Frequency conversion of an entangled state

Aihong Tan, Xiaojun Jia, and Changde Xie*
State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University Taiyuan 030006, People's Republic of China

(Received 10 January 2006; revised manuscript received 22 February 2006; published 23 March 2006)

The quantum characteristics of sum-frequency process in an optical cavity with an input signal optical beam, which is a half of entangled optical beams, are analyzed. The calculated results show that the quantum properties of the signal beam can be maintained after its frequency is conversed during the intracavity nonlinear optical interaction. The frequency-conversed output signal beam is still in an entangled state with the retained other half of the initial entangled beams. The resultant quantum correlation spectra and the parametric dependences of the correlations on the initial squeezing factor, the optical losses and the pump power of the sum-frequency cavity are calculated. The proposed system for the frequency conversion of the entangled state can be used in quantum communication network and the calculated results can provide direct references for the design of experimental systems.

DOI: 10.1103/PhysRevA.73.033817 PACS number(s): 42.50.Dv, 03.67.Mn, 03.65.Ud

I. INTRODUCTION

Quantum entanglement of amplitude and phase quadratures of optical fields, a typical continuous variable (CV) entanglement, has been extensively applied in the quantum information and communication [1]. The unconditionalness of CV entanglement, which is usually generated from the nonlinear optical interaction of a laser with a crystal in a determinant fashion for a given experimental system, is a valuable feature for efficiently exploiting the entanglement resource. The successful experiments of unconditional quantum teleportation, quantum dense coding, quantum entanglement swapping, and elementary quantum communication networks based on CV entanglement [2–8] enhance the interest to explore the schemes establishing a more complicated CV quantum communication network and developing CV quantum telecommunication. Recently, a direct quantum interface for photonic qubits at different wavelengths was experimentally demonstrated [9]. In their experiment, the energy-time entanglement of a photon at 1310 nm wavelength with a photon at 1550 nm, was coherently transferred to another photon at a wavelength of 710 nm via a process of sum-frequency generation (SFG). Since 710 nm wavelength is close to that of alkaline atomic transitions, the conversed photon can be considered to be used for the storage and processing of quantum information. It is important to preserve the initial entanglement after the wavelength of the light is conversed for building a complete quantum information network of CV using the entangled state of light. In 1990, Kumar proposed a scheme for quantum frequency conversion (QFC) and then experimentally proved that the nonclassical intensity correlation can be preserved after the frequency of one of the initial twin beams was conversed [10,11]. In Refs. [10,11], a mode-locked, Q-switched, and frequency-doubled Nd-doped yttrium-aluminum-garnet (Nd:YAG) laser was used as the pump laser for both twin-beam generation and the QFC in a nonlinear crystal.

On the other hand, the stable entangled state of amplitude and phase quadratures of continuous optical fields have been produced through the optical parametric amplification (OPA) processes of a continuous wave (CW) in an optical cavity and applied in a variety of CV quantum information [1–8]. Especially, CW nondegenerate OPA (NOPA) using type II nonlinear $\chi^{(2)}$ crystal can directly provide either the bright entangled optical beams with the correlated amplitude quadratures and the anticorrelated phase quadratures when it operates at amplification [12] or that with the anticorrelated amplitude and the correlated phase at deamplification [6–8]. In this paper, we will discuss the QFC of a CW, which is a half of the Eistain-Podolsky-Rosen (EPR) entangled optical beams produced from a CW NOPA. The optical process used for the QFC is an intracavity SFG. Our calculation shows that the quantum entanglement characteristics of the initial entangled beams can be preserved after the frequency of one of the entangled beams is conversed. The parameter dependences of the preserved quantum entanglement upon the initial squeezing factor, the quality of optical cavity for SFG and the pump power are calculated.

The paper is organized as follows. In the second section, a physical system for the QFC is summarized. Then in the third section, we recall the expressions of EPR entanglement between the amplitude and phase quadratures of the entangled optical beams. The process of SFG is described in the fourth section and the entanglement characteristics between the frequency conversed beam and the retained initial beam are discussed in the fifth section. Finally, we give a brief conclusion in the sixth section.

II. PHYSICAL SYSTEM FOR QUANTUM FREQUENCY CONVERSION

The schematic physical system for the QFC is shown in Fig. 1. At first, a pair of EPR entangled optical beams with degenerate frequency, $a_1(\omega_1)$ and $a_2(\omega_1)$, are produced from the NOPA via a frequency-down-conversion process of the

*Email: changde@sxu.edu.cn

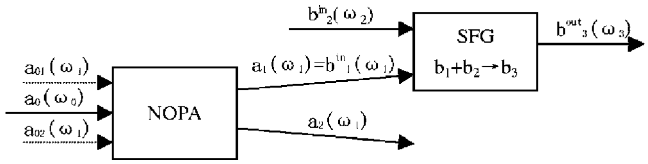

FIG. 1. Scheme of physical system for QFC.

pump field $a_0(\omega_0)$ at the frequency $\omega_0=2\omega_1$ [12]. The NOPA is implemented in an optical cavity involving a type-II phase-matching $\chi^{(2)}$ nonlinear crystal. The $a_{01}(\omega_1)$ and $a_{02}(\omega_1)$ in the coherent state are the injected signals which are polarized in same orientation with $a_1(\omega_1)$ and $a_2(\omega_1)$, respectively. We only consider the case of $|a_{01}|=|a_{02}|$ and $|a_1|=|a_2|$ for simplicity. The balance requirement is usually satisfied in experiments [6–8].

The input-output Heisenberg evolutions of the field modes of the NOPA operating in the state of amplification [the injected subharmonic signals, $a_{01}(\omega_1)$ and $a_{02}(\omega_1)$, and harmonic pump field $a_0(\omega_0)$ are in phase] are given by [13]

$$X_{a1} = X_{01}\cosh r + X_{02}\sinh r$$

$$Y_{a1} = Y_{01}\cosh r - Y_{02}\sinh r$$

$$X_{a2} = X_{02}\cosh r + X_{01}\sinh r$$

$$Y_{a2} = Y_{02}\cosh r - Y_{01}\sinh r, \tag{1}$$

where, X_{a1}, $X_{a2}(Y_{a1}, Y_{a2})$ denote the amplitude quadrature components (phase quadrature components) of the two output modes and X_{01}, X_{02} (Y_{01}, Y_{02}) are the corresponding quadrature components of the two injected fields. $r(0\leq r<\infty)$ is the squeezing factor which depends on the length and the effective second-order susceptibility of the nonlinear crystal used for NOPA, the losses of the optical cavity, as well as the intensity of pump field. From Eqs. (1), we can easily calculate the variances of the difference of amplitude quadratures and the sum of phase quadratures between $a_1(\omega_1)$ and $a_2(\omega_1)$:

$$\langle\delta^2(X_{a1}-GX_{a2})\rangle = \langle\delta^2(Y_{a1}+GY_{a2})\rangle = \frac{1}{\cosh(2r)}, \tag{2}$$

where the variances have been normalized to the shot noise limit (SNL) of the total beams $a_1(\omega_1)$ and $a_2(\omega_1)$, G is the optimum gain factor [13].

Then, one of the EPR beams (a_1) is injected in another optical cavity involving a nonlinear $\chi^{(2)}$ crystal (SFG). $b_1^{in}(\omega_1)=a_1(\omega_1)$ and $b_2^{in}(\omega_2)$ are the injected signal and pump field $(\omega_1\neq\omega_2)$ of the SFG, respectively. $b_1(\omega_1)$ and $b_2(\omega_2)$ stand for the corresponding intracavity-fields of b_1^{in} and b_2^{in}. An output field b_3 at the frequency $\omega_3=\omega_1+\omega_2$ is generated via a nonlinear sum-frequency process in SFG cavity. We will analyze the process of SFG and discuss the preserved entanglement between the output field $b_3^{out}(\omega_3)$ of SFG and the retained $a_2(\omega_1)$ in next sections.

III. SUM-FREQUENCY GENERATION

We calculate the quantum fluctuation characteristics of the SFG in an optical cavity using the semiclassical approach. The dynamics of small field fluctuations is described by linearizing the classical equations of motion in the vicinity of the stationary state. We consider that these field fluctuations are driven by the fluctuations of the input fields (including vacuum field) through the coupling mirrors. It has been theoretically demonstrated in Ref. [14] that the semiclassical approach can lead to the same results with the standard quantum methods [15] for the quantum fluctuations of the output fields from an optical cavity.

Under the case of perfect phase matching, zero detuning, small one-pass gain, and small losses, the equations of motion for the classical amplitudes β_1,β_2,β_3 of intracavity fields associated with the annihilation operators b_1,b_2,b_3 can be expressed by [16]

$$\tau\dot{\beta}_1(t) = -(\gamma_1+\rho_1)\beta_1(t) + \chi\beta_2^*(t)\beta_3(t) + \sqrt{2\gamma_1}\beta_1^{in}(t) + \sqrt{2\rho_1}c_1^{in}(t)$$

$$\tau\dot{\beta}_2(t) = -(\gamma_2+\rho_2)\beta_2(t) + \chi\beta_1^*(t)\beta_3(t) + \sqrt{2\gamma_2}\beta_2^{in}(t) + \sqrt{2\rho_2}c_2^{in}(t)$$

$$\tau\dot{\beta}_3(t) = -(\gamma_3+\rho_3)\beta_3(t) - \chi\beta_1(t)\beta_2(t) + \sqrt{2\gamma_3}\beta_3^{in}(t) + \sqrt{2\rho_3}c_3^{in}(t), \tag{3}$$

where, the round-trip time τ of light in the cavity is assumed to be the same for all the three fields. The γ_i and $\rho_i(i=1,2,3)$ stand for the single pass loss parameters corresponding to the transmission of the input and output couplers of the cavity and extra intracavity losses, respectively. The γ_i are directly related to the amplitude reflection and the transmission coefficients of the input and the output couplers of the optical cavity and the ρ_i to the amplitude transmission coefficient of the optical medium in the cavity. In Eqs. (3), γ_i and ρ_i express the losses during the single pass in the cavity, and is not the losses in a unit time as usual, thus the round-trip time τ appears in the equations [16]. The $\beta_i^{in}(i=1,2,3)$ are the classical amplitudes of b_i^{in} (b_3^{in} is vacuum field). The $c_i^{in}(i=1,2,3)$ denote the extra noise amplitudes in addition to the intracavity field b_i due to the internal loss mechanism. χ is the effective nonlinear coupling parameter, which is proportional to the second order susceptibility of the medium.

Assuming that the pump field b_2 is strong and can be considered to be undepleted, we have $\beta_2=\beta_2^*=E$ and we can linearize the evolution equations around the mean amplitudes:

$$\beta_1(t) = \langle\beta_1\rangle + \delta\beta_1(t) \quad \beta_1^{in}(t) = \langle\beta_1^{in}\rangle + \delta\beta_1^{in}(t)$$

$$\beta_3(t) = \langle\beta_3\rangle + \delta\beta_3(t) \quad \beta_3^{in}(t) = \delta\beta_3^{in}(t). \tag{4}$$

Substituting Eqs. (4) into Eqs. (3), we obtain the fluctuation dynamics equations,

$$\tau\delta\dot{\beta}_1(t) = -(\gamma_1+\rho_1)\delta\beta_1(t) + \chi E\delta\beta_3(t) + \sqrt{2\gamma_1}\delta\beta_1^{in}(t) + \sqrt{2\rho_1}c_1^{in}(t)$$

$$\tau\delta\dot{\beta}_3(t) = -(\gamma_3+\rho_3)\delta\beta_3(t) - \chi E\delta\beta_1(t) + \sqrt{2\gamma_3}\delta\beta_3^{in}(t) + \sqrt{2\rho_3}c_3^{in}(t). \quad (5)$$

After Fourier transformation, we have

$$(i\omega\tau+\gamma_1+\rho_1)\delta\beta_1(\omega) = \chi E\delta\beta_3(\omega) + \sqrt{2\gamma_1}\delta\beta_1^{in}(\omega) + \sqrt{2\rho_1}c_1^{in}(\omega)$$

$$(i\omega\tau+\gamma_3+\rho_3)\delta\beta_3(\omega) = -\chi E\delta\beta_1(\omega) + \sqrt{2\gamma_3}\delta\beta_3^{in}(\omega) + \sqrt{2\rho_3}c_3^{in}(\omega), \quad (6)$$

where ω is the analysis frequency.

Using the boundary condition on the output coupling mirror [17]

$$\delta\beta_3^{out} = \sqrt{2\gamma_3}\delta\beta_3 - \delta\beta_3^{in}, \quad (7)$$

we obtain the fluctuation of output field b_3^{out} in term of the input fluctuation,

$$\delta\beta_3^{out}(\omega) = \frac{1}{(i\omega\tau+\gamma_3+\rho_3)(i\omega\tau+\gamma_1+\rho_1)+(\chi E)^2} \times\Big\{[(i\omega\tau+\gamma_1+\rho_1)(-i\omega\tau+\gamma_3-\rho_3) - (\chi E)^2]\delta\beta_3^{in}(\omega) + 2\sqrt{\gamma_3\rho_3}(i\omega\tau+\gamma_1+\rho_1)c_3^{in}(\omega) - 2\chi E\sqrt{\gamma_1\gamma_3}\delta\beta_1^{in}(\omega) - 2\chi E\sqrt{\rho_1\gamma_3}c_1^{in}(\omega)\Big]. \quad (8)$$

IV. ENTANGLEMENT CHARACTERISTICS

From Eq. (8) and the definitions of the amplitude and phase quadratures, $X=\frac{1}{2}(b+b^+)$ and $Y=\frac{1}{2i}(b-b^+)$, the fluctuation spectra of the quadrature components of b_3^{out} are calculated:

$$\delta X_{b3}^{out} = \frac{1}{R}[A\delta X_{b1}^{in} + B\delta Y_{b1}^{in} + C\delta X_{b3}^{in} + D\delta Y_{b3}^{in} + GX_{c3}^{in} + HY_{c3}^{in} + MX_{c1}^{in} + NY_{c1}^{in}]$$

$$\delta Y_{b3}^{out} = \frac{1}{R}[A\delta Y_{b1}^{in} - B\delta X_{b1}^{in} + C\delta Y_{b3}^{in} - D\delta X_{b3}^{in} + GY_{c3}^{in} - HX_{c3}^{in} + M\delta Y_{c1}^{in} - N\delta X_{c1}^{in}], \quad (9)$$

where

$$R = [(\gamma_1+\rho_1)(\gamma_3+\rho_3) - (\omega\tau)^2 + (\chi E)^2]^2 + [\omega\tau(\gamma_1+\rho_1+\gamma_3+\rho_3)]^2$$

$$A = -2\chi E\sqrt{\gamma_1\gamma_3}[(\gamma_1+\rho_1)(\gamma_3+\rho_3) - (\omega\tau)^2 + (\chi E)^2]$$

$$B = -2\chi E\sqrt{\gamma_1\gamma_3}\omega\tau(\gamma_1+\rho_1+\gamma_3+\rho_3)$$

$$C = [(\gamma_1+\rho_1)(\gamma_3-\rho_3) + (\omega\tau)^2 - (\chi E)^2][(\gamma_1+\rho_1)(\gamma_3+\rho_3) - (\omega\tau)^2 + (\chi E)^2] + (\omega\tau)^2(\gamma_1+\rho_1+\gamma_3+\rho_3)(\gamma_3-\rho_3-\gamma_1-\rho_1)$$

$$D = \omega\tau[(\gamma_1+\rho_1)(\gamma_3-\rho_3) + (\omega\tau)^2 - (\chi E)^2](\gamma_1+\rho_1+\gamma_3+\rho_3) - \omega\tau[(\gamma_1+\rho_1)(\gamma_3+\rho_3) - (\omega\tau)^2 + (\chi E)^2](\gamma_3-\rho_3-\gamma_1-\rho_1)$$

$$G = 2\sqrt{\gamma_3\rho_3}\{[(\gamma_1+\rho_1)(\gamma_3+\rho_3) + (\chi E)^2](\gamma_1+\rho_1) + (\omega\tau)^2(\gamma_3+\rho_3)\}$$

$$H = 2\sqrt{\gamma_3\rho_3}(\omega\tau)[(\gamma_1+\rho_1)^2 + (\omega\tau)^2 - (\chi E)^2]$$

$$M = -2\chi E\sqrt{\rho_1\gamma_3}[(\gamma_1+\rho_1)(\gamma_3+\rho_3) - (\omega\tau)^2 + (\chi E)^2]$$

$$N = -2\chi E\sqrt{\rho_1\gamma_3}(\omega\tau)(\gamma_1+\rho_1+\gamma_3+\rho_3)$$

X_{bi}^{in}, $Y_{bi}^{in}(i=1,3)$ and X_{ci}^{in}, $Y_{ci}^{in}(i=1,3)$ denote the amplitude and phase quadratures of b_1^{in}, b_3^{in} and c_1^{in}, c_3^{in}, respectively.

For observing the optimum entanglement between b_3^{out} and a_2, we should implement the appropriate unitary transformation on a_2, in which the quantum properties of the optical field a_2 will not be changed. Generally, the amplitude and phase quadratures of a_2 are expressed by [18]

$$X_{a2}^{\theta} = \frac{1}{2}(a_2e^{-i\theta} + a_2^+e^{i\theta}), \quad Y_{a2}^{\theta} = \frac{1}{2i}(a_2e^{-i\theta} - a_2^+e^{i\theta}), \quad (10)$$

where, θ is the phase angle of X_{a2}^{θ} and Y_{a2}^{θ} rotated from the initial X_{a2} and Y_{a2}, and it can be conveniently completed by adjusting the phase of the local oscillator or using a phase-shifter in experiments.

Rewriting Eq. (10) in terms of the amplitude and phase quadratures X_{a2} and Y_{a2} of a_2, we obtain

$$X_{a2}^{\theta} = X_{a2}\cos\theta + Y_{a2}\sin\theta \quad (11)$$

and

$$Y_{a2}^{\theta} = -X_{a2}\sin\theta + Y_{a2}\cos\theta. \quad (12)$$

The correlation fluctuations of amplitude and phase quadratures between b_3^{out} and a_2^{θ} [a_2^{θ} is the transformed a_2 according to Eqs. (10)] are expressed by:

$$\delta X_{b3}^{out} - g\,\delta X_{a2}^{\theta} = \frac{1}{R}[A\delta X_{b1}^{in} + B\delta Y_{b1}^{in} + C\delta X_{b3}^{in} + D\delta Y_{b3}^{in} + GX_{c3}^{in} + HY_{c3}^{in} + MX_{c1}^{in} + NY_{c1}^{in}] - g\,\delta X_{a2}\cos\theta - g\,\delta Y_{a2}\sin\theta = \left(\frac{A}{R}\delta X_{b1}^{in} - g\cos\theta\delta X_{a2}\right) + \left(\frac{B}{R}\delta Y_{b1}^{in} - g\sin\theta\delta Y_{a2}\right) + \frac{1}{R}[C\delta X_{b3}^{in} + D\delta Y_{b3}^{in} + GX_{c3}^{in} + HY_{c3}^{in} + MX_{c1}^{in} + NY_{c1}^{in}], \quad (13)$$

$$\delta Y_{b3}^{out}+g\,\delta Y_{a2}^{\theta}=\frac{1}{R}[A\,\delta Y_{b1}^{in}-B\,\delta X_{b1}^{in}+C\,\delta Y_{b3}^{in}-D\,\delta X_{b3}^{in}-HX_{c3}^{in}+GY_{c3}^{in}+M\,\delta Y_{c1}^{in}-N\delta X_{c1}^{in}]-g\,\delta X_{a2}\sin\theta+g\,\delta Y_{a2}\cos\theta=\left(\frac{A}{R}\delta Y_{b1}^{in}+g\cos\theta\delta Y_{a2}\right)-\left(\frac{B}{R}\delta X_{a1}^{in}+g\sin\theta\delta X_{a2}\right)+\frac{1}{R}[-D\,\delta X_{b3}^{in}+C\,\delta Y_{b3}^{in}-HX_{c3}^{in}+GY_{c3}^{in}-NX_{c1}^{in}+MY_{c1}^{in}], \tag{14}$$

where, g is an adjustable gain factor. Since $b_1^{in}=a_1$ and the quantum fluctuation is not changed in the unitary transformation, we may substitute Eqs. (1) into Eqs. (13) and (14), and get

$$\delta X_{b3}^{out}-g\,\delta X_{a2}^{\theta}=\left[\frac{A}{R}\cosh r-g\cos\theta\sinh r\right]X_{01}+\left[\frac{A}{R}\sinh r-g\cos\theta\cosh r\right]X_{02}+\left[\frac{B}{R}\cosh r+g\sin\theta\sinh r\right]Y_{01}-\left[\frac{B}{R}\sinh r+g\sin\theta\cosh r\right]Y_{02}+\frac{1}{R}[C\,\delta X_{b3}^{in}+D\,\delta Y_{b3}^{in}+GX_{c3}^{in}+HY_{c3}^{in}+MX_{c1}^{in}+NY_{c1}^{in}], \tag{15}$$

and

$$\delta Y_{3}^{out}+g\,\delta Y_{2}^{\theta}=\left[\frac{A}{R}\cosh r-g\cos\theta\sinh r\right]Y_{01}-\left[\frac{A}{R}\sinh r-g\cos\theta\cosh r\right]Y_{02}-\left[\frac{B}{R}\cosh r+g\sin\theta\sinh r\right]X_{01}-\left[\frac{B}{R}\sinh r+g\sin\theta\cosh r\right]X_{02}+\frac{1}{R}[-D\,\delta X_{b3}^{in}+C\,\delta Y_{b3}^{in}-HX_{c3}^{in}+GY_{c3}^{in}-NX_{c1}^{in}+MY_{c1}^{in}]. \tag{16}$$

Then, the correlation variance of the difference of amplitude quadratures and the sum of phase quadratures are obtained

$$\langle\delta^2(X_{b3}^{out}-gX_{a2}^{\theta})\rangle=\langle\delta^2(Y_{b3}^{out}+gY_{a2}^{\theta})\rangle=\left[\frac{A}{R}\cosh r-g\cos\theta\sinh r\right]^2+\left[\frac{A}{R}\sinh r-g\cos\theta\cosh r\right]^2+\left[\frac{B}{R}\cosh r+g\sin\theta\sinh r\right]^2+\left[\frac{B}{R}\sinh r+g\sin\theta\cosh r\right]^2+\frac{1}{R^2}[C^2+D^2+G^2+H^2+M^2+N^2]=\frac{4(\chi E)^2\gamma_1\gamma_3}{R}\frac{e^{2r}+e^{-2r}}{2}-\frac{A}{R}g\cos\theta(e^{2r}-e^{-2r})+g^2\frac{e^{2r}+e^{-2r}}{2}+\frac{B}{R}g\sin\theta(e^{2r}-e^{-2r})+\frac{1}{R^2}[C^2+D^2+G^2+H^2+M^2+N^2]. \tag{17}$$

Taking

$$\tfrac{A}{\sqrt{R}}=2(\chi E)\sqrt{\gamma_1\gamma_3}\cos\varphi,$$

and $\frac{B}{\sqrt{R}}=2(\chi E)\sqrt{\gamma_1\gamma_3}\sin\varphi$, we have

$$\langle\delta^2(X_{b3}^{out}-gX_{a2}^{\theta})\rangle=\langle\delta^2(Y_{b3}^{out}+gY_{a2}^{\theta})\rangle=\frac{4(\chi E)^2\gamma_1\gamma_3}{R}\frac{e^{2r}+e^{-2r}}{2}-(2\chi E\sqrt{\gamma_1\gamma_3})\frac{1}{\sqrt{R}}g\cos(\varphi+\theta)(e^{2r}-e^{-2r})+g^2\frac{e^{2r}+e^{-2r}}{2}+\frac{1}{R^2}[C^2+D^2+G^2+H^2+M^2+N^2]. \tag{18}$$

Calculating the minimum value of Eq. (18) in term of g, we obtain the optimum gain:

$$g_{opt}=\frac{2(\chi E)\sqrt{\gamma_1\gamma_3}}{\sqrt{R}}\cos(\varphi+\theta)\frac{e^{2r}-e^{-2r}}{e^{2r}+e^{-2r}},$$

and the corresponding correlation variance equals to

$$S=\langle\delta^2(X_{b3}^{out}-g_{opt}X_{a2}^{\theta})\rangle_{\min}=\langle\delta^2(Y_{b3}^{out}+g_{opt}Y_{a2}^{\theta})\rangle_{\min}=\frac{2(\chi E)^2\gamma_1\gamma_3}{R}\left[e^{2r}+e^{-2r}-\cos^2(\theta+\varphi)\frac{(e^{2r}-e^{-2r})^2}{e^{2r}+e^{-2r}}\right]+\frac{1}{R^2}[C^2+D^2+G^2+H^2+M^2+N^2], \tag{19}$$

when $\theta=-\varphi$, S reaches the minimum value $S_{\min}$.

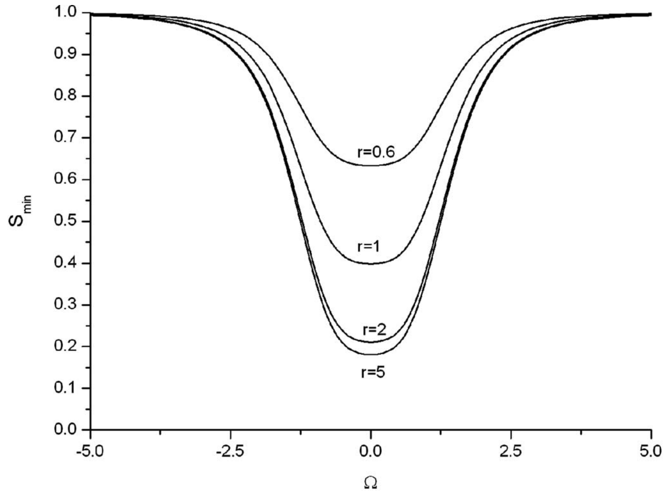

FIG. 2. The correlation fluctuation spectra $S_{\min}$ vs the normalized analysis frequency $\Omega(\Omega=\omega\tau/\gamma_1)$. Transmission of the output coupler $\gamma_3/\gamma_1=1$, extra intracavity losses $\rho_1/\gamma_1=\rho_3/\gamma_1=0.1$. Pump parameter $\chi E/\gamma_1=1$, gain factor g is chosen to be the optimum value.

Figure 2 shows the minimum correlation fluctuation spectra $S_{\min}$ versus the normalized analysis frequency $(\Omega=\omega\tau/\gamma_1)$ for different initial squeezing factor r. Obviously, at zero frequency $(\Omega=0)$, the maximum correlation is obtained $[S_{\min}(\Omega=0)$ reaches the minimum], and the larger the initial squeezing factor r is, the better the preserved correlation between a_2 and b_3^{out} is.

In Figs. 3 and 4, the dependences of $S_{\min}$ on the pump parameters $(\chi E/\gamma_1)$ are calculated for different relative transmissions of γ_3/γ_1 and different squeezing factors r, respectively. For the smaller pump parameters, the smaller γ_3/γ_1 is better ($S_{\min}$ is smaller), but for larger pump parameters the larger γ_3/γ_1 value corresponds to smaller $S_{\min}$. For a given γ_3/γ_1, we have an optimal pump parameter at which

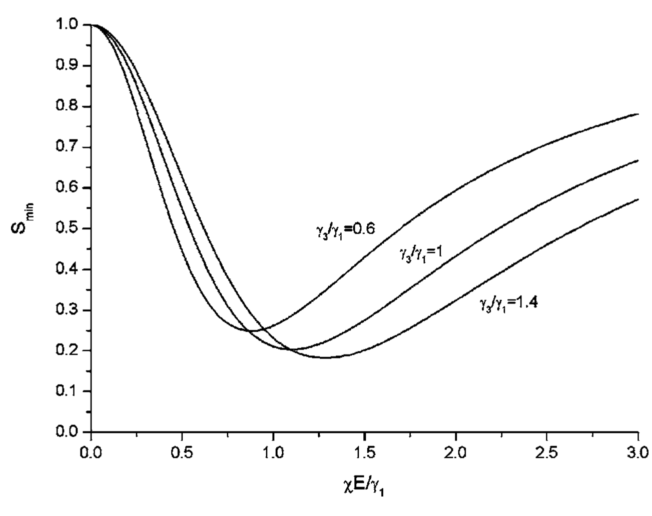

FIG. 3. The correlation fluctuation spectra $S_{\min}$ at $\Omega=0$ vs the pump parameter $\chi E/\gamma_1$ for different relative transmission $\gamma_3/\gamma_1=0.6, 1, 1.4$, extra intracavity losses $\rho_1/\gamma_1=\rho_3/\gamma_1=0.1$, gain factor g is chosen to be the optimum value, the squeezing parameter $r=2$.

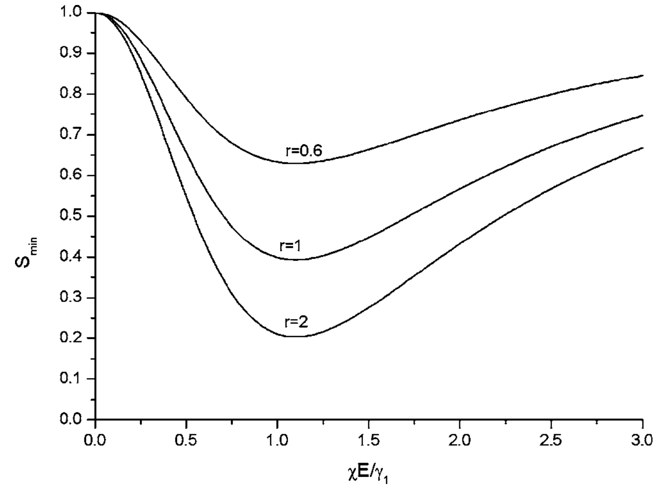

FIG. 4. The correlation fluctuation spectra $S_{\min}$ at $\Omega=0$ vs the pump parameter $\chi E/\gamma_1$ for different squeezing factor $(r=0.6, 1, 2)$. The normalized frequency $\Omega=0$, extra intracavity losses $\rho_1/\gamma_1=\rho_3/\gamma_1=0.1$, gain factor g is chosen to be the optimum value. Transmission of input-output couple $\gamma_3/\gamma_1=1$.

the $S_{\min}$ reaches to a minimum. If γ_3/γ_1 increases, the optimal pump parameter increases too. That is because for larger output transmission γ_3 of b_3^{out}, the higher pump power is needed to achieve the optimal SFG. Figure 4 shows, when the parameters of SFG cavity and initial squeezing are given, we should choose the optimal pump power to meet the smallest $S_{\min}$ value for successfully preserving the quantum correlation. In the ideal limit without any intracavity losses, if taking $g_{opt}=1$, the $S_{\min}$ will equal to the initial EPR correlation between a_1 and a_2. In this case $\theta=\pi$, it means that there is a phase difference of π between the input $(X_{a1}=X_{b1}^{in})$ and the output (X_{b3}^{out}) of SFG [19], thus when we measure the correlation between X_{b3}^{out} and X_{a2}, a phase shift of π should be added on X_{a2}.

In Fig. 5, the relation of the minimum correlation fluctuations $S_{\min}$ versus the initial squeezing factor r is drawn. The correlation variance $S_{\min}$ between the field a_2 and b_3^{out} decrease, i.e., the entanglement increases when the r increases. From Eq. (2) we know that the initial correlation variances of both amplitude and phase quadratures between a_1 and a_2 are smaller than the normalized SNL for $r>0$, thus the inseparability criterion of EPR entanglement state for continuous variables proposed by Duan [20] is satisfied, that is

$$\langle \delta^2(X_{a1}-GX_{a2})\rangle + \langle \delta^2(Y_{a1}+GY_{a2})\rangle < 2. \qquad (20)$$

In the case of $r>0$, the minimum correlation variances for the amplitude and phase quadratures between a_2 and b_3^{out} fields are equal [see Eq. (19)] and both smaller than 1 also (Fig. 5), so we have

$$\langle \delta^2(X_{b3}^{out}-g_{opt}X_{a2}^{\theta})\rangle_{\min} + \langle \delta^2(Y_{b3}^{out}+g_{opt}Y_{a2}^{\theta})\rangle_{\min} < 2.$$

It means, the correlation variables of the quadratures between the field a_2 and b_3^{out} satisfy the inseparability criterion for quantum entangled state. Once the initial entanglement between the field a_1 and a_2 exists $(r>0)$, the entanglement between the field a_2 and b_3^{out} also exists. The better the initial

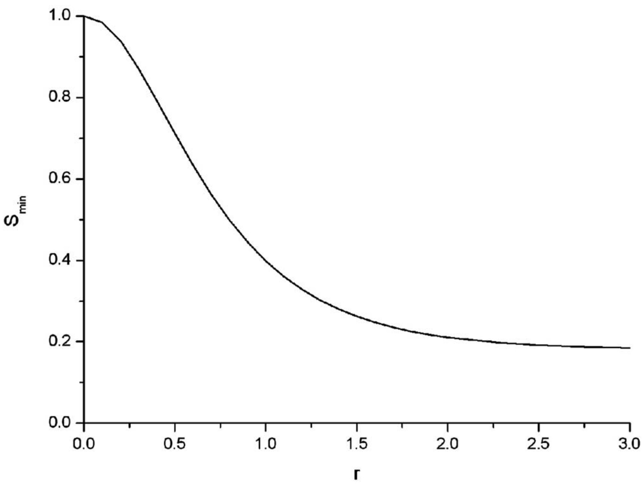

FIG. 5. The minimum correlation fluctuation $S_{\min}$ at $\Omega=0$ vs the initial squeezing factor r. Transmission of the output coupler $\gamma_3/\gamma_1=1$, extra intracavity losses $\rho_1/\gamma_1=\rho_3/\gamma_1=0.1$. Pump parameter $\chi E/\gamma_1=1$, gain factor g is chosen to be the optimum value.

entanglement is, the larger the remaining entanglement after the frequency conversion is. For an ideal SFG without the intracavity losses and taking $\Omega=0$ and $(\chi E)^2=\gamma_1\gamma_3$, the remaining correlation variances will equal to the initial variances from Eq. (19), thus the entanglement can be perfectly preserved. However, for any experimental system with the losses, the remaining entanglement is always worse than that of the initial state.

V. CONCLUSION

Our analyses theoretically proved that the initial EPR entanglement between the amplitude and phase quadratures of entangled beams can be preserved after the frequency of one of the beams is conversed via an intracavity nonlinear interaction of SFG. We calculated the dependences of the resultant correlation fluctuation spectra on the parameters of SFG system, the pump power and the initial squeezing. The squeezing parameter $r=0.6$ corresponds to the correlation fluctuation of the initial EPR beams is ≈ 5.2 dB below the SNL, which has been realized by experiments [6–8], in this case the preserved entanglement is about 2 dB below the SNL($S_{\min}\approx 0.63$) in the experimentally accessible systems. The frequency conversion of entangled optical beams is important in constructing complete quantum communication networks. The calculated results may be a useful reference for the design of quantum communication systems.

A.T. thanks J. Zhang for the helpful discussion. This research was supported in part by the National Natural Science Foundation of China (Grants Nos. 60238010, 60378014), the Major State Basic Research Project of China (Grant No. 2001CB309304).

[1] S. L. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2005).
[2] A. Furusawa, J. L. Sorensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282**, 706 (1998).
[3] H. Yonezawa, T. Aoki, and A. Furusawa, Nature (London) **431**, 430 (2004).
[4] N. Takei, H. Yonezawa, T. Aoki, and A. Furusawa, Phys. Rev. Lett. **94**, 220502 (2005).
[5] W. P. Bowen, N. Treps, B. C. Buchler, R. Schnabel, T. C. Ralph, H. A. Bachor, T. Symul, and P. K. Lam, Phys. Rev. A **67**, 032302 (2003).
[6] X. Y. Li, Q. Pan, J. T. Jing, J. Zhang, C. D. Xie, and K. C. Peng, Phys. Rev. Lett. **88**, 047904 (2002).
[7] J. T. Jing, J. Zhang, Y. Yan, F. G. Zhao, C. D. Xie, and K. C. Peng, Phys. Rev. Lett. **90**, 167903 (2003).
[8] X. J. Jia, X. L. Su, Q. Pan, J. R. Gao, C. D. Xie, and K. C. Peng, Phys. Rev. Lett. **93**, 250503 (2004).
[9] S. Tanzilli, W. Tittel, M. Halder, O. Alibart, P. Baldi, N. Gisin, and H. Zbinden, Nature (London) **437**, 116 (2005).
[10] P. Kumar, Opt. Lett. **15**, 1476 (1990).
[11] J. M. Huang and P. Kumar, Phys. Rev. Lett. **68**, 2153 (1992).
[12] Y. Zhang, H. Wang, X. Y. Li, J. T. Jing, C. D. Xie, and K. C. Peng, Phys. Rev. A **62**, 023813 (2000).
[13] M. D. Reid, Phys. Rev. A **40**, 913 (1989).
[14] S. Reynaud and A. Heidmann, Opt. Commun. **71**, 209 (1989).
[15] A. S. Lane, M. D. Reid, and D. F. Walls, Phys. Rev. A **38**, 788 (1988).
[16] C. Fabra, E. Giacobino, A. Heidmann, and S. Reynaud, J. Phys. (France) **50**, 1209 (1989).
[17] M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).
[18] B. Yurke, Phys. Rev. A **29**, 408 (1984).
[19] R. W. Boyd, *Nonlinear Optics* (Academic, New York, 1992), p. 72.
[20] L. M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000).