Continuous-variable multipartite unlockable bound entangled Gaussian states

Jing Zhang (张靖)*

State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan 030006, People's Republic of China

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We investigate continuous-variable (CV) multipartite unlockable bound entangled states. Comparing with the qubit multipartite unlockable bound entangled states, CV multipartite unlockable bound entangled states present some new and different properties. CV multipartite unlockable bound entangled states may serve as a useful quantum resource for new multiparty communication schemes. The experimental protocol for generating CV unlockable bound entangled states is proposed with a setup that is, at present, accessible.

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Quantum entanglement is a striking property of composite quantum systems that lies at the heart of the fundamental quantum information protocols, which has led to ongoing efforts for its quantitative and qualitative characterization. While entanglement of pure bipartite states is well understood, the entanglement of mixed and multipartite systems is still under intense research. Entanglement is a very fragile resource, easily destroyed by the decoherence processes to become mixed owing to unwanted coupling with the environment. Therefore, it is important to know which mixed states can be distilled to maximally entangled states from many identical copies by means of local operations and classical communication (LOCC). A surprising discovery in this area is that there exist mixed entangled states from which no pure entanglement can be distilled, and these states are called bound entangled states [1]. This new class of states is between separable and free entangled states. Much effort has been devoted to the characterization and detection of bound entanglement [2], and various properties and applications of bound entanglement have been found. The distillability of multipartite entangled states, however, is much more complicated than that of bipartite entangled states. Usually, a multipartite entangled state is bound entangled if no pure entanglement can be distilled between any two parties by LOCC when all the parties remain spatially separated from each other. However, a multipartite bound entangled state may be unlocked or activated. If all the parties are organized into several groups and we let each group join together and perform collective quantum operations, then pure entanglement may be distilled between two parties within some of the groups, and this state will be called an unlocked or activable bound entangled state. A famous class of multipartite unlockable bound entangled states is the Smolin state [3], which is a four-qubit state and was generalized recently into an even number of qubits [4,5]. These states have been applied in remote information concentration [6], quantum secret sharing [7], and superactivation [8,9]. Particularly, the link between multipartite unlockable bound entangled states and the stabilizer formalism was found [10]. The properties of the multipartite unlockable bound entangled states can be easily explained from the stabilizer formalism. Recently, the four-qubit unlockable bound entangled state (Smolin state) was demonstrated experimentally with polarization photons [11,12] and ions [13].

Most of the concepts of quantum information and computation have been initially developed for discrete quantum variables, in particular two-level or spin-$\frac{1}{2}$ quantum variables (qubits). In parallel, quantum variables with a continuous spectrum, such as the position and momentum of a particle or amplitude and phase quadrature of an electromagnetic field, in informational or computational processes have attracted a lot of interest and appear to yield very promising perspectives concerning both experimental realizations and general theoretical insights [14,15] due to relative simplicity and high efficiency in the generation, manipulation, and detection of continuous-variable (CV) states. Bound entanglement of bipartite states has also been considered for continuous variables, and the nontrivial examples of bound entangled states for CV have been constructed [16–19]. However, the research of CV bound entanglement far lags that for discrete-variable (DV). In this paper, we first exploit the stabilizer formalism to study the CV multipartite unlockable bound entangled states. Comparing with the qubit multipartite unlockable bound entangled states, CV multipartite unlockable bound entangled states present some different properties. We also study the four-mode multipartite unlockable bound entangled states in detail and present the experimental protocol for generating CV unlockable bound entangled states.

For CV systems, the Weyl-Heisenberg group [20], which is the group of phase-space displacements, is a Lie group with generators $\hat{x} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$ (quadrature amplitude or position) and $\hat{p} = -i(\hat{a} - \hat{a}^\dagger)/\sqrt{2}$ (quadrature phase or momentum) satisfying the canonical commutation relation $[\hat{x},\hat{p}] = i$ (with $\hbar = 1$). The single-mode Pauli operators (so termed in analogy with qubit systems) are defined as $X(s) = \exp[-is\hat{p}]$ and $Z(t) = \exp[it\hat{x}]$ with $s,t \in \mathbb{R}$. The Pauli operator $X(s)$ is a position-translation operator, which acts on the computational basis of position eigenstates $\{|q\rangle; q \in \mathbb{R}\}$ as $X(s)|q\rangle = |q+s\rangle$, whereas Z is a momentum-translation operator, which acts on the momentum eigenstates as $Z(t)|p\rangle = |p+t\rangle$. These operators are noncommutative and obey the identity $X(s)Z(t) = e^{-ist}Z(t)X(s)$. The Pauli operators for one mode can be used to construct a set of Pauli operators $\{X_i(s_i), Z_i(t_i); i = 1,\ldots,n\}$ for n-mode systems. The n-mode Pauli group is expressed as

$$G_n = \{X_1(s_1)Z_1(t_1) \otimes \cdots \otimes X_n(s_n)Z_n(t_n) : s_i,t_i \in \mathbb{R}\}. \quad (1)$$

*jzhang74@yahoo.com, jzhang74@sxu.edu.cn

The elements of this group can expressed in terms of a linear combination of the canonical operators $\hat{x}_i$ and $\hat{p}_i$ indexed by a vector $\mathbf{v} = (s_1, \ldots, s_n, t_1, \ldots, t_n) \in \mathbb{R}^{2n}$:

$$U(\mathbf{v}) = \exp\left[i \sum_{i=1}^{n} (-s_i \hat{p}_i + t_i \hat{x}_i)\right]. \tag{2}$$

The commutative relationship between any two element operators in an n-mode Pauli group is expressed as

$$U(\mathbf{v})U(\mathbf{v}') = e^{i\omega(\mathbf{v},\mathbf{v}')} U(\mathbf{v}')U(\mathbf{v}), \tag{3}$$

where $\omega(\mathbf{v},\mathbf{v}') = \sum_{i=1}^{n} (s_i' t_i - s_i t_i')$.

Suppose we choose commuting operators $U_1(\mathbf{v}_1)$, $U_2(\mathbf{v}_2)$, $\ldots$,$U_k(\mathbf{v}_k)$ from G_n, and thus, the k independent vectors $\mathbf{v}_1$, $\ldots$, $\mathbf{v}_k$ must satisfy $\omega(\mathbf{v}_i, \mathbf{v}_j) = 0$ for all i, j [see Eq. (3)]. Then we have an Abelian subgroup,

$$S = \left\{ U(\mathbf{u}) : \mathbf{u} = \sum_{i=1}^{k} a_i \mathbf{v}_i,\ a_i \in \mathbb{R} \right\}, \tag{4}$$

in which any two operators $U(\mathbf{u})$ and $U(\mathbf{u}')$ commute. The Abelian subgroup may be expressed by $S = \langle U_1(\mathbf{v}_1), U_2(\mathbf{v}_2), \ldots, U_k(\mathbf{v}_k) \rangle$, which denotes the Abelian subgroup generated by them. A state $|\psi\rangle$ is said to be stabilized by S, or S is the stabilizer of $|\psi\rangle$, if $U_i(\mathbf{v}_i)|\psi\rangle = |\psi\rangle$, where $i = 1,2,\ldots,k$. The stabilizer formalism for CV systems [21–24] has been used to study the CV graph state [25,26]. The Abelian subgroup S can be conveniently defined by its Lie algebra, $S' = \langle H_1, H_2, \ldots, H_k \rangle$. The operators $H_i = \mathbf{v}_i \mathbf{R}^T$ are the linear combination of the canonical operators $\mathbf{R} = (\hat{x}_1, \ldots, \hat{x}_n, \hat{p}_1, \ldots, \hat{p}_n)$. Any two operators H_i and H_j commute. S' is referred to as the nullifier of $|\psi\rangle$ since $H_i|\psi\rangle = 0$, $i = 1,2,\ldots,k$. Every nullifier is Hermitian and so is observable. Thus, the state $|\psi\rangle$ can be expressed in the simple nullifier representation.

All the states stabilized by S constitute a subspace denoted by V_S. There is a unique pure state for the n-mode system stabilized by S if S has n independent stabilizer generators [thus, $S = \langle U_1(\mathbf{v}_1), U_2(\mathbf{v}_2), \ldots, U_n(\mathbf{v}_n) \rangle$ are called a complete set of stabilizer generators]. When S has k independent stabilizer generators that are less than the total mode number of the n-mode system, the states in the subspace V_S will be more than one. Therefore, the maximally mixed state over V_S is expressed by $\rho_S = P_S/\mathrm{tr}(P_S)$, where

$$P_S = \int d\eta_1 \ldots d\eta_k U_1(\eta_1 \mathbf{v}_1) \ldots U_k(\eta_k \mathbf{v}_k) \tag{5}$$

is the projection operator onto V_S. Note that the stabilized subspace V_S is the subspace spanned by the simultaneous eigenstates of the stabilizer generator $\{U_1(\mathbf{v}_1), U_2(\mathbf{v}_2), \ldots, U_k(\mathbf{v}_k)\}$ with the eigenvalues $\{1,1,\ldots,1\}$ (corresponding to simultaneous eigenstates of the nullifier $\{H_1, H_2, \ldots, H_k\}$ with the eigenvalues $\{0,0,\ldots,0\}$). In general, any orthogonal subspaces $V_S^{\{\lambda_1,\ldots,\lambda_k\}}$ may be expressed by the simultaneous eigenstates of $\{U_1(\mathbf{v}_1), U_2(\mathbf{v}_2), \ldots, U_k(\mathbf{v}_k)\}$ with the eigenvalues $\{e^{i\lambda_1}, \ldots, e^{i\lambda_k}\}$ (corresponding to simultaneous eigenstates of the nullifier $\{H_1, H_2, \ldots, H_k\}$ with the eigenvalues $\{\lambda_1, \ldots, \lambda_k\}$). The corresponding maximally mixed state over $V_S^{\{\lambda_1,\ldots,\lambda_k\}}$ is $\rho_S^{\{\lambda_1,\ldots,\lambda_k\}} = P_S^{\{\lambda_1,\ldots,\lambda_k\}}/\mathrm{tr}(P_S^{\{\lambda_1,\ldots,\lambda_k\}})$, where

$$P_S^{\{\lambda_1,\ldots,\lambda_k\}} = \int d\eta_1 \ldots d\eta_k e^{i\lambda_1\eta_1} U_1(\eta_1 \mathbf{v}_1) \ldots e^{i\lambda_k\eta_k} U_k(\eta_k \mathbf{v}_k) \tag{6}$$

is the projection operator onto $V_S^{\{\lambda_1,\ldots,\lambda_k\}}$. All these subspaces have the same dimensions and form an orthogonal decomposition of the whole space.

A partition of the n-mode system $\{M_1, M_2, \ldots, M_n\}$ is defined as a set of its proper subsets $\{V_1, V_2, \ldots, V_m\}$, in which $V_i \cap V_j = \phi (i \neq j)$, $\cup_{i=1}^{m} V_i = \{M_1, M_2, \ldots, M_n\}$, and $|V_i|$ denotes the number of modes in V_i. The k independent stabilizer generators can be split into local stabilizer generators with respect to the partition $\{V_1, V_2, \ldots, V_m\}$**{**$\{U_1^{(V_1)}(\mathbf{v}_1), \ldots U_k^{(V_1)}(\mathbf{v}_k)\}, \{U_1^{(V_2)}(\mathbf{v}_1), \ldots U_k^{(V_2)}(\mathbf{v}_k)\}, \ldots, \{U_1^{(V_m)}(\mathbf{v}_1), \ldots U_k^{(V_m)}(\mathbf{v}_k)\}$**}:**

$$U_\beta^{(V_\alpha)}(\mathbf{v}_\beta) = \exp\left[i \sum_{j \in V_\alpha} (s_j \hat{p}_j + t_j \hat{x}_j)\right]. \tag{7}$$

If all local stabilizer generators commute each other, the maximally mixed state ρ_S for an n-mode system stabilized by $\{U_1(\mathbf{v}_1), U_2(\mathbf{v}_2), \ldots, U_k(\mathbf{v}_k)\}$ is said to be separable with respect to the partition $\{V_1, V_2, \ldots, V_m\}$ [10], which may be rewritten with the product form

$$\begin{aligned}\rho_S = &\int_{\left(\lambda^{U_1^{(V_1)}} + \cdots + \lambda^{U_1^{(V_m)}}\right)=0} d\lambda^{U_1^{(V_1)}} \cdots d\lambda^{U_1^{(V_m)}} \\ &\times \int_{\left(\lambda^{U_2^{(V_1)}} + \cdots + \lambda^{U_2^{(V_m)}}\right)=0} d\lambda^{U_2^{(V_1)}} \cdots d\lambda^{U_2^{(V_m)}} \\ &\times \int_{\left(\lambda^{U_k^{(V_1)}} + \cdots + \lambda^{U_k^{(V_m)}}\right)=0} d\lambda^{U_k^{(V_1)}} \cdots d\lambda^{U_k^{(V_m)}} \\ &\times \rho_{S^{(V_1)}}^{\{\lambda^{U_1^{(V_1)}}, \ldots, \lambda^{U_k^{(V_1)}}\}} \otimes \rho_{S^{(V_2)}}^{\{\lambda^{U_1^{(V_2)}}, \ldots, \lambda^{U_k^{(V_2)}}\}} \\ &\otimes \rho_{S^{(V_m)}}^{\{\lambda^{U_1^{(V_m)}}, \ldots, \lambda^{U_k^{(V_m)}}\}},\end{aligned} \tag{8}$$

where $\rho_{S^{(V_j)}}^{\{\lambda^{U_1^{(V_j)}}, \ldots, \lambda^{U_k^{(V_j)}}\}}$ is given in Eq. (6). Moreover, if ρ_S is separable with respect to the partition $\{V_1, V_2, \ldots, V_m\}$ and the local stabilizer generators $S^{(V_j)} = \langle U_1^{(V_j)}(\mathbf{v}_1), \ldots, U_k^{(V_j)}(\mathbf{v}_k) \rangle$ in one of the subsets V_j contain the number of the independent stabilizer generators equal to the number of modes in V_j ($S^{(V_j)}$ is a complete set of stabilizer generators on V_j), pure entanglement among the modes inside V_j can be distilled [10] by letting the modes in all other subsets $V_1, V_2, \ldots, V_{i \neq j}, \ldots, V_m$ join together and perform joint measurements. Thus, the maximally mixed state ρ_S for n-mode system stabilized subspace V_S is called an unlockable bound entangled state.

Now we consider a four-mode system with two independent stabilizers:

$$\begin{aligned} H_1 &= \hat{x}_1 + \hat{x}_2 + \hat{x}_3 + \hat{x}_4 \to 0 \\ U_1 &= Z_1(1)Z_2(1)Z_3(1)Z_4(1) \to 1, \\ H_2 &= \hat{p}_1 - \hat{p}_2 + \hat{p}_3 - \hat{p}_4 \to 0 \\ U_2 &= X_1(1)X_2(-1)X_3(1)X_4(-1) \to 1, \end{aligned} \tag{9}$$

which is analogous to the four-qubit unlockable bound entangled state, also called the Smolin state [3]. However, a CV four-mode unlockable bound entangled state has some distinct properties that compare with the counterpart of qubit. Considering the 2:2 partition $\{\{M_1,M_2\},\{M_3,M_4\}\}$, we have local stabilizer generators $\{\{U_1^{(\{1,2\})} = Z_1(1)Z_2(1), U_2^{(\{1,2\})} = X_1(1)X_2(-1)\}, \{U_1^{(\{3,4\})} = Z_3(1)Z_4(1), U_2^{(\{3,4\})} = X_3(1)X_4(-1)\}\}$, which commute each other. Therefore, the maximally mixed state $\rho_S^{(4)}$ stabilized by U_1 and U_2 may be expressed by the product form with respect to the partition $\{\{M_1,M_2\},\{M_3,M_4\}\}$:

$$\begin{aligned}\rho_S^{(4)} &= \frac{1}{\mathrm{tr}(P_S)}\int d\eta_1 d\eta_2 Z_1(\eta_1)Z_2(\eta_1)Z_3(\eta_1)Z_4(\eta_1)\\ &\quad\times X_1(\eta_2)X_2(-\eta_2)X_3(\eta_2)X_4(-\eta_2)\\ &= \int d\lambda_1 d\lambda_2 \rho_{S(\{M_1,M_2\})}^{\{\lambda_1,\lambda_2\}} \otimes \rho_{S(\{M_3,M_4\})}^{\{-\lambda_1,-\lambda_2\}},\end{aligned} \tag{10}$$

where $\rho_{S(\{M_1,M_2\})}^{\{\lambda_1,\lambda_2\}} = \int d\eta_1 d\eta_2 e^{i\lambda_1\eta_1} Z_1(\eta_1)Z_2(\eta_1)e^{i\lambda_2\eta_2}X_1(\eta_2)X_2(-\eta_2)$ and $\rho_{S(\{M_3,M_4\})}^{\{-\lambda_1,-\lambda_2\}}$ is similar to $\rho_{S(\{M_1,M_2\})}^{\{\lambda_1,\lambda_2\}}$. $\rho_S^{(4)}$ is separable with respect to the 2:2 partition $\{\{M_1,M_2\},\{M_3,M_4\}\}$. Furthermore, we consider the 2:2 partition $\{\{M_1,M_4\},\{M_2,M_3\}\}$, whose properties are the same as the partition $\{\{M_1,M_2\},\{M_3,M_4\}\}$. However, the partition $\{\{M_1,M_3\},\{M_2,M_4\}\}$ is quite different since its local stabilizer generators do not commute in the same subset. Thus, $\rho_S^{(4)}$ is inseparable with respect to the partition $\{\{M_1,M_3\},\{M_2,M_4\}\}$. Comparing the CV four-mode unlockable bound entangled state, the four-qubit unlockable bound entangled state is invariant under arbitrary permutation of the four qubits and is separable with respect to any 2:2 partition.

Nondistillabitily. When the four parties sharing four modes respectively are located in separated stations (thus, they cannot perform joint quantum operation), they cannot distill any pure entanglement by LOCC. This comes from the fact the state is separable with respect to the partitions $\{\{M_1,M_2\},\{M_3,M_4\}\}$ and $\{\{M_1,M_4\},\{M_2,M_3\}\}$. In detail, since the state is separable across $\{\{M_1,M_2\},\{M_3,M_4\}\}$, local measurements and classical communication for M_1 and M_3 (M_1 and M_4; M_2 and M_3; M_2 and M_4) cannot establish any entanglement between M_2 and M_4 (M_2 and M_3; M_1 and M_4; M_1 and M_3, respectively) since the amount of entanglement cannot be increased by local operations and classical communication [27,28]. Considering the state is separable across $\{\{M_1,M_4\},\{M_2,M_3\}\}$, local measurements and classical communication for M_1 and M_2 (M_1 and M_3; M_4 and M_2; M_4 and M_3) cannot establish any entanglement between M_4 and M_3 (M_4 and M_2; M_1 and M_3; M_1 and M_2, respectively). Thus, by definition, this state is called a multipartite bound entangled state.

Unlockability. Though this state is nondistillable under LOCC when all the modes remain spatially separated from each other, its entanglement can be unlocked. For example, considering the state is separable across $\{\{M_1,M_2\},\{M_3,M_4\}\}$, performing the joint Bell-basis measurement on M_1 and M_4 (M_2 and M_3) can establish pure entanglement between M_2 and M_3 (M_1 and M_4) [see Eq. (9)]. However, performing the joint Bell-basis measurement on M_1 and M_3 (M_2 and M_4) cannot establish any entanglement between M_2 and M_4 (M_1 and M_3) since the local stabilizer generators of $\{\{M_1,M_3\},\{M_2,M_4\}\}$ do not commute in the same subset. Considering that the state is separable across $\{\{M_1,M_4\},\{M_2,M_3\}\}$, the joint Bell-basis measurement performing on M_1 and M_2 (M_4 and M_3) can establish pure entanglement between M_4 and M_3 (M_1 and M_2). However, performing the joint Bell-basis measurement on M_1 and M_3 (M_4 and M_2) cannot establish any entanglement between M_4 and M_2 (M_1 and M_3).

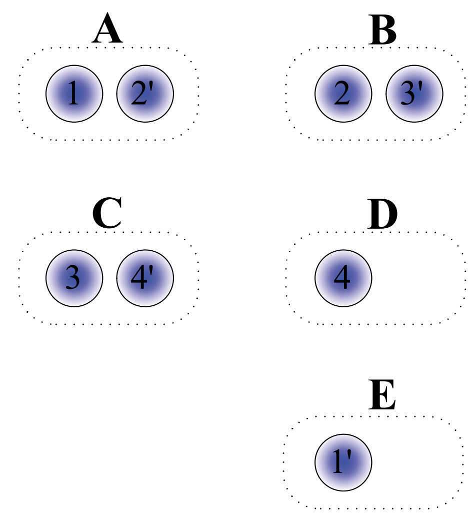

FIG. 1. (Color online) The state $\rho_{1,2,3,4}^{(S)} \bigotimes \rho_{1',2',3',4'}^{(S)}$ distilled into an EPR pair between D and E by local measurements.

The Einstein-Podolskyo-Rosen (EPR) entangled state can also be distilled by employing the tensor product of two identical CV four-mode unlockable bound entangled states, which is analogous to the distillation process for the four-qubit unlockable bound entangled state, also called superactivation of bound entanglement [8]. Two identical CV four-mode unlockable bound entangled states $\rho_{1,2,3,4}^{(S)}$ and $\rho_{1',2',3',4'}^{(S)}$ are assigned to five remote parties, A, B, C, D, and E, as shown in Fig. 1. Thus, parties A, B, C, and D each have one mode, 1, 2, 3, and 4, respectively, of state $\rho_{1,2,3,4}^{(S)}$, and similarly, parties A, B, C, and E each have one mode, $2', 3', 4'$, and $1'$, of state $\rho_{1',2',3',4'}^{(S)}$. Parties A, B, and C perform joint Bell-basis measurement respectively and then send their measured results to D. Party D translates the measurement results into mode 4, which is expressed by

$$\begin{aligned}\hat{x}_4' &= \hat{x}_4 + (\hat{x}_1 + \hat{x}_{2'}) + (\hat{x}_2 + \hat{x}_{3'}) + (\hat{x}_3 + \hat{x}_{4'})\\ &= \hat{x}_{2'} + \hat{x}_{3'} + \hat{x}_{4'},\\ \hat{p}_4' &= \hat{p}_4 - (\hat{p}_1 - \hat{p}_{2'}) + (\hat{p}_2 - \hat{p}_{3'}) - (\hat{p}_3 - \hat{p}_{4'})\\ &= \hat{p}_{2'} - \hat{p}_{3'} + \hat{p}_{4'}.\end{aligned} \tag{11}$$

Thus, an EPR pair between D and E is distilled with $\hat{x}_{1'} + \hat{x}_4' \to 0$ and $\hat{p}_{1'} - \hat{p}_4' \to 0$.

The CV four-mode unlockable bound entangled state may be generalized into $2n$ modes, whose nullifiers (stabilizer generators) are expressed by

$$\begin{aligned}H_1^{(2n)} &= \hat{x}_1 + \hat{x}_2 + \hat{x}_3 + \hat{x}_4 + \cdots + \hat{x}_{2n-1} + \hat{x}_{2n},\\ H_2^{(2n)} &= \hat{p}_1 - \hat{p}_2 + \hat{p}_3 - \hat{p}_4 + \cdots + \hat{p}_{2n-1} - \hat{p}_{2n}.\end{aligned} \tag{12}$$

It can easily be seen that the maximally mixed state $\rho_S^{(2n)}$ stabilized by U_1^{2n} and U_2^{2n} is not separable for any 2:2 : ... : 2

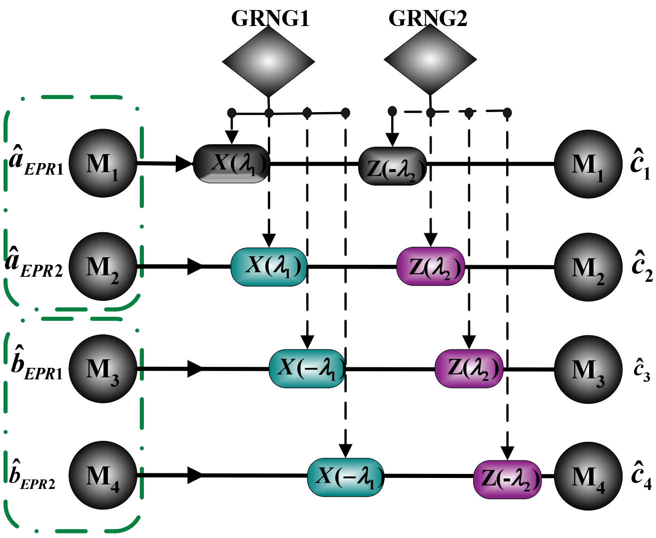

FIG. 2. (Color online) The generation of a four-mode unlockable bound entangled state. GRNG denotes the Gaussian random number generator. X and Z are the position- and momentum-translation Pauli operators, respectively.

partition. Applying this method, many more CV unlockable bound entangled states can be found and defined.

The above analyses of the CV multipartite unlockable bound entangled states based on the stabilizer formula require infinite energy and stand as an idealized limit. Now we investigate the four-mode unlockable bound entangled state with finite squeezing and present the protocol to generate it experimentally. As shown in Fig. 2, two pairs, $(\hat{a}_{\mathrm{EPR1}},\hat{a}_{\mathrm{EPR2}})$ and $(\hat{b}_{\mathrm{EPR1}},\hat{b}_{\mathrm{EPR2}})$, of EPR entangled states [also called the two-mode squeezed state $|\psi(r)\rangle=\sum_n\lambda^n\sqrt{1-\lambda^2}|n,n\rangle$ with $\lambda=\tanh r$, where r is the squeezing factor] are distributed into four stations, $M_1(\hat{a}_{\mathrm{EPR1}}),M_2(\hat{a}_{\mathrm{EPR2}}),M_3(\hat{b}_{\mathrm{EPR1}})$, and $M_4(\hat{b}_{\mathrm{EPR2}})$, respectively. The EPR entangled state has a very strong correlation property, namely, that both its sum-amplitude quadrature variance $\langle\delta^2(\hat{x}_{a(b)_{\mathrm{EPR1}}}+\hat{x}_{a(b)_{\mathrm{EPR2}}})\rangle=e^{-2r}$ and its difference-phase quadrature variance $\langle\delta^2(\hat{p}_{a(b)_{\mathrm{EPR1}}}-\hat{p}_{a(b)_{\mathrm{EPR2}}})\rangle=e^{-2r}$ are less than the quantum noise limit [29,30]. The position and momentum of two pairs, $(\hat{a}_{\mathrm{EPR1}},\hat{a}_{\mathrm{EPR2}})$ and $(\hat{b}_{\mathrm{EPR1}},\hat{b}_{\mathrm{EPR2}})$, are translated random using two Gaussian random number generators (GRNGs), as shown in Fig. 2. This random operation applied exhibits a Gaussian distribution; hence, the standard deviation of the GRNG σ_{GRNG} provides a complete characterization of its strength. The resulting state is expressed by

$$\begin{aligned}\hat{c}_1&=\hat{a}_{\mathrm{EPR1}}+x_{\mathrm{GRNG1}}-p_{\mathrm{GRNG2}},\\ \hat{c}_2&=\hat{a}_{\mathrm{EPR2}}+x_{\mathrm{GRNG1}}+p_{\mathrm{GRNG2}},\\ \hat{c}_3&=\hat{b}_{\mathrm{EPR1}}-x_{\mathrm{GRNG1}}+p_{\mathrm{GRNG2}},\\ \hat{c}_4&=\hat{b}_{\mathrm{EPR2}}-x_{\mathrm{GRNG1}}-p_{\mathrm{GRNG2}},\end{aligned}\tag{13}$$

and the correlation variances of two independent stabilizers of this state are $\langle\delta^2(\hat{x}_{c_1}+\hat{x}_{c_2}+\hat{x}_{c_3}+\hat{x}_{c_4})\rangle=2e^{-2r}$ and $\langle\delta^2(\hat{p}_{c_1}-\hat{p}_{c_2}+\hat{p}_{c_3}-\hat{p}_{c_4})\rangle=2e^{-2r}$. The output state will exactly become that in Eq. (9) [and will be expressed by the density operator Eq. (10)] when $r\to\infty$ and $\sigma_{\mathrm{GRNG}}\to\infty$. Note that λ_1 and λ_2 for GRNG1 and GRNG2 in Fig. 2 correspond to those of the density operator $\rho_S^{(4)}$ of Eq. (10). When $\lambda_1=\lambda_2=0$, it corresponds to two original input pairs of the EPR entangled state without performing GRNG, which may be expressed by the density operator $\rho^{\{0,0\}}_{S(\{M_1,M_2\})}=\int d\eta_1 d\eta_2 Z_1(\eta_1)Z_2(\eta_1)X_1(\eta_2)X_2(-\eta_2)$ (and $\rho^{\{0,0\}}_{S(\{M_3,M_4\})}$ is similar to $\rho^{\{0,0\}}_{S(\{M_1,M_2\})}$) or $\hat{x}_{a(b)_{\mathrm{EPR1}}}+\hat{x}_{a(b)_{\mathrm{EPR2}}}=\hat{p}_{a(b)_{\mathrm{EPR1}}}-\hat{p}_{a(b)_{\mathrm{EPR2}}}\to 0$. Here, due to finite squeezing, the strength of the GRNG does not need to be infinite but will have a lower limit value depending on the squeezing factor r.

Giedke *et al.* [31] give a necessary and sufficient condition for separability of Gaussian states of bipartite systems of arbitrarily many modes. The condition provides an operational criterion since it can be checked by simple computation with the covariance matrix (CM). The Wigner distribution of the Gaussian states can be constructed as $W(R)=\pi^{-N}\exp(-R^T\cdot\Gamma^{-1}\cdot R)$, where $R=(x_1,p_1,x_2,p_2,\ldots,x_N,p_N)^T$ is the vector of phase-space variables. This implicitly defines the elements of the CM Γ, which up to local displacements provides a complete description of the Gaussian states [14,15]. Thus, CM of the four-mode unlockable bound entangled state with finite squeezing can be derived by Eq. (3) and may be used to achieve the condition for separability [31].

The state $\hat{c}_1,\hat{c}_2,\hat{c}_3,\hat{c}_4$ with respect to the partition $\{\{M_1,M_2\},\{M_3,M_4\}\}$ is always separable independent of the strength of the GRNG, as seen in Fig. 2, since the amount of entanglement cannot be increased by local operations and classical communication [27,28]. However, the separability of the state $\hat{c}_1,\hat{c}_2,\hat{c}_3,\hat{c}_4$ with respect to the partition $\{\{M_1,M_4\},\{M_2,M_3\}\}$ depends on the strength of the GRNG. The state $\hat{c}_1,\hat{c}_2,\hat{c}_3,\hat{c}_4$ can be generated equivalently, as shown in Fig. 3. Thus, we may utilize directly the separability criterion in terms of measurable squeezing variances of two-mode states:

$$\langle\delta^2(\hat{x}_1+\hat{x}_2)\rangle+\langle\delta^2(\hat{p}_1-\hat{p}_2)\rangle\geqslant 2.\tag{14}$$

This is a sufficient criterion for separability for any two-mode state, expressed in a form suitable for experimental verification

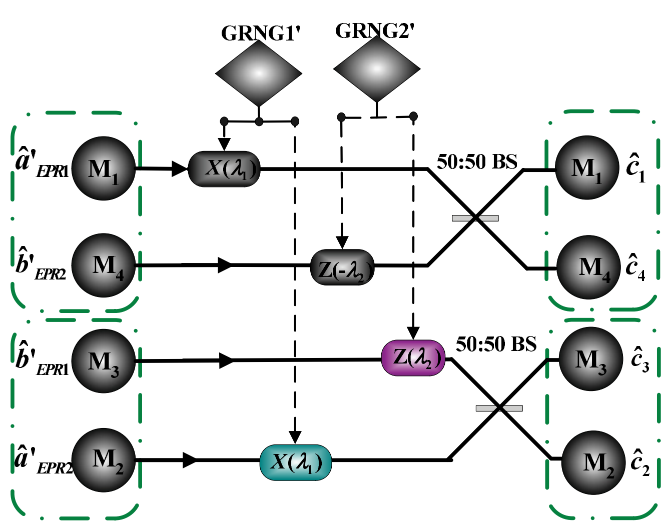

FIG. 3. (Color online) The four-mode unlockable bound entangled state in Fig. 1 is generated equivalently while considering the partition $\{\{M_1,M_4\},\{M_2,M_3\}\}$. BS indicates beam splitter.

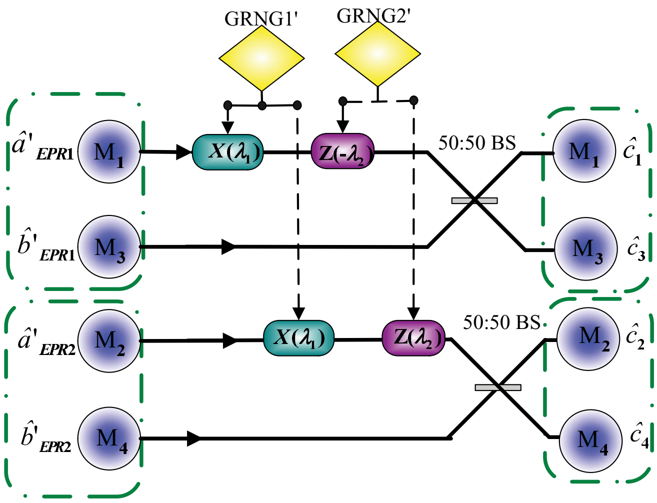

FIG. 4. (Color online) The four-mode unlockable bound entangled state in Fig. 2 is generated equivalently while considering the partition $\{\{M_1,M_3\},\{M_2,M_4\}\}$.

[29,30]. The lowest limit value of the strength of the GRNG1′ can be obtained by

$$\begin{aligned}&\langle\delta^2(\hat{x}_{a'_{\rm EPR1}}+\hat{x}_{a'_{\rm EPR2}})\rangle+4\langle\delta^2(\hat{x}_{\rm GRNG1'})\rangle\\&\quad+\langle\delta^2(\hat{p}_{a'_{\rm EPR1}}-\hat{p}_{a'_{\rm EPR2}})\rangle\geqslant 2\\&\Rightarrow\langle\delta^2(\hat{x}_{\rm GRNG1'})\rangle\geqslant(1-e^{-2r})/2.\end{aligned}\qquad(15)$$

The lowest limit value of the strength of the GRNG2′ can be also obtained with $\langle\delta^2(\hat{p}_{\rm GRNG2'})\rangle\geqslant(1-e^{-2r})/2$. Note that whether this lowest value for GRNG is the necessary and sufficient condition for separability of the state $\hat{c}_1,\hat{c}_2,\hat{c}_3,\hat{c}_4$ with respect to the partition $\{\{M_1,M_4\},\{M_2,M_3\}\}$ still must be further studied by applying the necessary and sufficient condition for separability of Gaussian states of bipartite systems of arbitrarily many modes to check the separability [31]. Considering the partition $\{\{M_1,M_3\},\{M_2,M_4\}\}$, the state $\hat{c}_1,\hat{c}_2,\hat{c}_3,\hat{c}_4$ in Fig. 2 can be generated equivalently, as shown in Fig. 4. It is easily seen that there is an EPR entangled state without any translation operations. Therefore, the four-mode unlockable bound entangled state is always entangled with respect to the partition $\{\{M_1,M_3\},\{M_2,M_4\}\}$. Moreover, the EPR entanglement between modes 1 and 2 (or 3 and 4) can be distilled by letting modes 3 and 4 (1 and 2) come together and performing the joint Bell-basis measurement, and the resulting EPR entanglement becomes $\langle\delta^2(\hat{x}_{\rm EPR1(3)}+\hat{x}_{\rm EPR2(4)})\rangle=2e^{-2r}<1$ and $\langle\delta^2(\hat{p}_{\rm EPR1(3)}-\hat{p}_{\rm EPR2(4)})\rangle=2e^{-2r}<1$. [Here the EPR entanglement is generated unconditionally by displacing the results of the joint Bell-basis measurement on mode 1 (or 2) with a gain factor of 1 [32]. Thus, a certain initial degree of squeezing $r>\ln(2)/2$ should be necessary in order to have entanglement activation.] However, the EPR entanglement between modes 1 and 3 (or 2 and 4) cannot be distilled by letting modes 2 and 4 (1 and 3) come together and performing the joint Bell-basis measurement.

Here we would like to emphasize that the definition of the multipartite bound entangled state in this paper is completely different from the bipartite bound entangled Gaussian states defined in Ref. [17]. In Ref. [17], they study the bound entangled Gaussian states for a bipartite system with an arbitrary number of modes in each party. The bipartite bound entangled Gaussian states are positive partial transpose and thus are undistillable, and they are not separable. According to the definition of the bipartite bound entangled Gaussian states defined in Ref. [17], the CV four-mode unlockable bound entangled state in this paper has three possibilities for bipartition: $\{\{M_1,M_2\},\{M_3,M_4\}\}$, $\{\{M_1,M_4\},\{M_2,M_3\}\}$, and $\{\{M_1,M_3\},\{M_2,M_4\}\}$. The bipartition $\{\{M_1,M_2\},\{M_3,M_4\}\}$ (and $\{\{M_1,M_4\},\{M_2,M_3\}\}$) of the CV four-mode unlockable bound entangled state is separable, which does not hold for a bipartite bound entangled Gaussian state. The bipartition $\{\{M_1,M_3\},\{M_2,M_4\}\}$ of the CV four-mode unlockable bound entangled state is inseparable and also nonpositive partial transpose; thus, it is entangled and distillable.

In conclusion, we have introduced CV multipartite unlockable bound entangled states. It is interesting to further investigate the relationship between the finite squeezing and the strength of the GRNG for more complex CV multipartite unlockable bound entangled states, which relates to the separability problem. CV multipartite unlockable bound entangled states may serve as a useful quantum resource for multiparty communication schemes in the continuous-variable field, such as remote information concentration, quantum secret sharing, and superactivation. We believe that this work will contribute to deeper understanding of CV entanglement.

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