Optimization of the noise property of delayed light in electromagnetically induced transparency

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One of the important applications of electromagnetically induced transparency (EIT) is to delay and store light in atomic ensembles. In this paper, the noise spectrum of the delayed quantum light throughout an EIT medium is investigated. With zero detection frequency, we can have minimum noise of delayed light in two-photon resonance of EIT, and the noise is larger than the minimum noise at off two-photon resonance due to the phase-to-amplitude noise conversion. It is shown that the noise for nonzero detection frequency can be suppressed by operating the system at off two-photon resonance, even when the unavoidable dephasing is included.

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I. INTRODUCTION

It is well known that the photon is an ideal carrier of quantum information, but the qubits with the photons are called flying qubits, because they cannot be stored for a long time. On the other hand, the atoms may be able to convert the flying qubits into storage qubits, since atoms can be kept in a single location for a long time without changing their quantum states [1]. The research on quantum memory is directed toward developing reliable registering and exchanging of quantum information, the basic computational elements, from atoms to photons or vice versa [2–4], which attracted a lot of attention in recent years [5–7]. The most far reaching is the neutral atom quantum computer, for which quantum theory predicts a different type of computing logic [8], available in some cases, to outrun the present classical computers by many orders of magnitude in processing time.

Most of the investigations about information storage were carried out in atomic systems, in which atomic coherence was generated by two distinct optical fields due to the electromagnetically induced transparency (EIT) [9]. The storing and releasing of a weak light pulse was theoretically and experimentally demonstrated in EIT medium [10,11]. Recently the significant progress towards the quantum memory in mapping a quantum state of light onto a long-lived atomic state has been reported theoretically [12–15]. The quantum storage in atoms and the retrieving of coherent states [3], single photon state [6], and Einstein-Podolsky-Rosen (EPR) entanglement beams [16], have been demonstrated. Schemes to investigate quantum entanglement with atomic memory [17,18] were also reported.

Using atoms as the quantum memory for quantum information processing requires efficient means for the preparation, manipulation, and storage of qubits inscribed into atoms as well as for the readout of the information of quantum state from atomic memory. One of the ways to verify the faithful atomic memory is the measurement of quantum correlations or noise of the retrieved states [18]. The experiment of EIT with a squeezed vacuum state demonstrated that the squeezing could be extracted through the EIT medium [19]. Very recently, the experimental results showed that the excess noise, which has not yet been explained, was added to the delayed light in the slow-light EIT medium [20]. Quantum mechanical treatment for both atomic and field fluctuations were used in the discussion of the quantum noise of delayed light, which showed that the quantum states can be well preserved in EIT medium under the resonance or in two-photon Raman resonance [21,22]. On the other hand, the generation and enhancement of squeezing for the fluorescent light were presented in three- and four-level atomic systems, and it was concluded that the effect of spontaneous generated coherence plays a crucial role in the squeezing spectrum of fluorescent light [23,24]. These studies show that the atomic coherences can provide a possible approach not only for the slowing and storage of light, but also for the noise suppressing of light. In this paper, we study the quantum noise spectrum of the delayed light in EIT medium with two-photon detuning. The optimum condition for minimizing the quantum noise of the delayed state at the nonzero detection frequency in a realistic EIT scheme can be achieved by tuning the two-photon detuning under the condition of quantum coherence of EIT. Although the dispersion of EIT coherence converts the laser phase noise into amplitude noise in the transmitted probe beam, the large phase-to-amplitude noise can be suppressed when a proper two-photon detuning and a detection frequency are chosen.

II. THE THEORETICAL MODEL

Consider a closed three-level Λ-type system, as shown in Fig. 1. A strong coupling laser of frequency ωc interacts with

![FIG. 1. Schematic of the Λ-type system.](image-url)
|c⟩→|a⟩ transition, while a weak quantum probe beam with frequency ωp interacts with transition |b⟩→|a⟩. δ0 = ωac − ωc is one-photon detuning, δ2 = ωab − ωp − δ0 is two-photon detuning. For this A-type EIT system, the evolution equations for both the slowly varying atomic operators and the slowly varying annihilation operator of quantum probe field ̂a are given by

\begin{align*}
\hat{σ}_{ba} &= - \left[ \gamma_1 - i(\delta_0 + \delta_0) \right] \hat{σ}_{ba} + ig\hat{a}(\hat{σ}_{bb} - \hat{σ}_{aa}) + i\Omega\hat{σ}_{bc} + \hat{F}_{ba}, \\
\hat{σ}_{bc} &= - (\gamma_0 - i\delta_0)\hat{σ}_{bc} - ig\hat{σ}_{ac} + i\Omega^*\hat{σ}_{ba} + \hat{F}_{bc}, \\
\hat{σ}_{ac} &= - (\gamma_2 + i\delta_0)\hat{σ}_{ac} - ig\hat{σ}_{ac} + i\Omega^*(\hat{σ}_{aa} - \hat{σ}_{cc}) + \hat{F}_{ac}, \\
\hat{σ}_{bb} &= \gamma_1\hat{σ}_{aa} - ig\hat{σ}_{ab} + ig\hat{σ}_{ba} + \hat{F}_{bb}, \\
\hat{σ}_{cc} &= \gamma_2\hat{σ}_{aa} - i\Omega\hat{σ}_{ac} + i\Omega^*\hat{σ}_{ca} + \hat{F}_{cc},
\end{align*}

(1)

where the atomic dipole operator at position z in the rotating frame is defined by locally averaging over a transverse slice containing many atoms ̂σµ(z,t) = \frac{1}{nA} \sum_{z_k \in S} e^{-i(\omega_p - \omega_c)(z - z_{k})/\mu} (|\mu\rangle \langle \nu|)_z, where n is the atomic density and A is the cross section area of the light beam. γ1 and γ2 are the atomic dipole decay rates from the excited state of |a⟩ and the ground state of |b⟩ and |c⟩, respectively. Y0 is the dephasing rate for the two ground states of the A-like system, g is the coupling constant between atom and probe field, Ω is the Rabi frequency of the coupling field, N refers to the number of atoms per unit volume. The ̂σµ’s are δ-correlated Langevin noise operators caused by reservoir noisy fluctuations. The correlation functions can be calculated via the quantum regression theorem [25,26]

\begin{align}
\langle \hat{F}_{\mu}(z_1,t) \hat{F}_{\alpha\beta}(z_2,t_2) \rangle &= \frac{L}{N} \langle D(\hat{σ}_{\mu\nu}) \hat{σ}_{\alpha\beta} \rangle - \langle D(\hat{σ}_{\mu\nu}) \hat{σ}_{\alpha\beta} \rangle \\
&\quad - \hat{σ}_{\mu\nu} \langle D(\hat{σ}_{\alpha\beta}) \delta(z_2 - z_1) \delta(t_2 - t_1) \rangle.
\end{align}

(2)

Here \langle D(\hat{σ}_{\mu\nu}) \rangle is the expression obtained from the Heisenberg-Langevin equation for ̂σµν, but without the Langevin force term. The Dirac δ function in Eq. (2) represents the short memory of the reservoir of vacuum modes responsible for the Langevin forces. L is the length of the medium interacting with the lights.

In the following analysis, we study the noise property of the probe light under conditions of EIT. Making the usual assumption that the quantum probe field is much less than the coupling field and assuming that all the atoms are initially in the ground state of |b⟩, we can simplify Eq. (1) to the following by keeping ̂a to the lowest order and Ω to all orders,

\begin{align*}
\hat{σ}_{ba} &= - \left[ \gamma_1 - i(\delta_0 + \delta_0) \right] \hat{σ}_{ba} + ig\hat{a} + i\Omega\hat{σ}_{bc} + \hat{F}_{ba}, \\
\hat{σ}_{bc} &= - (\gamma_0 - i\delta_0)\hat{σ}_{bc} - ig\hat{σ}_{ac} + i\Omega^*\hat{σ}_{ba} + \hat{F}_{bc}, \\
(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z})\hat{a} &= ig^\dagger N\hat{σ}_{ba},
\end{align*}

(3)

In order to solve Eq. (3), we convert the equations into Fourier form in the frequency domain via

\begin{equation}
\hat{F}(z,\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \hat{F}(z,t)\exp(i\omega t)dt,
\end{equation}

(4)

then solving Eq. (3), the annihilation operator of delayed light ̂a(z,t) has the Fourier transform of

\begin{equation}
\frac{d}{dz}\hat{a}(z,\omega) = - \Lambda(\omega)\hat{a}(z,\omega) + \frac{g^\dagger N}{C}\hat{F}(z,\omega),
\end{equation}

(5)

where

\begin{equation}
\Lambda(\omega) = \frac{|g|^2 N}{c} \left( \frac{\gamma_0 - i(\delta_0 + \omega)}{\gamma_0 - i(\delta_0 + \omega) + [\gamma_0 - i(\delta_0 + \omega)] + \Omega^2} \right)
\end{equation}

and \omega is the detection frequency.

Performing formal integration over z to obtain the delayed output probe beam throughout the EIT medium gives

\begin{align}
\hat{a}(L,\omega) &= e^{-\Lambda(0)L}\hat{a}(0,\omega) + \frac{g^\dagger N}{C} \int_0^L e^{-\Lambda(\omega)(t-z)}\hat{F}(z,\omega)dz.
\end{align}

(6)

Introducing amplitude and phase quadratures [X(z,t) and ̂Y(z,t)] of probe light, which are defined by

\begin{align}
\hat{X}(z,t) &= \hat{a}(z,t) + \hat{a}^+(z,t), \\
\hat{Y}(z,t) &= -i[\hat{a}(z,t) - \hat{a}^+(z,t)],
\end{align}

(7)

using the definition of the quadrature spectrum

\begin{align}
\langle \hat{X}(L,\omega)\hat{X}(L,\omega') \rangle &= \frac{2\pi L}{c} \delta(\omega + \omega')s_X(L,\omega), \\
\langle \hat{Y}(L,\omega)\hat{Y}(L,\omega') \rangle &= \frac{2\pi L}{c} \delta(\omega + \omega')s_Y(L,\omega),
\end{align}

(8)

the output amplitude noise spectrum of probe light is then obtained as
\begin{align}
S_X(L, \omega) \delta(\omega + \omega') &= S_1(\omega) + S_2(\omega) + \left( \frac{|g|^2 N L^2}{c L} \right) \int \int ds ds' e^{-[\lambda(\omega) + \lambda'(\omega')]|L^{-1}|} \hat{F}(s, \omega) \hat{F}(s', \omega') \\
&+ \int \int ds ds' e^{-[\lambda(\omega) + \lambda'(\omega')]|L^{-1}|} \hat{F}(s, \omega) \hat{F}^*(s', -\omega') + \int \int ds ds' e^{-[\lambda'(\omega) + \lambda'(\omega')]|L^{-1}|} \hat{F}^*(s, -\omega) \hat{F}(s', \omega') \\
&+ \int \int ds ds' e^{-[\lambda'(\omega) + \lambda'(\omega')]|L^{-1}|} \hat{F}^*(s, -\omega) \hat{F}^*(s', -\omega') . \tag{9}
\end{align}

The correlation function of \( \hat{F}(s, \omega) \) can be calculated via the quantum regression theorem of Eq. (2) when we take \( \gamma_1 = \gamma_2 = \gamma \).

\[
\langle \hat{F}_{h_1}(z_1, \omega_1) \hat{F}^*_{h_2}(z_2, \omega_2) \rangle = \frac{L \delta(z_1 - z_2) \delta(\omega_1 + \omega_2)}{N} 2 \gamma, \tag{10}
\]

Substituting Eq. (10) into Eq. (9), one obtains the normalized output amplitude spectrum

\[
S_X(L, \omega) = S_1(\omega) + S_2(\omega) + S_3(\omega) \tag{11}
\]

with

\[
S_1(\omega) = \left[ S_X(0, \omega)/4 \right] \left[ \exp\left[ -\left[ \Lambda(\omega) + \Lambda(-\omega) \right] L \right] \\
+ \exp\left[ -\left[ \Lambda^\prime(\omega) + \Lambda^\prime(-\omega) \right] L \right] \right] L \right] \\
+ \exp\left[ -\left[ \Lambda(\omega) + \Lambda^\prime(\omega) \right] L \right] \\
+ \exp\left[ -\left[ \Lambda(-\omega) + \Lambda^\prime(-\omega) \right] L \right],
\]

\[
S_2(\omega) = \left[ S_X(0, \omega)/4 \right] \left[ -\exp\left[ -\left[ \Lambda(\omega) + \Lambda(-\omega) \right] L \right] \\
- \exp\left[ -\left[ \Lambda^\prime(\omega) + \Lambda^\prime(-\omega) \right] L \right] \right] L \right] \\
+ \exp\left[ -\left[ \Lambda(\omega) + \Lambda^\prime(\omega) \right] L \right] \\
+ \exp\left[ -\left[ \Lambda(-\omega) + \Lambda^\prime(-\omega) \right] L \right],
\]

\[
S_3(\omega) = 1 - \exp\left[ -2 \text{Re} \Lambda(\omega) L \right].
\]

The output amplitude noise spectrum in Eq. (11) consists of three contributions. The first part, \( S_1(\omega) \), is related to the amplitude noise spectrum of the input probe beam \( S_X(0, \omega) \); the second part, \( S_2(\omega) \), is related to the phase noise spectrum of the input probe beam \( S_X(0, \omega) \), which contributes to the output amplitude noise of probe light due to the phase-to-amplitude noise conversion when the light pass through a coherence medium [27,28]. The third part, \( S_3(\omega) \), arises from the Langevin atomic noise resulted from the random decay process of atoms. In general, the phase-to-amplitude conversion noise may add excess noise to the quantum state of probe light during the quantum storage, except in the case of resonant EIT, for which the term of \( S_3(\omega) \) takes the value of zero; however, our following discussion will show that this excess noise can be well suppressed by choosing suitable detuning and detection frequency.

### III. The Output Noise of Probe Beam at Zero Spectral Frequency

The spectral component of the probe beam at \( \omega=0 \) for a 3 dB squeezed input beam throughout the system is plotted in Fig. 2 as a function of the two-photon detuning \( \delta_p \), with \( |g|^2 N L/c = 25 \), \( \gamma_0/\gamma = 0 \), \( \Omega/\gamma = 3.6 \), \( \delta_p = 0 \). For a 3 dB amplitude-squeezed state of light, the normalized spectrum of the two quadrature components are given as \( S_X(0, 0) = 0.5, S_1(0, 0) = 2 \). For the sake of seeing the noise squeezing within the EIT window clearly, we also plot the probe absorption versus \( \delta_p \), i.e., EIT (the thin solid line), the resonant EIT happens at \( \delta_p = 0 \). The thick solid line shows the total output amplitude noise spectral component \( S_X(L, 0) \) of probe beam, the dashed line, dotted line, and dash-dotted line represent the noise spectral components of \( S_1(0) \), \( S_2(0) \), and \( S_3(0) \), respectively. Note that \( S_X(L, 0) = 1 \) represents the shot noise level (SNL) of the output field. It is clear that at the EIT resonance (i.e., \( \delta_p = \delta_p^* = 0 \)), the contribution to the amplitude noise from the phase-to-amplitude conversion is al-

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**FIG. 2.** The output amplitude noise (the left axis) and the probe absorption (the right axis) vs the probe detuning. The thick solid line shows the total output amplitude noise \( S_X(L, \omega=0) \) of the probe light, the dashed, dotted, and dash-dotted lines are the noises of \( S_1(0) \), \( S_2(0) \), and \( S_3(0) \). The thin solid line is the probe absorption vs the probe detuning, i.e., EIT. The parameters are \( |g|^2 N L/c = 25 \), \( \gamma_0/\gamma = 0 \), \( \Omega/\gamma = 3.6 \), \( \delta_p = 0 \).
most zero and the noise of the output probe beam equals that of input probe, that is, the squeezing of probe beam throughout the atomic medium can be well maintained only at the EIT resonance, which is the recovery result discussed earlier [20,21]. When the probe beam is tuned off the EIT resonance \((\delta_p \neq 0, \delta_a = 0)\), the phase-to-amplitude noise of \(S_2(\omega)\) and the Langevin noise from the atom \(S_1(\omega)\) are gradually responsible for the output amplitude noise spectrum of the probe field. It can be seen in Fig. 2 that although the noise of \(S_1(\omega)\) reduces to zero at a particular detuning, the noise of \(S_2(\omega)\) reaches its maximum or vice versa. The total output noise (see the thick solid line) at nonzero detuning (near the EIT resonance) is always larger than that at the resonance mainly due to the phase-to-amplitude noise conversion. It is obviously that this phase-to-amplitude noise conversion introduces excess noise to output beam, and it would affect the output squeezing, so that the bandwidth of squeezing is much less than that of the EIT window.

In the preceding section we have discussed the noise spectrum of output probe light throughout an EIT medium under the relative ideal situation of \(\gamma_0 / \gamma = 0\), \(\omega / \gamma = 0\). For the physically realistic scheme, although the ground dephasing rate \((\gamma_0)\) is very small, it still cannot be neglected due to atomic collisions and atoms drifting out of the interaction region; and on the other hand, though the best squeezing occurs at zero detection frequency \((\omega / \gamma = 0)\), the relaxation oscillation of the laser at low frequency [29] prevents us from detecting the squeezing at the zero frequency in the process of squeezing measurement. Thus it is necessary to consider the influence of these parameters on the results.

**IV. THE OUTPUT NOISE OF PROBE BEAM AT NONZERO SPECTRAL FREQUENCY**

Now we turn our attention to the effects of detection frequency on the probe beam in Fig. 3. To illustrate these effects, as an example, we still consider the output noise for 3 dB input squeezing. It is noted that the noise profile in Fig. 3 is asymmetrical, the noise of probe light at the probe detuning of \(\delta_p / \gamma = -1\) is even less than that at the resonant EIT.

The reason is that, in this case, the atomic noise at the detuning of \(\delta_p / \gamma = -1\) is smaller than the noise at the resonant condition, and it almost reduces to zero (corresponding to the lowest absorption). In order to explain the reason clearly, let us consider the atomic noise term of \(S_3(\omega)\) in Eq. (11). For simplicity, we neglect the contribution from \(\gamma_0\) (because of

![FIG. 3. The output amplitude noise for nonzero detection frequency. The parameter values are \(\omega / \gamma = 1.0, \gamma_0 / \gamma = 0.01, |g|^2 NL/c = 25, \Omega / \gamma = 3.6\).](image)

![FIG. 4. The amplitude noise spectra for different two-photon detuning. Solid line: \(\delta_p / \gamma = -1.0\); dashed line: \(\delta_p / \gamma = 0\). (a), (b), and (c) are the results for the ground dephasing rate of \(\gamma_0 / \gamma = 0, \gamma_0 / \gamma = 0.01, and \gamma_0 / \gamma = 0.05\), respectively. The other parameters are as in Fig. 3.](image)
\( \gamma_0 / \gamma \), and for the one photon resonance we have \( \delta_0 = \omega_a - \omega_e = 0 \), and the atomic noise term, \( S_3(\omega) \), can be simplified to

\[
S_3(\omega) = 1 - \exp \left( -\frac{\lambda (\delta_p + \omega)^2}{([\Omega^2 - (\delta_p + \omega)]^2 + [\gamma (\delta_p + \omega)]^2)} \right).
\]

(12)

Equation (12) tell us that the atomic noise reduces to zero at \( \delta_p + \omega = 0 \), that is to say, the EIT window is shifted to an off-resonance frequency, \( \delta_p = -\omega \). Thus in the realistic measurement system with nonzero detection frequency, we can adjust the two-photon detuning so that almost no atomic noise is added to the output probe beam.

V. THE NOISE SPECTRA WITH TWO-PHOTON DETUNING

As discussed above, the two-photon detuning in the EIT system makes it possible for the quantum state of storage to be read out with low noise while simultaneously having the relative higher detection frequency. The dependences of the output noise on the detection frequency for different two-photon detuning and ground dephasing rate are plotted in Figs. 4(a)–4(c). The noise spectra show that, if we detect the output probe beam in resonant EIT with the detection frequency of \( \omega / \gamma = 1 \), the output noise would increase to 0.65 from the input 0.5 [dashed line in Fig. 4(a)]; however, for the nonresonant situation, the output noise could be lower than 0.65, see the solid line in Fig. 4(a), the output amplitude noise at the spectral frequency, \( \omega / \gamma = -\delta_p / \gamma = 1.0 \), reaches its minimum, and is almost the same as that before the process of storage. Figures 4(b) and 4(c) show the results for nonzero ground dephasing rates for \( \gamma_0 / \gamma = 0.01 \) and \( \gamma_0 / \gamma = 0.05 \). It is obvious that both the dephasing rate \( \gamma_0 \) and the detection frequency have negative effects on the maintenance of the squeezing of the quantum state (dashed lines), taking a proper two-photon detuning can minimize the output noise and consequently optimize the quantum property of output state of quantum memory.

This result gives an instructive suggestion that in the real observable scheme of state storage, one might optimize the quantum behavior of delayed light by choosing a proper two-photon detuning.

In conclusion, we have investigated the noise spectrum of amplitude-squeezed input probe light in the EIT medium. The amplitude noise spectrum of the delayed squeezed light throughout the EIT medium is determined not only by the amplitude noise of the input light, but also the phase-to-amplitude converted noise and the atomic noise. It is shown that the optimum condition to minimize the quantum noise of the delayed light in a realistic EIT scheme with a finite detection frequency can be obtained by tuning from the two-photon resonance.

The EIT medium can slow down and even store the transmitted probe light, which can also be used to implement quantum phase gates for quantum computing. In order to obtain the cross-phase modulation for the phase gates, one needs to shift from the EIT resonance to off two-photon resonance [30,31]. Preserving a quantum state efficiently under two-photon detuning discussed above has the potential application in quantum phase gates in EIT medium. Further study including the higher order of the probe intensity as discussed in Ref. [30] is needed.

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