Continuous-variable quantum-information distributor: Reversible telecloning

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We first classify the quantum clone into irreversible and reversible types from the perspective of quantum-information distribution. We propose a scheme of continuous-variable reversible telecloning, which broadcast the information of an unknown state without loss from a sender to several spatially separated receivers exploiting multipartite entanglement as quantum channels. In this scheme, the quantum information of an unknown state is distributed into M optimal clones and $M-1$ anticlones using $2M$-partite entanglement. For the perfect quantum-information distribution that is optimal cloning, $2M$-partite entanglement is required to be a maximum two-party entanglement.

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I. INTRODUCTION

One of the main tasks in quantum-information processing and quantum computation is the distribution of quantum information encoded in the states of quantum systems. Perfect distribution does not allow losing any quantum information of the transmitted unknown state, which means this process is reversible and the unknown state can be reconstructed in a quantum system again. It is now well known that quantum information cannot be exactly copied [1]. Although exact cloning is impossible, one can construct approximate cloning machines. Buzek and Hillery proposed a universal quantum cloning machine for an arbitrary quantum state where the copying process is independent of the input states [2]. In recent years, quantum information and communication have been extended to the domain of continuous variables (CVs) [3], due to the relative simplicity and high efficiency in generation, manipulation, and detection of the CV state. To date, CV local cloning has been studied intensively [4–10]. In this paper, we first classify the quantum clone into irreversible and reversible types from the perspective of quantum-information distribution.

Quantum nonlocal cloning (telecloning) has been intensively studied; it is a combination of quantum cloning and teleportation performed simultaneously. The aim of telecloning is to broadcast information of an unknown state from a sender to several spatially separated receivers exploiting multipartite entanglement as quantum channels. For qubits, Bruss *et al.* first proposed $1\rightarrow 2$ telecloning, which uses nonmaximum tripartite entanglement (here it is named described as irreversible teleclone states) [11]. In this case, the anticlones (phase-conjugate clones, or time-reversed states) are lost; thus, quantum channels do not require maximum entanglement. This kind of telecloning is called irreversible telecloning and is regarded as an imperfect nonlocal distributor of quantum information. More generally, $1\rightarrow M$ irreversible teleclone states, which is $(M+1)$-partite entanglement, are given in Ref. [12]. Later, Murao *et al.* proposed a new $1\rightarrow M+(M-1)$ telecloning scheme, in which quantum information of an input qubit is distributed into M optimal clones and $M-1$ anticlones using $2M$-partite entanglement [13]. This kind of telecloning is called reversible telecloning and is regarded as a perfect nonlocal distributor of quantum information. Because there is no loss of quantum information, $2M$-partite entanglement is required to be maximum two-partite entanglement. For continuous variables, Loock and Braunstein proposed optimal $1\rightarrow M$ telecloning of coherent states via an $(M+1)$-partite entangled state [14]. It is emphasized in the protocol that optimal telecloning can be achieved by exploiting nonmaximum bipartite entanglement between the sender and all receivers. This result is not surprising since anticlones are not produced in this protocol and partial quantum information about the unknown state is lost in the distribution process. This scheme is regarded as a CV irreversible telecloning and corresponds to the irreversible telecloning in the domain of discrete variables [11,12]. Furthermore, CV irreversible telecloning was studied in a noisy environment [15]. Recently, irreversible telecloning of optical coherent states was demonstrated experimentally [16]. In this paper, we propose a scheme of CV reversible telecloning, which broadcasts the information of an unknown state without loss from a sender to several spatially separated receivers, exploiting multipartite entanglement as quantum channels. Compared with the quantum telecloning proposed by Loock and Braunstein [14], this protocol produces anticlones of the unknown quantum state and thus keeps all information about an unknown state. Further, we generalize $1\rightarrow M+(M-1)$ quantum telecloning to the $N\rightarrow M+(M-N)$ case and also provide an explicit design for asymmetric reversible telecloning. As discussed for quantum teleportation, we give the lower and upper bounds to achieve quantum telecloning in the case when only imperfect quantum entanglement is available.

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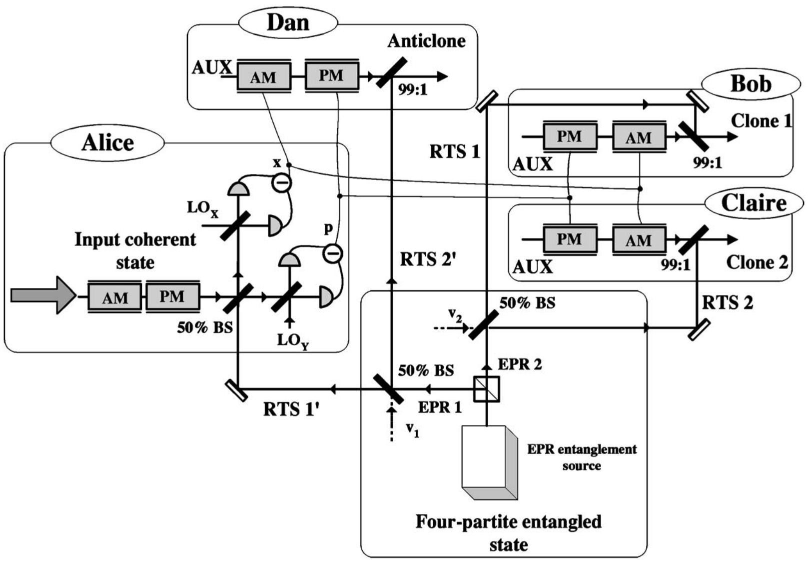

FIG. 1. A schematic diagram of $1\to 2+1$ telecloning. BS, beam splitter; LO, local oscillator; AM, amplitude modulator; PM, phase modulator; and AUX, auxiliary beam.

II. $1\to 2+1$ TELECLONING

A schematic setup for CV $1\to 2+1$ telecloning is depicted in Fig. 1. The quantum states we consider in this paper are described with the electromagnetic field annihilation operator $\hat{a}=(\hat{X}+i\hat{Y})/2$, which is expressed in terms of the amplitude $\hat{X}$ and phase $\hat{Y}$ quadratures with the canonical commutation relation $[\hat{X},\hat{Y}]=2i$. Without any loss of generality, the quadrature operators can be expressed in terms of a steady-state and a fluctuating component as $\hat{A}=\langle\hat{A}\rangle+\delta\hat{A}$, which have variances of $V_A=\langle\delta\hat{A}^2\rangle$ ($\hat{A}=\hat{X}$ or $\hat{Y}$). The heart of quantum telecloning is the multipartite entanglement shared among the sender and receivers. Without multipartite entanglement, it is only possible to perform the corresponding two-step protocol: the sender produces clones and anticlones locally, and then (bipartitely) teleports them to each receiver. The two-step protocol would require $2M-1$ bipartite entanglement for teleportation. Continuous-variable $1\to 2+1$ telecloning only needs one bipartite entanglement. The bipartite entangled state of CVs is a two-mode Gaussian entangled state [Einstein-Podolsky-Rosen (EPR) entangled state], which can be obtained directly by type-II parametric interaction [17] or indirectly by mixing two independent squeezed beams on a beam splitter [18]. The EPR entangled beams have very strong correlation, such that both their difference-amplitude quadrature variance $\langle\delta(\hat{X}_{a_{EPR1}}-\hat{X}_{a_{EPR2}})^2\rangle=2e^{-2r}$ and their sum-phase quadrature variance $\langle\delta(\hat{Y}_{a_{EPR1}}+\hat{Y}_{a_{EPR2}})^2\rangle=2e^{-2r}$ are less than the quantum noise limit, where r is the squeezing factor. The EPR entangled beams are divided into two beams at 50-50 beam splitters. The output modes $\hat{a}_{RTS1'}$, $\hat{a}_{RTS2'}$, $\hat{a}_{RTS1}$, and $\hat{a}_{RTS2}$ are expressed as

$$\hat{a}_{RTS1'}=\frac{\sqrt{2}}{2}(\hat{a}_{EPR1}+\hat{v}_1),\quad \hat{a}_{RTS2'}=\frac{\sqrt{2}}{2}(\hat{a}_{EPR1}-\hat{v}_1),$$

$$\hat{a}_{RTS1}=\frac{\sqrt{2}}{2}(\hat{a}_{EPR2}+\hat{v}_2),\quad \hat{a}_{RTS2}=\frac{\sqrt{2}}{2}(\hat{a}_{EPR2}-\hat{v}_2),\qquad (1)$$

where $\hat{v}_1$ and $\hat{v}_2$ refer to the annihilation operators of the vacuum noise entering the beam splitters. This output state is exactly the Gaussian analog of the $1\to 2+1$ reversible telecloning state of a qubit when $r\to\infty$. The $1\to 2+1$ telecloning state is partitioned into two sets $\{\hat{a}_{RTS1'},\hat{a}_{RTS2'}\}$ and $\{\hat{a}_{RTS1},\hat{a}_{RTS2}\}$. The parties in the same set come from one of the EPR entangled pair, so each party is in a thermal state and shows excess noise without any quantum entanglement between them. However, for any two parties in different sets there is bipartite entanglement. By using four-partite entangled modes, the sender Alice can perform quantum $1\to 2+1$ telecloning of a coherent state input to three receivers to produce two clones and one anticlone at their sites.

For quantum $1\to 2+1$ telecloning, Alice first performs a joint (Bell) measurement on her entangled mode $\hat{a}_{RTS1'}$ and an unknown input mode $\hat{a}_{in}$. The Bell measurement consists of a 50-50 beam splitter and two homodyne detectors as shown in Fig. 1. Alice's measurement results are labeled as $x=(\hat{X}_{RTS1'}-\hat{X}_{in})/\sqrt{2}$ and $p=(\hat{Y}_{RTS1'}+\hat{Y}_{in})/\sqrt{2}$. Receiving these measurement results from Alice, Bob, Claire, and Dan modulate the amplitude and phase of an auxiliary beam (AUX) at there site via two independent modulators with the scaling factors $g_x^{B(C,D)}$ and $g_p^{B(C,D)}$, respectively. The modulated beams are combined with Bob, Claire, and Dan's modes ($\hat{a}_{RTS1}$, $\hat{a}_{RTS2}$, and $\hat{a}_{RTS2'}$) at 1-99 beam splitters. The output modes produced by the telecloning process are expressed as

$$\hat{a}_{out}^B=\hat{a}_{in}+\frac{\sqrt{2}}{2}(\hat{a}_{EPR2}-\hat{a}_{EPR1}^\dagger)+\frac{\sqrt{2}}{2}(\hat{v}_2-\hat{v}_1^\dagger),$$

$$\hat{a}_{out}^C=\hat{a}_{in}+\frac{\sqrt{2}}{2}(\hat{a}_{EPR2}-\hat{a}_{EPR1}^\dagger)-\frac{\sqrt{2}}{2}(\hat{v}_2+\hat{v}_1^\dagger),$$

$$\hat{a}_{out}^D=\hat{a}_{in}^\dagger-\sqrt{2}\hat{v}_1,\qquad (2)$$

where we have taken $g_x^B=g_x^C=g_x^D=-\sqrt{2}$ and $g_p^B=g_p^C=-g_p^D=\sqrt{2}$. From these equations, we can see that Bob and Claire, whose entangled light lies in a different set from with Alice's, achieve the cloned states. The cloned states have additional noise terms to the input mode [4]. This noise is minimized in the case $r\rightarrow\infty$ corresponding to perfect EPR entanglement. These are the optimal clones of the coherent state input. The entangled light possessed by Dan is in the same set as Alice's, so he achieves the anticloned state, which is the complex conjugate of the input state and has additional noise. This additional noise is independent of the EPR entanglement. It always is the optimal anticlone of the coherent state input. In the case with perfect EPR entanglement, the unknown input state is completely unknown not only to Alice but to anyone in the process of telecloning. Thus quantum information of the unknown state is partitioned and distributed completely to Bob, Claire, and Dan. The optimal two clones and anticlone of Bob, Claire, and Dan may be reversed to the original unknown state of Alice by the same reversible telecloning state. Bob, Claire, and Dan perform the joint (Bell) measurement, respectively, on their entangled modes and clones (anticlone). Receiving these measurement results from Bob, Claire, and Dan, Alice displaces her entangled mode and can generate the original unknown state. However, the unknown state cannot be reconstructed only with two optimal clones. It is worth noting that the optimal two clones and anticlone of Bob, Claire, and Dan constitute a tripartite entangled state, which exactly corresponds to the $1\rightarrow 2$ CV irreversible telecloning state [14].

In a real experiment, a maximally EPR entangled state is not available because of finite squeezing and inevitable losses. Similarly as in teleportation, we apply the fidelity criterion $\mathcal{F}\equiv\langle\psi^{in}|\hat{\varrho}^{out}|\psi^{in}\rangle$ [18] to assess the quality of telecloning. In the case of unity gain, the fidelity for the Gaussian states is simply given by $\mathcal{F}=2/\sqrt{(1+\langle\delta\hat{X}^2_{out}\rangle)(1+\langle\delta\hat{Y}^2_{out}\rangle)}$. For the classical case of $r=0$, i.e., the EPR beams were replaced by uncorrelated vacuum inputs, the fidelity of Bob and Claire's outputs is found to be $\mathcal{F}_{class}=1/2$ [19], which corresponds to the classical limit for coherent state cloning. The fidelity of Dan's anticlone is $\mathcal{F}_{antic}=1/2$, which is independent of the quantum entanglement. When they share quantum entanglement $r>0$, the fidelity of the clones of Bob and Claire is $\mathcal{F}_{clone}=2/(3+e^{-2r})$. It clearly shows that Bob and Claire get clones with fidelity $\mathcal{F}_{clone}>1/2$; thus the quantum $1\rightarrow 2+1$ telecloning of coherent states is deemed successful. Note that the optimal fidelity of $1\rightarrow 2+1$ coherent state reversible telecloning is $2/3$ for the clones and $1/2$ for the anticlone, which requires the maximally EPR entangled state.

III. $1\rightarrow M+(M-1)$ TELECLONING

We now generalize $1\rightarrow 2+1$ quantum telecloning to $1\rightarrow M+(M-1)$, which produces M clones and $M-1$ anticlones from a single input state using $2M$-partite entanglement. We first generate the $2M$-partite entanglement by a sequence of a EPR entangled beams and $2(M-1)$ beam splitters with appropriately adjusted transmittances and reflectances as illustrated in Fig. 2. The modes $\hat{v}_{j,in}$ and $\hat{v}'_{j,in}$ are in the vacuum state. The EPR entangled modes $\hat{a}_{EPR1}$ and $\hat{a}_{EPR2}$

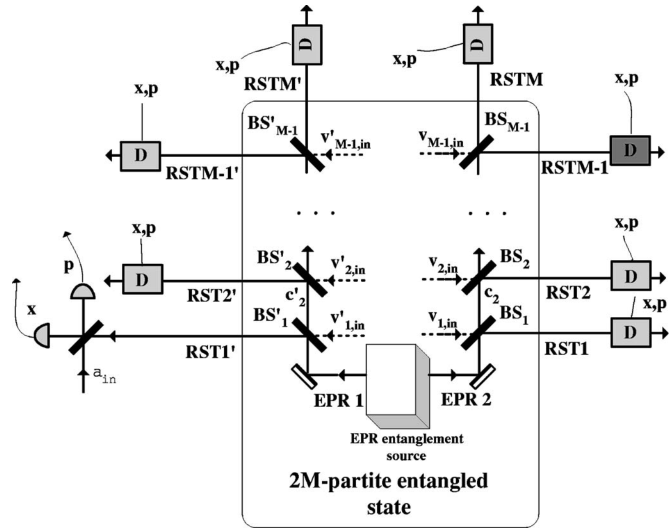

FIG. 2. A schematic diagram of $1\rightarrow M+(M-1)$ telecloning.

are mixed with $\hat{v}_{1,in}$ and $\hat{v}'_{1,in}$ at the beam splitters BS'_1 and BS_1, respectively. The mode $\hat{a}_{RTS1'}$ ($\hat{a}_{RTS1}$) contains the EPR entangled mode $\hat{a}_{EPR1}$ ($\hat{a}_{EPR2}$) by a factor of $1/\sqrt{M}$. The output $\hat{c}'_2$ ($\hat{c}_2$) is split at the BS'_2 (BS_2) and so on successively, until it arrives at the last beam splitter BS'_{M-1} (BS_{M-1}). The transformation performed by the jth beam splitter can be written as

$$\hat{a}_{RTSj^{(\prime)}}=\sqrt{\frac{1}{M-j+1}}\hat{c}^{(\prime)}_j+\sqrt{\frac{M-j}{M-j+1}}\hat{v}^{(\prime)}_{j,in},$$

$$\hat{c}^{(\prime)}_{j+1}=\sqrt{\frac{M-j}{M-j+1}}\hat{c}^{(\prime)}_j-\sqrt{\frac{1}{M-j+1}}\hat{v}^{(\prime)}_{j,in}, \quad (3)$$

where $\hat{c}'_1=\hat{a}_{EPR1}$, $\hat{c}_1=\hat{a}_{EPR2}$, and $\hat{a}_{RTSM^{(\prime)}}=\hat{c}^{(\prime)}_M$. It is clearly shown that each $2M$-partite entangled mode $\hat{a}_{RTSj^{(\prime)}}$ (or $\hat{a}_{RTSj}$) contains a $1/M$ portion of the EPR entangled mode $\hat{a}_{EPR1}$ (or $\hat{a}_{EPR2}$) and an $(M-1)/M$ portion of the vacuum noise. The entanglement structure of the $2M$-partite telecloning state is also divided into two sets $\{\hat{a}_{RTS1'},\hat{a}_{RTS2'},\ldots,\hat{a}_{RTSM'}\}$ and $\{\hat{a}_{RTS1},\hat{a}_{RTS2},\ldots,\hat{a}_{RTSM}\}$. There is no quantum entanglement in the parties in the same set; however, there is bipartite entanglement in any two parties in different sets.

For quantum $1\rightarrow M+(M-1)$ telecloning, the sender chooses any one of $2M$ modes of the telecloning state and performs a joint measurement on his entangled mode and an unknown input mode $\hat{a}_{in}$. Then the sender gives the measured results x and p to the other parties. After receiving these measurement results from sender, each party displaces his/her entangled mode by means of a 1-99 beam splitter with an auxiliary beam, the amplitude and phase of which have been modulated by the received x and p signals, respectively. The parties in the different sets from the sender produce clones with $-g_x=g_p=\sqrt{2}$ and the parties in the same sets as the sender produce anticlones with $g_x=g_p=-\sqrt{2}$. The fidelities of M clones and $M-1$ anticlones are given, respectively, by

$$\mathcal{F}_{clone}^{1\to M+(M-1)} = \frac{M}{2M-1+e^{-2r}},$$

$$\mathcal{F}_{antic}^{1\to M+(M-1)} = \frac{1}{2}. \tag{4}$$

The classical limit of the fidelity for $1\to M+(M-1)$ quantum telecloning is $\mathcal{F}_{class}=1/2$. The fidelity of the anticlones is $\mathcal{F}_{antic}=1/2$, which is independent of the quantum entanglement. When $r>0$, the fidelitiy of the clones is larger than $1/2$; thus the quantum $1\to M+(M-1)$ telecloning of coherent states is successful. The $1\to M+(M-1)$ coherent state telecloning becomes reversible and optimal with the fidelity $M/(2M-1)$ for the clones and $1/2$ for the anticlone when the EPR entangled state is perfect.

IV. $N\to M+(M-N)$ TELECLONING

We now address the most complicated case, the $N\to M+(M-N)$ quantum telecloning, which produces M clones and $M-N$ anticlones from N original replicas of a coherent state using $2M$-partite entanglement. The same multipartite entanglement Eq. (3) is used for the quantum channels. The N replicas of a coherent state are stored in the N modes $\hat{a}_{in,1},\ldots,\hat{a}_{in,N}$. In this scheme, we may consider using a sender who holds the N input replicas and N entangled modes in the same set of the $2M$-partite reversible telecloning state, or N senders each of whom holds one of N input replicas and of the entangled modes in the same set. By performing a joint measurement of each input replica and entangled mode, the sender(s) generate(s) N amplitude- and phase-quadrature measurement results $(x_1,p_1),\ldots,(x_N,p_N)$ and then inform(s) other parties. After receiving these measurement results, each party first combines the measurement results $x_s=\frac{\sqrt{2}}{N}(x_1+\ldots+x_N)$ and $p_s=\frac{\sqrt{2}}{N}(p_1+\ldots+p_N)$, and then displaces the self-entangled mode. The parties in the different set from the sender produce M clones with $-g_x=g_p=1$ and the parties in the same set as the sender produce $M-N$ anticlones with $g_x=g_p=-1$. The fidelities of M clones and $M-N$ anticlones are given by

$$\mathcal{F}_{clone}^{N\to M+(M-N)} = \frac{NM}{NM+M-N+Ne^{-2r}},$$

$$\mathcal{F}_{antic}^{N\to M+(M-N)} = \frac{N}{N+1}. \tag{5}$$

The classical limit for $N\to M+(M-N)$ quantum telecloning is $\mathcal{F}_{class}=N/(N+1)$. The fidelity of the anticlones is $\mathcal{F}_{antic}=N/(N+1)$, which is independent of the quantum entanglement. When $r>0$, the fidelity of the clones is larger than $N/(N+1)$; thus the quantum $N\to M+(M-N)$ telecloning of coherent states is successful. $N\to M+(M-N)$ reversible telecloning requires the maximum EPR entanglement, which is an optimal cloner with fidelity $MN/(MN+M-N)$ for the clones and $N/(N+1)$ for the anticlone [5].

V. ASYMMETRIC REVERSIBLE TELECLONING

Let us now demonstrate how to make the reversible telecloning asymmetric. This is particularly interesting in the context of quantum cryptography where it enables Eve to choose a trade-off between the quality of her copy and the unavoidable noise that is added to the copy sent to the receiver. Here we only concentrate on $1\to 2+1$ asymmetric telecloning. The scheme of $1\to 2+1$ asymmetric telecloning is similar to symmetric telecloning as in Fig. 1, in which only the vacuum noises $\hat{v}_1$ and $\hat{v}_2$ entering the beam splitters are replaced by other EPR entangled beams $\hat{b}_{EPR1}$ and $\hat{b}_{EPR2}$. Bob and Claire produce the clones and Dan the anticlone, whose fidelities are expressed, respectively, by

$$\mathcal{F}_{clone}^{B} = \frac{2}{2+e^{-2r}+e^{-2r_b}},$$

$$\mathcal{F}_{clone}^{C} = \frac{2}{2+e^{-2r}+e^{2r_b}},$$

$$\mathcal{F}_{antic}^{D} = \frac{2}{2+e^{-2r_b}+e^{2r_b}}, \tag{6}$$

where r_b is the squeezing factor of the EPR entangled beams $\hat{b}_{EPR1}$ and $\hat{b}_{EPR2}$. It clearly shows that $\mathcal{F}_{clone}^{B}>2/3>\mathcal{F}_{clone}^{c}$ and $\mathcal{F}_{antic}^{D}<1/2$ when $r\to\infty$, corresponding to reversible and optimal telecloning. This means that the more information of the unknown state is obtained by Bob, the less is obtained by Claire and Dan. The amount of information distributed to the remote receivers is controlled by the squeezing factor r_b. This confirms that the device indeed realizes the optimal asymmetric Gaussian telecloning of coherent states.

VI. CONCLUSION

We have introduced a scheme of CV reversible telecloning, which broadcast the information of an unknown state without loss from a sender to several spatially separated receivers by exploiting multipartite entanglement as the quantum channels. To the best of our knowledge, we are the first to classify the quantum clone from the perspective of quantum-information distribution and to propose a scheme that distributes quantum information perfectly by telecloning. This scheme of implementing quantum state distribution nonlocally helps to deepen our understanding of the properties of quantum communication systems enhanced by entanglement. On the other hand, its flexibility might have remarkable application in quantum communication and computation.

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