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Letter

Deterministic and multiuser quantum teleportation network of continuous-variable polarization states

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Quantum teleportation is widely applied as a key ingredient in quantum information science, and quantum network provides unique abilities for upscaling its applications. Polarization state of light is a vital degree of freedom with the advantages of remote transfer and direct light-atom interaction. To construct a multiuser quantum network, deterministic quantum teleportation of polarization state in more users' network remains challenging. Compared with photonic quantum state, the deterministic quantum teleportation network of polarization state is realized with a single set of continuous-variable entangled states. The key techniques include only one set of quadrature entangled network, single quadrature controller, and active polarization controller. Arbitrary polarization state is deterministically and controllably teleported in such a system involving four or three users. This system is deterministic and scalable with user number, which enables the potential application in deterministic metropolitan quantum network.

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Quantum teleportation plays a key role in applications of quantum communication, quantum computation, and quantum precision measurement [1–4]. Recent remarkable advances in developing quantum networks have led to a plethora of groundbreaking applications [5–7]. Quantum network consisting of remote atomic nodes [8,9] and more users [5,6] acts as the building block of a range of fields, ranging from quantum precision measurement network [10,11] and quantum communication network [12] to measurement-based quantum computation [13–16]. Much more substantial advantages of quantum teleportation network are expected when teleportation of quantum logic gates based on multiparty entangled network underlies the proposals of distributed quantum computation [17]. In multiuser quantum network, polarization state of light has emerged as one of the promising degrees of freedom for remote transferring of quantum information [18–20] and direct interaction with spin state in atomic node [21]. The teleportation of polarization states of single photons has been achieved in probabilistic manner. Polarization states of single photons are teleported onto not only remote photonic qubits [22] but also atomic nodes [23]. Furthermore, groundbreaking results in teleportation network with four and three users are achieved using photonic polarization states [24] and spin states [25]. Although the discrete variable polarization state has marked progress, its achievable efficiency is restricted by intrinsic probability [26]. As the user number increases, the success probability decreases [24].

On the other hand, the continuous variable (CV) quantum states of optical modes can realize the deterministic quantum teleportation network, due to the unconditional generation, manipulation, and measurements. In this scheme, the deterministic quantum teleportation network of polarization states in four-user network is implemented by utilizing the CV Greenberger-Horne-Zeilinger (GHZ) state. Thus, the CV polarization state is transferred instantly rather than probabilistically, and this protocol is efficient. The CV GHZ entangled state is by analogy with the discrete variable GHZ state [27]. It usually contains a total correlation relationship among the one component of participant and the relative correlation relationship in the other orthogonal component. The one of the ideal quadrature GHZ state is the eigenstate of zero total position ($\delta\hat{X}_1 + \delta\hat{X}_1 + \dots + \delta\hat{X}_n = 0$) and zero relative momentum ($\delta\hat{P}_j - \delta\hat{P}_k = 0$, where $j, k = 1, 2, \dots, n, i \neq j$). Because of finite squeezing and unavoidable losses in the experiment, when the quantum correlation variances of both quadrature amplitude sum $[\Delta^2(\delta\hat{X}_1 + \delta\hat{X}_1 + \dots + \delta\hat{X}_n)]$ and quadrature phase difference $[\Delta^2(\delta\hat{P}_j - \delta\hat{P}_k)]$ are lower than the corresponding quantum noise limits (QNL), the state can be seen as a n partite fully inseparable GHZ state [28]. Besides quadrature components [29,30], different CV degrees of freedoms are required for practical applications [26,31,32]. Especially, the CV polarization state of light is a vital degrees of freedom in multiuser quantum network [33–36], which enables the combination of advantages of the convenient local oscillation-free measurement for remote distribution [37], and direct light-atom interaction for storing quantum state and sensing weak magnetic field [38,39]. Quantum teleportation

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between two users has been proposed [40]. With the development of quantum network, it is demanded to involve more users, and thus we propose deterministic quantum teleportation of the CV polarization state in multiple user network at minimum cost of quantum resource.

Here, we propose deterministic and multiuser quantum teleportation of the CV polarization state in multiple user network. We experimentally present the four-user and three-user teleportation network for CV polarization state. The experimental fidelity of 0.49 and 0.53 are achieved with four and three user, both of which are beyond the classical limit of 0.41. Controlled quantum teleportation with more users is the main aim for practical application. The controlled quantum teleportation has been demonstrated [27,41]. The CV GHZ state is used as quantum channels, and there exist quantum correlations among all users $[\Delta^2(\delta\hat{X}_1 + \delta\hat{X}_1 + ... + \delta\hat{X}_n) = ne^{-2r} < n$, where r is the squeezing parameter and larger than 0]. While parts of users (n-m users) take part in teleportation, the quantum correlations $[\Delta^2(\delta\hat{X}_1 + \delta\hat{X}_1 + ... + \delta\hat{X}_{n-m}) = (n-m)^2e^{-2r}/n + (2nm - m^2)e^{2r}/n > n$ with larger r] will be destroyed, and input state cannot be reconstructed by the receiver. With and without the controllers' results, the success or failure of teleportation is controlled. Fidelity will surpass the classical limit when the correlation is lower than QNL with the controllers' measurement results; otherwise, fidelity will be destroyed by thermal state correlation noise in the absence of controllers' measurement results. In this scheme, the controlled quantum teleportation for the CV polarization state is implemented via a single set of the quadrature GHZ state, together with single quadrature controller [40]. In controlled quantum teleportation for the quadrature state [27,41], a pair of conjugate quadrature operators are needed to describe the quantum state, and thus it is possible to transfer two quadrature operators via one set of quadrature entangled states [2]. The polarization components are represented by the Stokes operators, and their sideband noise can be eliminated in classical optics. However, the unavoidable vacuum fluctuations of the Stokes operator sideband cannot be eliminated in quantum optics [42]. The sideband variances of the Stokes operators are restricted by uncertainty relations, so that they cannot be simultaneously measured because of quantum nature. The Stokes operators $\hat{S}_j$ ($j = 0$, 1, 2, 3) can be written as

$$
\begin{aligned}
\hat{S}_0 &= \hat{n}_H + \hat{n}_V \\
&= \alpha_H^2 + \alpha_H\delta\hat{X}_H + \delta\hat{a}_H^\dagger\delta\hat{a}_H + \alpha_V^2 + \alpha_V\delta\hat{X}_V + \delta\hat{a}_V^\dagger\delta\hat{a}_V, \\
\hat{S}_1 &= \hat{n}_H - \hat{n}_V \\
&= \alpha_H^2 + \alpha_H\delta\hat{X}_H + \delta\hat{a}_H^\dagger\delta\hat{a}_H - \alpha_V^2 - \alpha_V\delta\hat{X}_V - \delta\hat{a}_V^\dagger\delta\hat{a}_V, \\
\hat{S}_2 &= \hat{n}_D - \hat{n}_{\overline{D}} \\
&= 2\alpha_H\alpha_V\cos\theta + \alpha_V(\delta\hat{X}_H\cos\theta + \delta\hat{P}_H\sin\theta) + \delta\hat{a}_H^\dagger\delta\hat{a}_V e^{i\theta} \\
&\quad + \alpha_H(\delta\hat{X}_V\cos\theta - \delta\hat{P}_V\sin\theta) + \delta\hat{a}_V^\dagger\delta\hat{a}_H e^{-i\theta}, \\
\hat{S}_3 &= \hat{n}_R - \hat{n}_L \\
&= 2\alpha_H\alpha_V\sin\theta + \alpha_V(\delta\hat{X}_H\sin\theta - \delta\hat{P}_H\cos\theta) - i\delta\hat{a}_H^\dagger\delta\hat{a}_V e^{i\theta} \\
&\quad + \alpha_H(\delta\hat{X}_V\sin\theta + \delta\hat{P}_V\cos\theta) + i\delta\hat{a}_V^\dagger\delta\hat{a}_H e^{-i\theta}, \qquad (1)
\end{aligned}
$$

where $\hat{n}$ is the photon number operator with subscript of polarization mode $H(V)$, D $(\overline{D})$, $R(L)$ that represents horizontal (vertical), diagonal (antidiagonal), and right (left) circular polarization, $\hat{a}(\hat{a}^\dagger)$ is the annihilation (creation) operator with classical amplitude α, $\delta\hat{X}(\delta\hat{P})$ is quantum fluctuation of amplitude (phase) quadrature, θ is the phase between H and V polarization mode, and i is an imaginary unit; it can be seen that the four Stokes operators are a linear combination of conjugate quadrature operators with horizontal and vertical polarizing modes. The coherent state is a state with equal uncertainties, and the quantum nature of the CV coherent state is visualized by an appropriate sphere instead of one point in the Poincare sphere [43]. The coherent state is usually used as the input state in CV quantum information. Thus, the CV polarization coherent state is used as the input state of the controlled quantum teleportation system. Furthermore, it is possible to employ the CV polarization squeezed and entangled states as the input state of this system. It is a direct approach to transfer or store the four Stokes operators via dual-rail strategy, where two sets of quantum systems with orthogonal polarization are employed [44,45]. In fact, $\hat{S}_0$ commutes with other Stokes operators $\hat{S}_1$, $\hat{S}_2$, and $\hat{S}_3$ ($[\hat{S}_0, \hat{S}_j] = 0$, where $j = 1, 2, 3$), which can be measured without the penalty on the remaining three operators. Three Stokes operators $\hat{S}_1$, $\hat{S}_2$, and $\hat{S}_3$ are used to construct the Poincare sphere and completely describe the CV polarization state. Particularly, $\hat{S}_1$ commutes with $\hat{S}_2$ and $\hat{S}_3$ ($[\hat{S}_1, \hat{S}_2] = 0$, $[\hat{S}_1, \hat{S}_3] = 0$), if the horizontal polarizing mode has zero mean value. Based on the above commuting relations of Stokes operators, it is possible to simultaneously transfer $\hat{S}_2$ and $\hat{S}_3$ with the horizontal polarizing mode via a single set of CV quadrature GHZ state; while $\hat{S}_1$ with a vertical polarizing mode is independently transferred by another set of classical systems, where a single quadrature measurement feedback system is used. Therefore, only a single set of quantum entangled system is required in this scheme, and we claim that the setup is at a minimum cost of the quantum resource. For four-user network, the required number of nondegenerate optical parametric amplifiers (NOPAs) for CV polarization state teleportation can be reduced from four to two, which corresponds to a single set of quadripartite entangled source.

Besides quantum teleportation network of a fixed polarization, an active polarization controller is required to shift an arbitrary carrier polarization to this fixed polarization, and return it to the original polarization. According to sideband mode of quantum optics, the quantum state is represented by both carrier and sideband fluctuation ($\hat{a} = \alpha + \delta\hat{a}$, where α is classical amplitude and $\delta\hat{a}$ is quantum fluctuation), where quantum property is determined by the last term. In quadrature teleportation, the sideband fluctuations of quadrature components are transferred, while the input carrier amplitude is unimportant [2]. When the unknown quantum state of three Stokes operators, including both carrier polarization and sideband fluctuation, is used as the input state, the sideband fluctuations $\delta\hat{S}_1$, $\delta\hat{S}_2$, and $\delta\hat{S}_3$ determine quantum property. In such a system, the quantum property of $\delta\hat{S}_1$ can be independently transferred using the single quadrature controller ($\delta\hat{S}_1 = -\alpha_V\delta\hat{X}_V$). It is solely required to transfer the $\delta\hat{S}_2$ and $\delta\hat{S}_3$ via a single set of quadrature GHZ states ($\delta\hat{S}_2 = \alpha_V\delta\hat{P}_H$, $\delta\hat{S}_3 = \alpha_V\delta\hat{X}_H$). Like the phase locking approach of squeezed

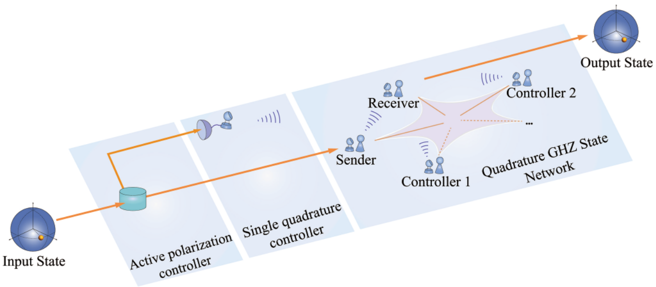

FIG. 1. Schematic of the deterministic and multiuser quantum teleportation network of the CV polarization state. Active polarization controller shifts an arbitrary polarization to vertical polarized state, and outputs its horizontal and vertical components. Single quadrature controller teleports $\hat{S}_1$ through the vertical component. Quadrature GHZ state teleports $\hat{S}_2$ and $\hat{S}_3$ by making use of the horizontal component. According to the measurement results in the active polarization controller, the output polarization is returned to its original state.

states [2], the carrier polarization can be measured and controlled. In the active polarization controller, the intensity of the weak transmitted part from the high-reflection mirror is used to measure the polarization, and the measurement result is fed forward to control the polarization via rotating the half-wave plates and quarter-wave plates. Through the physical explanations of the sideband model, the input sideband fluctuations $\delta\hat{S}_1$, $\delta\hat{S}_2$, and $\delta\hat{S}_3$ will only be properly transferred via single quadrature controller and a single set of quadrature GHZ states, provided the carrier polarization can be controlled by the active polarization controller. Four different polarized coherent states, including horizontal, vertical, diagonal, and right circular polarized states, are used as the input states. Each user can play any role of a sender, a receiver, or a controller, due to the symmetric structure of quantum correlations in GHZ states. In both four- or three-user network, the achieved averaged fidelity F_{ave} of 0.49 ± 0.01 or 0.53 ± 0.01 is deterministically beyond the classical limit of F_C of 0.41 under control.

Start from our scheme of controlled quantum teleportation for the CV polarization state (Fig. 1), where an arbitrary polarization coherent state can be deterministically teleported from one user to the designated user in a multiuser network. At first, arbitrary input polarization is rotated into the required vertical polarization by the active polarization controller. Then, the bright vertical polarizing mode is totally reflected by a polarizing beam splitter (PBS) and enters the single quadrature controller with all the light power. In the single quadrature controller the $\delta\hat{S}_1$ is measured and fed forward to the amplitude modulation of a vertically polarized coherent state with high power, which enables the polarization state to recover for the $\delta\hat{S}_1$. Meanwhile, both $\delta\hat{S}_2$ and $\delta\hat{S}_3$ are teleported under control, where the vacuum horizontal polarizing mode is transmitted from the PBS and transferred via a nonlocal multipartite quadrature GHZ state with low power. Not only the sender complete the joint measurement of his submode and the horizontal components of the input polarization state, but also all the controllers measure the submodes to determine the success of quantum teleportation. The controllers can determine the success of teleportation by restricting the access to their measurement results, because the controllers have quantum correlation with the sender and receiver. The transfer of both $\delta\hat{S}_2$ and $\delta\hat{S}_3$ is successful by processing the receiver's submode with the all the other users' measurement results. Otherwise, the receiver's quantum state will be destroyed by quantum noises. At the end, both the vertical and horizontal modes are recombined via another PBS, and the output polarization is rotated back to its original polarization by the active polarization controller.

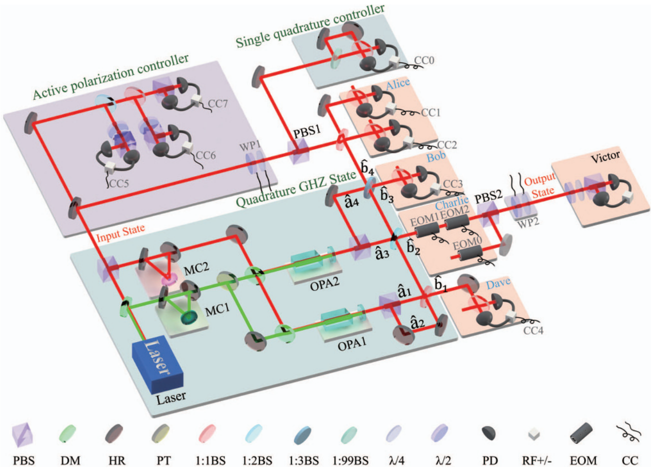

FIG. 2. The experimental setup of deterministic and multiuser quantum teleportation network of the CV polarization state. MC: mode cleaner, OPA: optical parametric amplifier, PBS: polarized beam splitter, DM: dichroic mirror, HR: high reflection mirror, PT: partial transmission mirror, BS: beam splitter, $\lambda/2$: half-wave plate, $\lambda/4$: quarter-wave plate, EOM: electro-optical modulator, CC: classical channel, PD: photodetector, and RF+/-: radio frequency adder/subtracter.

Next, we experimentally demonstrate the superiority of a controlled quantum teleportation of polarization state in both four- or three-user networks. An experimental setup of controlled quantum teleportation for the CV polarization state is shown in Fig. 2, including four parts. Part I is the active polarization controller, which contains the photodiodes to obtain the polarization and two pairs of wave plates to control the polarization. The arbitrary polarization coherent state can be chosen as the input state, since the active polarization controller transforms the arbitrary input polarization to be the vertical polarization, and return it back. Part II is the single quadrature controller to transfer the $\delta\hat{S}_1$ with vertical polarization, and includes a balanced homodyne detector (BHD) measurement and electro-optic modulator (EOM) feedback system. The quadrature amplitude is measured by BHD and fed forward to the auxiliary coherent state of EOM. Part III is the quadrature GHZ state to transfer the $\delta\hat{S}_2$ and $\delta\hat{S}_3$ with horizontal polarization. The quadripartite GHZ state for four users is obtained from minimum quantum resource of two NOPAs and a beam splitter (BS) network. Each NOPA consists of an α-cut type-II KTP crystal and a concave mirror. Through a parametric down-conversion process of type-II noncritical phase matching, the two-mode squeezed state is produced and converted to a pair of spatially separated amplitude quadrature and phase squeezed states via PBS, respectively. Two amplitude quadrature squeezed states $[\hat{a}_{1(3)} = e^{-r}\hat{X}^{(0)}_{1(3)} + ie^{r}\hat{P}^{(0)}_{1(3)}$, where superscript (0) represents vacuum state], and two phase quadrature squeezed states [$\hat{a}_{2(4)} = e^r\hat{X}^{(0)}_{2(4)} + ie^{-r}\hat{P}^{(0)}_{2(4)}$] are obtained from two NOPAs, where the squeezing parameters r are identical due to the same configurations and conditions of both NOPAs. For the four-user network, four squeezed states $\hat{a}_{1,2,3,4}$ are coupled in three BSs (with reflectivity of 1/4, 1/3, and 1/2) to generate the quadripartite entangled state for teleportation. For the three-user network, the OPA2 provide only one squeezed state $\hat{a}_3$ while OPA1 provide two squeezed states $\hat{a}_{1,2}$. By coupling these three squeezed states on two BSs (with reflectivity of 1/3 and 1/2), the tripartite entangled state is generated. The user number is changed by manipulating the number of squeezed states and the construction of the BS network. For implementing n-user network, n squeezed states and a linear BS network with $n - 1$ BSs (the reflectivity are 1/2, 1/3, ..., and $1/n$, respectively) are required. In our setup, to change the configure from three to four users, the BS with reflectivity of 1/4 (1:3BS) and squeezed state ($\hat{a}_4$) are added and shown in Fig. 2. All the users share the submodes of a GHZ state with the quadrature correlation variances of 5.70 ± 0.11 dB below the QNL, which enables the controlled quantum teleportation in a four-user network. Thus, the $\delta\hat{S}_2$ and $\delta\hat{S}_3$ can be transferred under the control. Part IV is the verification system for the recovered polarization state. The output noise powers are measured by BHD to evaluate the performance. The four-user controlled quantum teleportation system includes the minimum quantum resources of two NOPAs, together with the single quadrature controller and active polarization controller.

The measured noise levels of the output state at an analysis frequency of 3.0 MHz are shown in Fig. 3, where Fig. 3(a) [3(d)], Fig. 3(b) [3(e)], and Fig. 3(c) [3(f)] correspond to Stokes operators $\delta\hat{S}_1$, $\delta\hat{S}_2$, and $\delta\hat{S}_3$ with four (three) users, respectively. To realize the controlled quantum teleportation, the measurement results by sender and controllers are transferred to receiver with optimal gain of 0.96 ± 0.04. It is natural that the noise power with the controllers' results is lower than classical approach, due to quantum correlation among all the submodes; otherwise, it is worse than classical approach.

Fidelity is used to quantify the performance of quantum teleportation [46], and is obtained by the overlap of the input and output quantum states [2]. In general, the fidelity can be expressed by $F = \langle\Psi_{\text{in}}|\hat{\rho}_{\text{out}}|\Psi_{\text{in}}\rangle$, where $|\Psi_{\text{in}}\rangle$ is the input state and $\hat{\rho}_{\text{out}}$ is the density operator of output state. While fidelity is unity, quantum state is perfectly teleported. If fidelity is zero, the input and the output states are orthogonal. For a CV polarization state teleportation, the fidelity is the product of those at both horizontal and vertical polarizing modes, and written as

$$F = \frac{4}{\sqrt{\left(1 + V_H^{\text{Pout}}\right)\left(1 + V_H^{\text{Xout}}\right)\left(1 + V_V^{\text{Pout}}\right)\left(1 + V_V^{\text{Xout}}\right)}}, \quad (2)$$

where subscript of variance represents the polarization mode, and superscript is quadrature mode. The classical limit for the polarization coherent state is $F_C = 0.41$, which can be obtained in the absence of entanglement. If the fidelity surpasses this limit, the quantum nature of teleportation is verified. The expression of the actual fidelity of controlled quantum teleportation should be dependent on the experimental results [Eq. (2)]. By substituting the measured values into Eq. (2), the fidelities with and without the controllers' information are 0.49 ± 0.01 (0.53 ± 0.01) and 0.36 ± 0.01 (0.37 ± 0.01) in four (three) user networks, respectively. Thus, the four users can take part in CV polarization state teleportation, and their recovered fidelity is above the classical limit when the controllers cooperate. Otherwise, their fidelities are below the classical limit. Therefore, the experimental results demonstrate controlled implementation of quantum teleportation for the CV polarization state. In this system the fidelity can reach 0.82 with the single quadrature controller, because the system offers the fidelity of 0.82 for the Stokes operator $\hat{S}_1$, while the perfect GHZ state provides the unity fidelity. Furthermore, the fidelity can approach unity fidelity by increasing the cost of quantum resources.

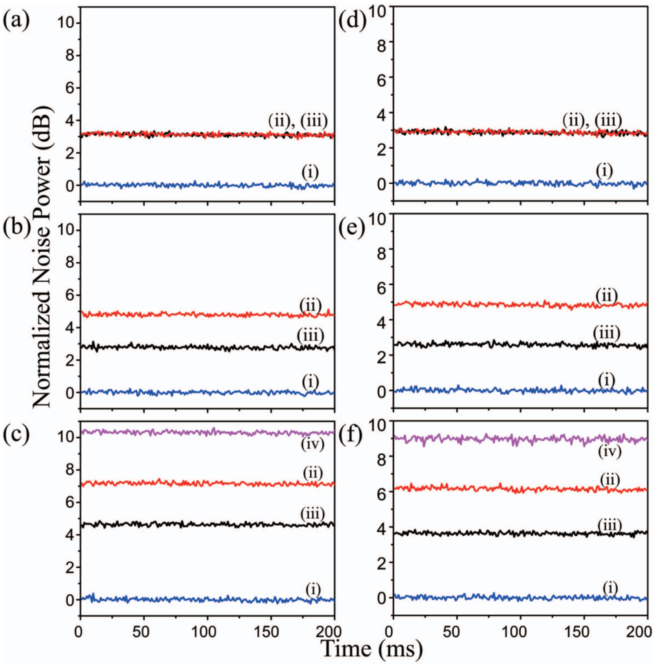

FIG. 3. The measured noise power of three Stokes operators (a) [(d)] $\hat{S}_1$, (b) [(e)] $\hat{S}_2$, and (c) [(f)] $\hat{S}_3$ in four (three)-user network. The QNLs [blue curves (i)] are measured by blocking all signal modes. The noise powers of teleported Stokes operator $\delta\hat{S}_1$, $\delta\hat{S}_2$, and $\delta\hat{S}_3$ for classical approach [red curves (ii)] are 3.12 ± 0.09 dB (2.89 ± 0.10 dB), 4.80 ± 0.09 dB (4.85 ± 0.13 dB), and 7.16 ± 0.09 dB (6.15 ± 0.13 dB) above the corresponding QNLs in four (three)-user scenarios, respectively. With the GHZ entangled state, the noise powers for controlled quantum teleportation [black curves (iii)] are 0.01 ± 0.12 dB (0.02 ± 0.12 dB), 2.02 ± 0.13 dB (2.26 ± 0.12 dB), and 2.53 ± 0.13 dB (2.51 ± 0.13 dB) below the noise levels of the corresponding classical approach for Stokes operators $\delta\hat{S}_1$, $\delta\hat{S}_2$, and $\delta\hat{S}_3$ in four (three)-user scenarios, respectively. Without the controllers' results, the power noise [pink trace (iv)] in four (three)-user network is 3.15 ± 0.12 dB (2.82 ± 0.16 dB) above the corresponding noise level of the classical approach for $\delta\hat{S}_3$ and will destroy the teleported polarization state.

To show the flexibility of controlled quantum teleportation, Table I is the fidelity in the three-user network with different user combinations, when the horizontal, vertical, diagonal, and right circular polarization are used as input states, respectively. These results demonstrate that the four

TABLE I. The fidelity of teleportation between any user pairs with different input polarization state including horizontal, vertical, diagonal, and right circular.

User pair \ Fidelity \ Polarization	Horizontal	Vertical	Diagonal	Right circular	Average
Alice - Bob	0.53 ± 0.01	0.52 ± 0.01	0.54 ± 0.01	0.54 ± 0.01	0.53 ± 0.01
Alice - Charlie	0.52 ± 0.01	0.52 ± 0.01	0.53 ± 0.01	0.54 ± 0.01	0.53 ± 0.01
Charlie - Bob	0.54 ± 0.01	0.53 ± 0.01	0.53 ± 0.01	0.53 ± 0.01	0.53 ± 0.01

different CV polarization states have been transferred with achieved fidelities beyond the classical limit, because all the carrier polarization can be converted to the required vertical polarization. Moreover, we implement controlled quantum teleportation with three different user combinations, which are the cases from Alice to Bob, from Alice to Charlie, and from Charlie to Bob, respectively. We find all the measured fidelities between any pair are beyond the classical limit of 0.41. The flexible quantum teleportation between any user pair can be implemented, due to the symmetric structure of quantum correlations among three users. Thus, controlled quantum teleportation for different CV polarization states can be flexibly performed between any user pair.

The CV quantum states of optical modes can realize the deterministic quantum teleportation, due to the deterministic generation, manipulation, and measurements of quantum states. Comparing with the previous photonic entangled protocols of polarization state quantum teleportation, quantum teleportation network of any unknown CV polarization states in four- and three-user network is deterministic by utilizing the CV GHZ state benefits from the minimum quantum resource. For the teleportation of a CV polarization state, the input polarization state, which composes two orthogonal polarization directions, can be transferred with two sets of entangled systems, respectively. For four-user network, it requires four NOPAs to generate two sets of entangled states. In our experiment, only single entangled states consisting of two NOPAs is required for four users to transfer the horizontal mode; meanwhile, a single quadrature controller is used to transfer the vertical mode. Thus, the minimum quantum resource is claimed. The controllers can determine the success of teleportation. Under the control, the input CV polarization state of the sender is reconstructed by the receiver. It is different from the quantum secret sharing, which the input state (secret) does not directly send to any user [7]. High fidelity teleportation is important for practical application. The fidelity mainly depends on squeezing factor, because the CV GHZ entangled state is obtained by coupling the squeezed states. It is possible to achieve higher fidelity with a larger squeezing factor, which can be obtained by reducing intracavity loss and enlarging the squeezing purity [47]. Furthermore, the nonlinear amplifier is an effective approach to further improve fidelity by overcoming the unavoidable transmission loss during long distance distribution of entangled states [48]. The entangled level can be well manipulated to improve its entanglement level by means of coherent feedback control and the cascaded entanglement enhancement system [49,50]. It is also helpful to employ the auxiliary squeezing channel for further improvement [40]. The CV polarization coherent state is used as an input state to test the performance of teleportation. Furthermore, the polarization squeezed and polarization entangled states can be used as input states of teleportation, and it is possible to teleport the non-Gaussian state by combining the photon subtraction technique [51]. The scalable user number is shown based on experimental demonstration of four users and theoretical analyses of 13 users (shown in the Supplemental Material [52]). The eight spatially separate entangled modes have been prepared [53], which can be directly applied in this scheme. Moreover, the time-domain two-dimensional cluster state containing more than ten thousand submodes has been generated [54,55], and it is possible to a realize teleportation network with a larger user number after separating these entangled submodes. The user distance can be extended by distributing the CV polarization state through an urban free space or fiber channel [37,56], and even longer distance may be achieved by developing quantum repeaters [57,58]. By combining quantum frequency conversion, the polarization state can interact with multiple atomic nodes for further storage and process [59]. Together with these techniques, such a system may perform a high-performance quantum teleportation of the CV polarization state in a more user network, and enables the remote distribution and deterministic light-atom interface. The techniques behind this result will be a fundamental element for light-atom hybrid networks, such as remote creation of hybrid entanglement [60] and quantum information scrambling [61]. Such a quantum teleportation network, relevant for distributed quantum computation, can benefit from these techniques, where the teleportation network of quantum logic gates is a key element [62,63]. The CV polarization state can be transferred in intercity distance of 20 km [56]. Therefore, it is possible to teleport the polarization state across fiber networks [12]. Besides, the other advantage of direct interaction with atomic nodes is important for quantum network. Thus, this system has the potential to pave an interesting avenue for light-atom hybrid metropolitan networks [60].

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