



# Deterministic manipulation of steering between distant quantum network nodes

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**Abstract:** Multipartite Einstein-Podolsky-Rosen (EPR) steering is a key resource in a quantum network. Although EPR steering between spatially separated regions of ultracold atomic systems has been observed, deterministic manipulation of steering between distant quantum network nodes is required for a secure quantum communication network. Here, we propose a feasible scheme to deterministically generate, store, and manipulate one-way EPR steering between distant atomic cells by a cavity-enhanced quantum memory approach. While optical cavities effectively suppress the unavoidable noises in electromagnetically induced transparency, three atomic cells are in a strong Greenberger-Horne-Zeilinger state by faithfully storing three spatially separated entangled optical modes. In this way, the strong quantum correlation of atomic cells guarantees one-to-two node EPR steering is achieved, and can preserve the stored EPR steering in these quantum nodes. Furthermore, the steerability can be actively manipulated by the temperature of the atomic cell. This scheme provides the direct reference for experimental implementation for one-way multipartite steerable states, which enables an asymmetric quantum network protocol.

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## 1. Introduction

Quantum network consists of quantum nodes and quantum channels. While quantum channels are used to transmit quantum information and connect quantum nodes, quantum nodes can store and process quantum information. Light is an ideal carrier of quantum information, and optical fiber or space channels can be employed as quantum channels [1,2]. Single atoms [3,4], atomic ensembles [5–10], trapped ions [11–13], superconductors [14], solid-state systems [15–18], optomechanics [19–22], and so on have been used as quantum nodes. And atomic ensembles are one of the candidates for quantum nodes due to the advantage of the collective enhancement of light and atom interaction [5–10]. Entanglement is a fundamental concept of quantum physics, has been used in quantum network [23,24], such as quantum teleportation networks [25], controlled dense coding [26], and quantum secret sharing [27]. The entangled quantum nodes are required for the applications of quantum networks [28–35], and especially the multipartite entanglement [36,37] among three [38,39] and four [40] atomic ensembles has been experimentally demonstrated.

Einstein-Podolsky-Rosen (EPR) steering is not only an intriguing feature of quantum mechanics, but also a valuable quantum resource in secure quantum communication networks [41–45]. EPR steering is defined in terms of violations of a local hidden state model, and is an intermediate type between Bell nonlocality [46] and entanglement [47] which allows one party to steer the state of a distant party [48]. If the correlations are sufficiently strong, local measurements in one party can change the quantum state in a distant party. The asymmetric steerability of two directions between the distant parties has been demonstrated in photonic and optical systems [49–51], which provides a way to demonstrate quantum correlation without the trustworthy requirement

of quantum users. Especially, the one-way EPR steering is important for its application where the levels of trust at different parties are highly asymmetric. Due to the asymmetric feature, EPR steering can be potentially applied for one-sided device-independent (1sDI) quantum key distribution [52–54], 1sDI quantum secret sharing protocol [55], and secure quantum teleportation [56]. With the development of quantum networks, the multipartite EPR steering which has been demonstrated in optical [57,58] and photonic networks [59–61], such as 1sDI secret sharing [55]. In atomic-ensemble-based quantum network, EPR steering between spatially separated regions of an ultracold atomic system has been observed [62,63], however, deterministic generation, storage and manipulation of steering among multiple distant quantum nodes still remain a challenge up to now.

Here, we propose a feasible scheme that an one-way EPR steering in three distant atomic cells can be deterministically generated, stored and manipulated based on the cavity-enhanced quantum memory approach. Greenberger-Horne-Zeilinger (GHZ) state plays an important role in quantum communication networks, which is generated by coupling three squeezed optical modes, generated from three optical parametric amplifiers (OPAs), are coupled on a beam splitter network. These three entangled optical modes are distributed and stored in three atomic cells via electromagnetically induced transparency (EIT) interaction. At the same time, the four-wave-mixing (FWM) noise is the main noise in quantum memory process. The optical cavity can obviously increase the EIT interaction, when the signal mode is resonant with the cavity; meanwhile, it enables to effectively suppress FWM noise. The cavity-enhanced approach can suppress excess noise in quantum memory [64], when the memory interaction is enhanced [65]. Based on cavity-enhanced quantum memory approach, the resulting entanglement among three atomic cells is strong enough for EPR steering, and one-to-two node EPR steering among three nodes is achieved. By reconstructing the covariance matrix of quantum state, we quantify the EPR steering. The stored EPR steering can be preserved in these quantum nodes. The temperature of atomic cell is the key factor in quantum steering manipulation scheme. When the memory noise is close to the shot noise limit (SNL) level, the steerability can be actively manipulated by controlling the temperature of atomic cell. This scheme provides the direct reference for experimental implementation.

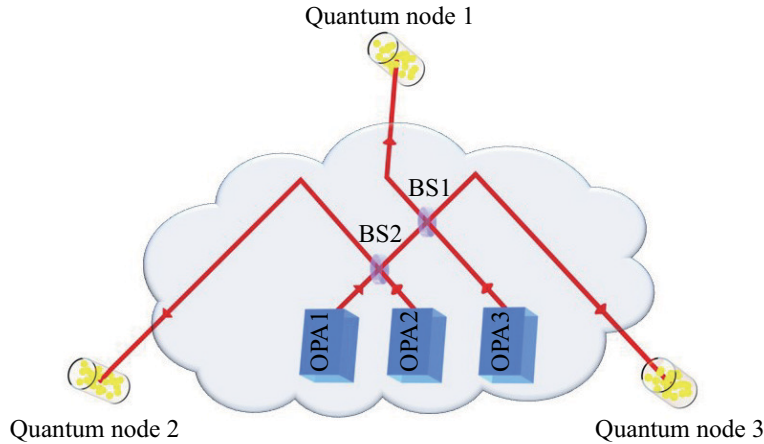
## 2. Manipulation of steering among three distant atomic cells

Our protocol consists of the entanglement source of tripartite entangled states which is an important resource for constructing quantum networks and three atomic cells as the quantum nodes, the schematic for the manipulation of EPR steering among three distant quantum nodes is shown in Fig. 1. In the entanglement source, three squeezed states of light are generated from three OPAs and coupled on a beam splitter network. Then the entanglement is distributed into three quantum nodes consisting of atomic cells.

The tripartite entangled state is prepared deterministically by coupling two amplitude-squeezed states of light ( $\hat{a}_1$  and  $\hat{a}_3$ ) and a phase-squeezing state of light ( $\hat{a}_2$ ) on a beam splitter network which consists of two beam splitters with transmittance of  $T_1 = 1/3$  and  $T_2 = 1/2$ , respectively. The quadrature amplitude and phase squeezed states are produced by OPAs, and the expression of input squeezed states are

$$\begin{aligned}\hat{a}_1 &= \frac{1}{2} \left( e^{-r_1} \hat{x}_1^{(0)} + i e^{r_1} \hat{p}_1^{(0)} \right), \\ \hat{a}_2 &= \frac{1}{2} \left( e^{r_2} \hat{x}_2^{(0)} + i e^{-r_2} \hat{p}_2^{(0)} \right), \\ \hat{a}_3 &= \frac{1}{2} \left( e^{-r_3} \hat{x}_3^{(0)} + i e^{r_3} \hat{p}_3^{(0)} \right),\end{aligned}\tag{1}$$

where  $r_i (i = 1, 2, 3)$  is the squeezing parameter, the amplitude and phase quadratures of an optical field  $\hat{a}$  are  $\hat{x} = \hat{a} + \hat{a}^\dagger$  and  $\hat{p} = (\hat{a} - \hat{a}^\dagger)/i$ , respectively. The losses at the beam splitter network for



**Fig. 1.** The schematic for the manipulation of EPR steering among three distant quantum nodes.

the generation of entangled optical fields are unavoidable and should be considered. These losses are modeled with transmission efficiency  $\eta$  in the beam splitter network, whose output modes are

$$\begin{aligned}\hat{A} &= \sqrt{\eta}(\sqrt{\frac{2}{3}}\hat{a}_1 + \sqrt{\frac{1}{3}}\hat{a}_2) + \sqrt{1-\eta}\hat{v}_A, \\ \hat{B} &= \sqrt{\eta}(-\sqrt{\frac{1}{6}}\hat{a}_1 + \sqrt{\frac{1}{3}}\hat{a}_2 + \sqrt{\frac{1}{2}}\hat{a}_3) + \sqrt{1-\eta}\hat{v}_B, \\ \hat{C} &= \sqrt{\eta}(-\sqrt{\frac{1}{6}}\hat{a}_1 + \sqrt{\frac{1}{3}}\hat{a}_2 - \sqrt{\frac{1}{2}}\hat{a}_3) + \sqrt{1-\eta}\hat{v}_C,\end{aligned}\quad (2)$$

where  $\hat{v}_{i(i=A,B,C)}$  are vacuum noises coupled into the signal channels. Then the multipartite entanglement are distributed into three quantum nodes. Each user can store the distributed entangled state in the atomic cell to establish entangled quantum nodes. In cavity-enhanced quantum memory, three atomic cells can be entangled by storing the entangled states. Atomic cells which are filled with  $^{87}\text{Rb}$  atoms transmit quantum information. The tripartite entangled optical modes are distributed into three atomic cells via EIT interaction. The distribution of quantum steering in the three atomic cells are theoretically studied, and the important part of our protocol is to analyze the properties of tripartite steering scheme.

EIT is the result of quantum interference between coherent electromagnetic field and multi-level atomic system which includes three energy configurations,  $\Lambda$ -type energy structure is applied in a wide range of quantum memory. The  $\Lambda$ -type three-level system of  $^{87}\text{Rb}$  atom  $D1$  line, which constitutes a ground state  $|g\rangle$ , a meta-stable state  $|m\rangle$  and an excited state  $|e\rangle$ . The signal mode is near-resonance with the transition between a ground state  $|g\rangle$  and an excited state  $|e\rangle$ , while the control mode is near-resonance with the transition between a meta-stable state  $|m\rangle$  and an excited state  $|e\rangle$ . Quantum memory based on EIT is an essential mechanism in quantum networks. When the collective atomic spin wave  $\hat{S}(t)$  interacts with the signal field  $\hat{a}(t)$  via EIT dynamics, the quantum state of signal mode and the atomic ensemble can be transferred to each other, because the effective Hamiltonian  $\hat{H}_{EIT}$  of light-atom interaction can be described by the beam-splitter solution, as  $\hat{H}_{EIT} = \hbar\kappa\hat{a}^\dagger\hat{S} + \hbar\kappa\hat{S}^\dagger\hat{a}$ , where  $\kappa$  is the effective interaction between signal mode and atomic ensemble. The quantum memory process of entangled state includes three stages which are writing, storage and reading. In the writing process ( $-\infty < t < 0$ ), both the weak input entangled optical mode and the strong control mode interact with an atomic medium.

The entangled optical mode and the strong control mode have different wavelength, so that the atomic medium becomes transparent for the entangled optical mode and the group velocity for the entangled optical field is reduced. In the storage process ( $0 < t < T_0$ ), when the whole entangled optical field is totally compressed into the atomic medium, the control mode is adiabatically switched off, and the quantum state is stored and preserved in atomic medium. In the reading process ( $T_0 < t < \infty$ ), the control mode is adiabatically switched on again, and the quantum state can be transferred from the atomic medium to the released optical mode.

High-performance quantum memory with the necessary features of both high writing efficiency and low excess noise is a prerequisite building block of quantum steering manipulation scheme. In order to improve the performance of quantum memory, we present a cavity-enhanced quantum EIT memory with warm atomic cell. The cavity with a bow tie-type ring configuration consists of two plano mirrors and two concave mirrors, which enables to enhance the light-atom interaction and suppress the excess noise, the warm atomic cell is placed between the two plano mirrors. The input signal mode  $\hat{A}(t)^{in}$  can be coupled into the cavity mode  $\hat{a}$  through the input-output mirror with the coupling rate to the cavity of the input mode  $\gamma_1 = T/2\tau$ , where the  $T$  is the transmission of input-output mirror and  $\tau$  is the round-trip time of the optical mode. The other mirrors are highly reflective for the signal mode and one of them is mounted on the piezoelectric transducer for scanning or locking the cavity length. By adjusting the appropriate cavity length, the signal mode is resonant with the optical cavity and the control mode is near-resonance with the optical cavity, while the FWM optical mode is anti-resonant. Thus, the memory interaction is enhanced and the scattered FWM mode is suppressed in the cavity. In this way, the high-fidelity quantum memory can be realized. In the further research, both the signal and control modes can resonant with the optical cavity and FWM noise is totally off resonant with the optical cavity by employing the birefringent crystal [64]. Also, the power of control mode can also be obviously reduced. The intracavity loss  $L$  is unavoidable as a result of imperfect optical components and the corresponding decay rate of the intracavity loss is  $\gamma_2 = L/2\tau$ , which introduces the vacuum noise  $\hat{A}(t)_v^{in}$ . The spin wave decoherence rate is  $\gamma_0$ , which couples the noise of the atomic medium  $\hat{S}(t)_v$  into the cavity mode  $\hat{a}$ . Quantum Langevin equations describing evolution of observable operators for the cavity mode  $\hat{a}(t)$  and collective atomic spin wave  $\hat{S}(t)$  are shown as

$$\frac{d\hat{a}(t)}{dt} = -\gamma\hat{a}(t) - i\kappa(t)\hat{S}(t) + \sqrt{2\gamma_1}\hat{A}(t)^{in} + \sqrt{2\gamma_2}\hat{A}(t)_v^{in}, \quad (3)$$

$$\frac{d\hat{S}(t)}{dt} = -\gamma_0\hat{S}(t) - i\kappa(t)\hat{a}(t) + \sqrt{2\gamma_0}\hat{S}(t)_v, \quad (4)$$

where  $\gamma = \gamma_1 + \gamma_2$  corresponds the sum of the coupling rate and the decay rate of cavity. By solving quantum Langevin equations with the proper input temporal mode function, the writing efficiency  $\eta(T_0)_W$  at the user-controlled storage time  $T_0$  from input signal mode to the collective atomic spin wave is given by

$$\eta(T_0)_W = \frac{\gamma_1\kappa^2 e^{-\gamma_0 T_0}}{(\gamma_0 + \gamma)(\kappa^2 + \gamma_0\gamma)}. \quad (5)$$

The writing efficiency mainly depends on the input-output mirror transmission  $T$  and intracavity loss  $L$ . The writing efficiency of entangled optical modes in cavity-enhanced quantum memory can be obtained by this equation.

Due to the existence of the unavoidable excess noises in the experiment, the noise values of the measured states are always higher than the quantum noise limit (QNL). In the processing of atom-light interaction with a  $\Lambda$ -type energy configuration, the coupling of the control mode on the signal mode transition will induce unwanted FWM noise, which is the main noise source of

quantum memory. The FWM process is a third-order nonlinearity interaction process between medium and optical mode, the Hamiltonian of the parametric down-conversion process [66] is given by  $\hat{H} = X^{(3)}A_C^2\hat{a}_S^+\hat{a}_F^+ + H.c.$ , where the parameter  $X^{(3)}$  is associated with the nonlinear susceptibility coefficient of gain medium,  $A_C$  is the strong control mode,  $\hat{a}_S$  is the signal mode, and  $\hat{a}_F$  stands for FWM mode. In cavity-enhanced quantum memory process, the FWM noise can be suppressed by resonant cavity. In this case, the phases of FWM and signal modes are  $\pi$  and 0, respectively. We can calculate the cavity length is 500 mm and the noise level of quantum memory is close to QNL. The optical cavity can enhance the light-atom interaction and suppress the excess noise, thus we can obtain high-performance quantum memory and effectively establish quantum steering among three atomic cells.

By reconstructing the covariance matrix of three atomic cells with a GHZ state, the amount of EPR steering is quantified. The properties of a  $(n_A + m_B)$ -mode Gaussian state  $\rho_{AB}$  of a bipartite system can be determined by its covariance matrix

$$\sigma_{AB} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}, \quad (6)$$

with elements  $\sigma_{ij} = \langle \hat{\xi}_i \hat{\xi}_j + \hat{\xi}_j \hat{\xi}_i \rangle / 2 - \langle \hat{\xi}_i \rangle \langle \hat{\xi}_j \rangle$ , where  $\hat{\xi} \equiv (\hat{x}_1^A, \hat{p}_1^A, \dots, \hat{x}_n^A, \hat{p}_n^A, \hat{x}_1^B, \hat{p}_1^B, \dots, \hat{x}_m^B, \hat{p}_m^B)$  is the vector of the amplitude and phase quadratures of optical modes. The submatrices  $A$  and  $B$  are corresponding to the reduced states of Alice's and Bob's subsystems, respectively.

The necessary and sufficient criterion for steering of Gaussian states is in terms of the symplectic eigenvalues  $\bar{v}_j^{AB|A}$  of the Schur [67] complement. Based on the covariance matrix of the quantum state, the steerability of Bob by Alice ( $A \rightarrow B$ ) for a  $(n_A + m_B)$ -mode Gaussian state can be quantified by [67]

$$\mathcal{G}^{A \rightarrow B}(\sigma_{AB}) = \max\{0, -\sum_{j: \bar{v}_j^{AB|A} < 1} \ln(\bar{v}_j^{AB|A})\}, \quad (7)$$

where  $\bar{v}_j^{AB|A}$  ( $j = 1, \dots, m_B$ ) are the symplectic eigenvalues of  $\sigma_{AB|A} = B - C^T A^{-1} C$ , derived from the Schur complement of  $A$  in the covariance matrix  $\sigma_{AB}$ . The steerability of Alice by Bob [ $\mathcal{G}^{B \rightarrow A}(\sigma_{AB})$ ] can be obtained by swapping the roles of  $A$  and  $B$ .

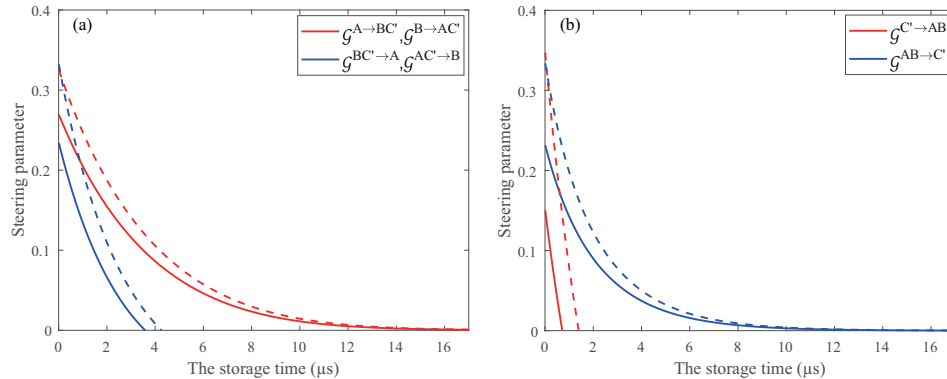
In this protocol, the tripartite entangled states can be stored in three atomic cells by modulating the light-atom interaction with control modes. It is necessary to calculate three covariance matrices for each quantum state and get the steering parameters from the corresponding values. The steerability of three atomic cells will be influenced by the writing efficiency of tripartite entangled optical modes and extra noise. Based on the covariance matrices for three spin waves, we can analyze the steerabilities between three atomic cells.

### 3. Results and discussions

EPR steering is a strict subset of entanglement and superset of Bell nonlocality. In the view of quantum information processing, EPR steering can be regarded as a verifiable entanglement distribution by an untrusted party, while entangled states need both parties to trust each other, and the Bell nonlocality is valid on the premise that they distrust each other. In Ref. [38], tripartite entanglement of atomic ensembles has been observed. However, in our scheme, the tripartite steering of atomic cells is investigated. We first prepare the tripartite entanglement in three atomic cells. Then, we analyze tripartite steering in the three atomic cells based on tripartite entanglement. In our system, we have assumed that three squeezed states have the identical squeezing parameter  $r = r_1 = r_2 = r_3 = 0.576$  in theoretical prediction, which corresponding to  $-5$  dB squeezing, and all the losses of beam splitters are summed to be 1%. We can calculate the writing efficiency of entangled optical mode according to the theoretical arithmetic. When

the atomic cell is heated to around  $95^{\circ}\text{C}$ , the transmission of the input-output mirror is 0.3 and the intracavity loss is 0.02, both the writing efficiency of  $A(T_0)$  and  $B(T_0)$  modes at the storage time of 100 ns is 88%. In this case, both the extra noise of  $A(T_0)$  and  $B(T_0)$  modes is 0.0002. We confirm the writing efficiency and extra noise of  $A(T_0)$  and  $B(T_0)$  modes, and analyze the steerability between different parties by changing the parameter in cavity-enhanced quantum memory of  $C'$  mode.

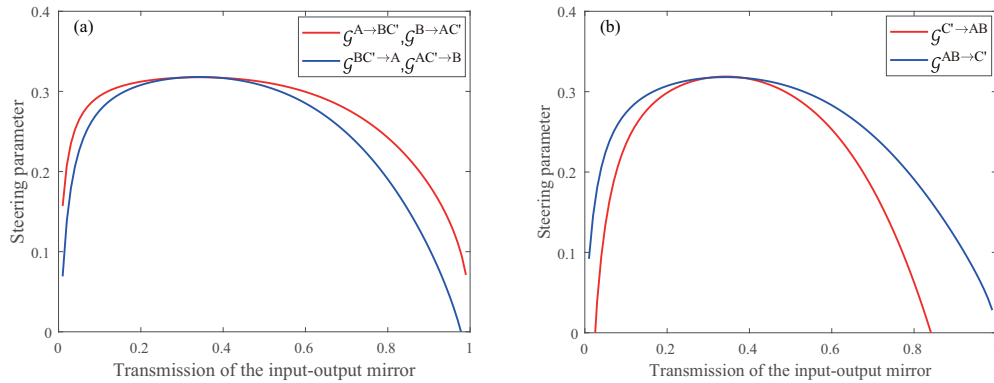
The conditions for the generation of one-way, two-way steering and entanglement are different from each other. We analyze the steerability between (1 + 2)-node and (2 + 1)-node partitions vs the storage time in different atomic cell temperature of cavity-enhanced quantum memory of  $C'$  mode in Fig. 2. We can see that any two quantum nodes can collectively steer the third node. Inequal steerability between (1 + 2)-node and (2 + 1)-node is shown due to the manipulation of the temperature of atomic cell. When the temperature of atomic cell is  $50^{\circ}\text{C}$ , two-way EPR steering of  $\mathcal{G}^{A\rightarrow BC'}$ ,  $\mathcal{G}^{B\rightarrow AC'}$  ( $\mathcal{G}^{BC'\rightarrow A}$ ,  $\mathcal{G}^{AC'\rightarrow B}$ ) and of  $\mathcal{G}^{C'\rightarrow AB}$ ,  $\mathcal{G}^{AB\rightarrow C'}$  are observed at the storage time range from 0 to  $3.6\ \mu\text{s}$ , and from 0 to  $0.7\ \mu\text{s}$ , respectively. One-way EPR steering  $\mathcal{G}^{A\rightarrow BC'}$ ,  $\mathcal{G}^{B\rightarrow AC'}$  ( $\mathcal{G}^{BC'\rightarrow A}$ ,  $\mathcal{G}^{AC'\rightarrow B}$ ) and of  $\mathcal{G}^{C'\rightarrow AB}$ ,  $\mathcal{G}^{AB\rightarrow C'}$  are observed at the storage time from  $3.6$  and  $15\ \mu\text{s}$  and from  $0.7$  to  $12\ \mu\text{s}$ . If the temperature is increased to be  $95^{\circ}\text{C}$ , the storage time when two-way EPR steering of  $\mathcal{G}^{A\rightarrow BC'}$ ,  $\mathcal{G}^{B\rightarrow AC'}$  ( $\mathcal{G}^{BC'\rightarrow A}$ ,  $\mathcal{G}^{AC'\rightarrow B}$ ) and of  $\mathcal{G}^{C'\rightarrow AB}$ ,  $\mathcal{G}^{AB\rightarrow C'}$  are changed to one-way EPR steering is  $4.3\ \mu\text{s}$  and  $1.4\ \mu\text{s}$ , respectively. The positive partial transposition (PPT) criterion is a necessary and sufficient condition for quantum entanglement of Gaussian state [68,69]. According to PPT criterion, the tripartite entanglement is investigated. Although the PPT values get larger from 0.46 and 0.49 with the increase of storage time, they can be smaller than the boundary of 1 beyond the storage time of  $15\ \mu\text{s}$ .



**Fig. 2.** EPR steering between one and two nodes vs the storage time in different atomic cell temperature. (a) The steerability between  $\mathcal{G}^{A\rightarrow BC'}$ ,  $\mathcal{G}^{B\rightarrow AC'}$  and  $\mathcal{G}^{BC'\rightarrow A}$ ,  $\mathcal{G}^{AC'\rightarrow B}$ . (b) The steerability between  $\mathcal{G}^{C'\rightarrow AB}$  and  $\mathcal{G}^{AB\rightarrow C'}$ . The solid line and dashed line stand for the steerability in  $50^{\circ}\text{C}$  and  $95^{\circ}\text{C}$  of atomic cell, respectively.

Furthermore, we study quantitatively the optimal experimental condition of EPR steering in atomic cells. We analyze the steerability by changing transmission of the input-output mirror  $T$  in cavity-enhanced quantum memory of optical mode  $C'$ , the steerability between (1 + 2)-node and (2 + 1)-node partitions vs the transmission of input-output mirror  $T$  in Fig. 3. Figure 3 (a) shows the steerability between  $\mathcal{G}^{A\rightarrow BC'}$ ,  $\mathcal{G}^{B\rightarrow AC'}$  and  $\mathcal{G}^{BC'\rightarrow A}$ ,  $\mathcal{G}^{AC'\rightarrow B}$ . Figure 3 (b) shows the steerability between  $\mathcal{G}^{C'\rightarrow AB}$  and  $\mathcal{G}^{AB\rightarrow C'}$ . There is an optimal transmission to make the maximize entanglement for steerability. In this system, the transmission of the input-output mirror  $T$  is 0.3.

From these results, we demonstrate that the steerability exists between the (1 + 2)-node and (2 + 1)-node partitions. With the same method as the (1 + 2)-node and (2 + 1)-node partitions of



**Fig. 3.** EPR steering between one and two nodes vs the transmission of the input-output mirror in cavity-enhanced quantum memory. (a) The steerability between  $\mathcal{G}^{A \rightarrow BC'}$ ,  $\mathcal{G}^{B \rightarrow AC'}$  and  $\mathcal{G}^{BC' \rightarrow A}$ ,  $\mathcal{G}^{AC' \rightarrow B}$ . (b) The steerability between  $\mathcal{G}^{C' \rightarrow AB}$  and  $\mathcal{G}^{AB \rightarrow C'}$ .

the three atomic cells, the steerability between (1 + 1)-node can be calculated. In EPR steering between (1 + 1)-node, all the steerabilities between (1 + 1)-node are below 0 with the optimized experimental conditions. Thus, there is no steering between any (1 + 1)-node of the three atomic cells. These results confirm that EPR steering among three entangled atomic cells can be generated, stored and manipulated.

#### 4. Conclusions

In summary, we propose a feasible scheme to generate, store and manipulate EPR steering in three entangled atomic cells based on cavity-enhanced quantum memory approach. More importantly, this EPR steering can be preserved in atomic cells. The optimal conditions, including squeezed parameter of OPA and the transmission of input-output mirror of cavity, for realizing the steerability over (1 + 2)-node and (2 + 1)-node partitions have been obtained. By controlling the temperature of atomic cell, the steerability can be actively manipulated. Furthermore, the main factors that limit the steerability between distant nodes are the squeezed degree of non-classical light, the memory efficiency and the noise of atomic cell. By optimizing the generation, transmission and detection of non-classical light, the squeezed degree can be improved [70,71]. By selecting optimal the transmission of input-output mirror of cavity based on the mode matching mechanism, and improving the coating of atomic cell, the memory efficiency can be improved. The noise can be more effectively suppressed if the birefringent crystal technique is employed [64]. The mature quantum optical technology can be used for realizing the quantum steerability of more nodes in quantum networks. These results demonstrate that the EPR steering of multiple distant three quantum nodes is a key resource of asymmetric quantum information network protocol.

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**Disclosures.** The authors declare no conflicts of interest.

**Data availability.** Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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