta_b^2 + (G - 1) \eta_b (1 - \eta_b) \}, \quad (22)$$

where

$$f_2 = \sqrt{4 + \frac{6}{(N_e + 2)G - 2}} (G - 1) \eta_b.$$

Figure 3 shows the noise figures of the theoretical calculation when the losses are involved. We can optimize the squeezing by adjusting the losses on the two output beams, which satisfies Eq. (17) ($N_e = 0$) or Eq. (20) ($N_e > 0$).

3. Experiment

The experimental setup is shown in Fig. 4. The output of the Ti:sapphire laser tuned to 795 nm is divided into two parts. One is used as the pump laser, the other as the probe laser is quadruple-passed through an 800-MHz AOM (Brimrose: TEF-800-200-795). The AOM driver consists of an amplifier (Mini-circuits model: ZFL-2-12) and the radio-frequency source (Agilent: N5183A). The pump and the probe laser cross at a small angle (4 mrad) in a naturally mixed rubidium vapor cell. The cell is 1.5-cm long and has a 2.5-cm diameter, the temperature of which is controlled by a heater. The waist of the pump is about 800 µm at the crossing point, and the probe is about 360 µm. The power of the pump before the cell is 510 mW, and the power of the probe is about 40 µW. The probe and the conjugate beams are measured by two pairs of the self-balanced detection systems, as shown in Fig. 4. We employ a variable electronic attenuator placed on the arm of the probe beam, which represents the classical gain discussed in the theoretical analysis.
We first measure the noise spectrum of the probe laser without the Rb cell. The probe light is measured by one of the two self-balanced detection systems (D1 and D2). The red curve $a$ in Fig. 5(a) is the noise power spectrum of the sum of the photocurrents of the two detectors, which corresponds to the quadrature amplitude noise of the probe laser. The black curve $b$ is the noise power spectrum of the difference of the photocurrents, which corresponds to the noise of the vacuum field coupled from the other port of the beam-splitter. The two noise curves coincide with each other in the analysis frequency regime, which demonstrate that the extra noise is not introduced into the signal input field by the AOM, which presents a good coherent light. We apply this probe laser to the experiment and set the power to 45 µW. We tune the pump laser to the transition $F = 2 \rightarrow F'$ of the $^{85}$Rb D1 line with about 800 MHz–1200 MHz. The detuning of the probe is about $-3$ GHz from the pump laser and the temperature of the cell is 148 °C.

The single photon detuning will change the position of the pump and the probe relative to the absorption profile of the naturally mixed Rb atomic vapor. Therefore, we increase the detuning of the single photon detuning in order to decrease the loss on the probe beam. However, the gain of the two output beams (the probe and the conjugate beams) decreases simultaneously when the detuning is increased. Therefore, the loss and the gain need to be balanced and optimized. At the same time, the two-photon detuning can be slightly adjusted to keep the gain large. The details of the adjustment of the experimental parameters in Ref. [21] have been discussed. In our experiment, the diameter of the pump laser is larger than that in Ref. [21], so the intensity of the pump is small, and we have to increase the temperature (atom number) in order to enhance the nonlinearity. Consequently, the absorption for the two beams is increased. So we tune the pump laser by an amount of about $+1.1$ GHz to the transition $F = 2 \rightarrow F'$ of the $^{85}$Rb D1 line in order to obtain the largest gain and decrease the losses. Finally, by balancing the losses and the gain in the FWM and optimizing the classical gain (electronic attenuator), we measure the $-4.1$ dB squeezing when the single photon detuning is about $+1.1$ GHz and the two-photon detuning is several MHz, as shown in Fig. 5(b). The power of the probe is about 155 µW and the conjugate is about 125 µW after the Rb cell. The gain ($G$) of the probe laser is 3.4. The losses of the probe and the conjugate are about 10% and 4%, respectively. The value of the attenuator is 0.3 dB. These parameters are basically coincident with the theoretical calculational results of the largest squeezing. The squeezing value is much smaller than the theoretical estimation and the recent experimental results,[9,21] because of the low transmission efficiency of the light after the cell in our experiment. The transmissivity of the Glans-Taylor prism for the 795-nm laser is 78% (the extinction ratio is about $7 \times 10^{-4}$). The total loss of the reflect mirror, the lens, and the PBS before the detectors is about 10%. The quantum efficiency of the photodiode is 84%, so the total transmission efficiency is less than 60%. The losses will decrease the squeezing by an amount of about 4 dB. In addition, the spontaneous emission and the decay of the two ground states are increased and losses on the two beams become large in the high temperature, they also decrease the squeezing.

Next, we investigate the influence of the extra noise. We replace the 800-MHz AOM with a 1.5-GHz AOM (Brimrose model-GPF-1500-200-795). The probe is double-passed through the 1.5-GHz AOM. We measure the intensity noise spectrum of the probe beam after the AOM and find the extra noise is introduced by this AOM system. In Fig. 5(c), the intensity noise (red curve) is 3 dB higher than SNL (the black curve) with the power of 42 µW. Thus, the probe beam has the extra 3-dB quadrature amplitude noise which is coupled into the laser from the AOM driver. We employ this probe beam in our experiment. At first, the experimental parameters are the same as these discussed above. We measure a gain ($G$) of 3.4. However, the squeezing is smaller than $-3$ dB due to the influence of the extra noise. In order to cancel the extra noise and maximize the squeezing, we optimize the experimental parameters further. According to the theoretical calculation, increasing the gain ($G$) can improve the squeezing. Therefore, we decrease the detuning of the pump laser in order to increase the gain. However, at the same time, the losses on the two beams become large because they get closer to the absorption profiles of the naturally mixed Rb atomic medium. The large losses will decrease the squeezing. Here we can use the losses to minimize the influence of the extra noise according to the theoretical analysis (Eqs. (20) and (21)). Through balancing the losses and the gain and adjusting the electronic attenuator, we get the maximum squeezing of $-3.7$ dB when the single photon detuning is about $+1$ GHz and the two photon detuning is slightly different with the above. The squeezing is 0.4 dB smaller than the above case, and the sum of intensity noise is about 20 dB larger than the SNL due to the extra noise as shown in Fig. 5(d). In this case, the power of the probe is about 150 µW and the conjugate is about 130 µW after the cell. The gain ($G$) is 3.6 and the losses for the probe and the conjugate beams are about 16% and 5%. And the value of the attenuator is 0.5 dB. These parameters are coincided with the theoretical analysis of the largest squeezing when the extra noise is introduced into the probe laser. The squeezing value is also much smaller than the theoretical calculational result; the reason for this has been discussed above.
4. Conclusion

In conclusion, we present the study of the generation of twin beams and the measurement of the quadrature amplitude squeezing via the non-degenerate FWM process in the Rb cell. Two pairs of self-balanced detection systems are employed to measure the probe and the conjugate beams, respectively. We easily obtain the quantum shot noise limit, and the squeezed and the amplified intensity noise power spectra. We also study the influence of the extra noise of the seed probe beam and introduce an optimal classical gain to achieve the maximum squeezing. The experiment will help us to investigate the related nonlinear process and the quantum entanglement.

References