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Quantum phase estimation with a stable squeezed state*

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Abstract. Phase estimation of optical field is a vital measurement strategy that can be used to perform accurate measurements of various physical quantities. Phase estimation precision can be greatly enhanced by using the nonclassical state such as squeezed state and entangled state. Therefore, we addressed the generation of a stable squeezed state through locking the pump laser of non-degenerate optical parametric amplifier to a high finesse Fabry–Perot cavity based on an improved cascade Pound–Drever–Hall frequency stabilization strategy. Then the obtained squeezed state is used as the probe state of quantum phase estimation and its precision is enhanced from 9.3 E-5 (shot-noise-limit) to 4.2 E-5 at the optimal phase with a given average photon number.

1 Introduction

Phase estimation protocol provides a fundamental benchmark for quantum enhanced metrology and it is a powerful measurement strategy to perform accurate measurements of various physical quantities including length, velocity and displacements [1–4]. The current studies on phase estimation are generally divided into two scenarios [5,6]. One is an interferometer with a phase shift in one arm and the total photon number is considered as a resource [7–10]. Phase sensitivity is usually more concerned than the true value of the phase shift and generally called as phase sensing. The other is a phase shift which is unknown and time-varying on a single-mode, the phase shift is measured by projective Von Neumann measurement relative to a strong local oscillator. The true value of the phase shift and phase sensitivity can be deduced, which is known as phase tracking [11,12]. Now only the photon number of the mode with the phase shift is considered as the resource. These two kinds of phase estimation strategies are both important but not equivalent.

The common measurement results for quantum mechanics are statistical, and the results are affected by the statistical error. The error comes not only from the imperfections of experiments, but also the limit of the uncertainty principle of quantum mechanics in essence. In order to reduce the influence of statistical error, the classical method is increasing the number of repeated measurements under the same conditions and then get the average result. Quantum mechanics gives a limit for parameter measurement accuracy in such process and the fluctuation of the parameter to be measured is $\propto 1/\sqrt{n}$ (where n is the mean photon number of the resource), which is so-called standard quantum limit (SQL) [13]. This is the attainable maximum accuracy for classical source. The SQL is not unbreakable theoretically, it can be broken by carefully designing the estimation procedures and many strategies have been proposed in the last several decades.

In 1981, Caves [14] proposed that a squeezed vacuum state can improve the phase estimation accuracy and break the limit of granular noise. Except for the squeezed state [15–18], an entangled state, some other state [19–21] can also be used to break the SQL in phase estimation. And there are lots of research groups focussing on how to improve the measurement strategies, such as homodyne detection [22], heterodyne detection [23], parity detection [24,25] and so on. In addition, the introduction of nonlinear effect in the parametric coupling process can also break the SQL and some nonlinear elements are introduced in linear interferometers to enhance resolution and sensitivity [26–28]. The fundamental property of quantum mechanics – Heisenberg uncertainty principle, sets a lower limit for quantum phase estimation. The lower limit is called Heisenberg limit (HL) $\propto 1/n$ and it's the ultimate precision limit for phase estimation [29].

In this paper, we address a simple and stable scheme of high precision phase estimation with a squeezed state via homodyne detection. A solid state laser is locked to a high finesse Fabry–Perot (F-P) cavity to improve the

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long-term frequency stability by an improved cascade Pound–Drever–Hall (PDH) frequency stabilization. The generated stable squeezed state undergoing an unknown phase shift is detected via homodyne detection with the help of a strong local oscillator beam. The phase estimation accuracy is enhanced from 9.3 E-5 to 4.2 E-5 at the optimal phase with a −5.99 dB squeezed state of light at a given photon number.

2 Estimation of a phase shift

The true value of phase cannot be directly measured due to the non Hermitian of the phase operator and a problem is to find a proper quantum counterpart to the classical phase observable. According to the early work of Dirac [30], the phase of optical field $\hat{\phi}$ and the number operator $\hat{n}$ should be a pair of conjugate variables and the corresponding uncertainty relation is $\Delta\hat{\phi}\Delta\hat{n} \geq 1$. The number of photon cannot be measured with infinite precision, it goes up and down around an average value because of the vacuum fluctuation of quantized electromagnetic fields. This inequality can be used to estimate the accuracy of phase estimation. If the photon number fluctuation is $\Delta\hat{n} = \sqrt{n}$, the phase estimation accuracy is satisfied $\Delta\hat{\phi} \geq 1/\sqrt{n}$. $1/\sqrt{n}$ is a standard to distinguish classical from non-classical measurements.

Phase estimation theory provides the bounds of any unbiased estimator at the given number of measured samples N in the form of Cramér–Rao theorem [2,31,32]:

$$\Delta^2\hat{\phi} \geq \frac{1}{NF(\phi)} \geq \frac{1}{NH}, \tag{1}$$

where $F(\phi)$ is the Fisher information (FI), which is the amount of information about the unknown quantity phase shift, and the quantum Fisher information (QFI) H is the maximized FI over all detection strategies. For coherent state, the QFI is proportional to the number of photons, $H = 4n$, and the quantum Cramér–Rao (QCR) bound of phase estimation is $V = 1/(4Nn)$, which is the well-known result SQL. The more number of photons of the mode with the phase shift, the more accurate of the estimation results. For squeezed state or entangled state, the phase estimation accuracy will be greatly enhanced beyond the SQL.

3 Experimental setup and results

A schematic of our experimental setup is illustrated in Figure 1. The laser source is a continuous wave intra-cavity frequency-doubled tunable single-frequency Nd:YAP/LBO solid state laser with the wavelength outputs of both 1080 nm and 540 nm, which is provided by YuGuang company (CDPSSFG-VIB). The two output beams are separated by a dichroic beam spliter (DBS) coated with the films of high reflection (HR) at 540 nm and antireflection (AR) at 1080 nm. The light at the wavelength of 540 nm is transmitted through a mode cleaner (MC) which is an optical ring cavity consisting of three mirrors to reduce extra amplitude and phase fluctuations. And the cleaned laser at 540 nm is injected as the pump field of non-degenerate optical parametric amplifier (NOPA). To improve the solid-state laser' long-term stability, we select a spherical ultra-low expansion (ULE) F-P cavity as the frequency reference standard. Due to the high finesse (50 000) of the ULE F-P cavity, it is necessary to use a fast feedback actuator that can quickly realize the locking of the F-P cavity based on the standard PDH frequency stabilization technique [33–35]. The AOM is used as the fast feedback actuator to improve the response bandwidth of the locking system. The 1080 nm firstly passes an optical isolator (ISO) and an acousto-optic modulator (AOM). The frequency-shifted first-order light of the AOM is divided into two parts, one part is coupled to a single-mode polarization maintaining fiber and then injected into the ULE F-P cavity. A piezo-electric transducer (PZT) attached on a cavity mirror of the laser is used as a slow feedback actuator to improve the long-term stability of the system. The laser frequency stabilization to a high finesse F-P cavity is achieved by using the combination of PZT and AOM as the slow and fast feedback actuators, respectively. The frequency drift of the laser is less than 7.72 MHz in 4 h in this condition, which proves an obviously improvement at frequency stability. This helps us to obtain a stable squeezed state by the NOPA. Through locking the laser to the ULE F-P cavity, the laser system can work continuously for more than

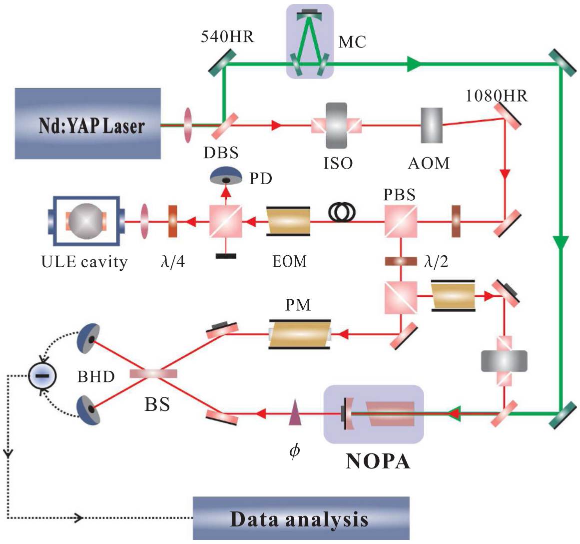

Fig. 1. Schematic of the experimental setup. Nd:YAP laser: intra-cavity frequency-doubled tunable single-frequency laser. DBS: dichroic beam spliter. 540 HR: high reflection at 540 nm. 1080 HR: high reflection at 1080 nm. MC: mode cleaner. ISO: isolator. AOM: acousto-optic modulator. $\lambda/2$: half-wave plate. $\lambda/4$: quarter-wave plate. PBS: polarization beam splitter. EOM: electro-optic modulator. PD: power detector. PM: phase modulator. ULE cavity: ultralow expansion cavity. NOPA: nondegenerate optical parametric amplifier. BS: 50/50 beam splitter. ϕ: electric phase controller. BHD: balanced homodyne detector.

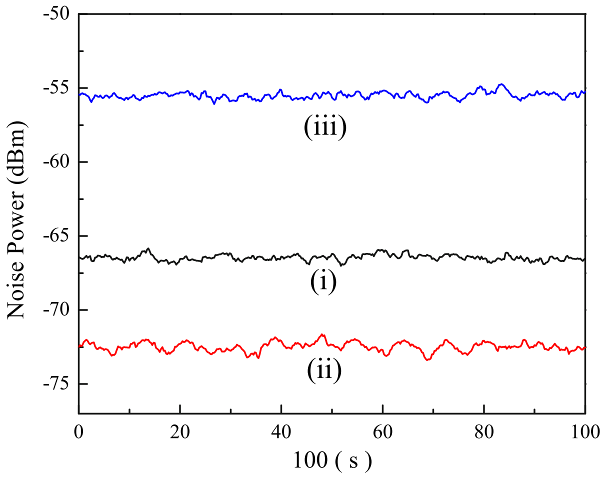

Fig. 2. The noise power spectra of the generated squeezed state. The trace (i), (ii) and (iii) are the corresponding SQL, the noise of the suppression amplitude quadrature and the noise of the amplified phase quadrature, respectively. The analysis frequency is 3 MHz, and the noise suppression of amplitude quadrature is measured as -5.99 dB relative to the SQL while the variance of the phase quadrature is amplified to $+10.95$ dB.

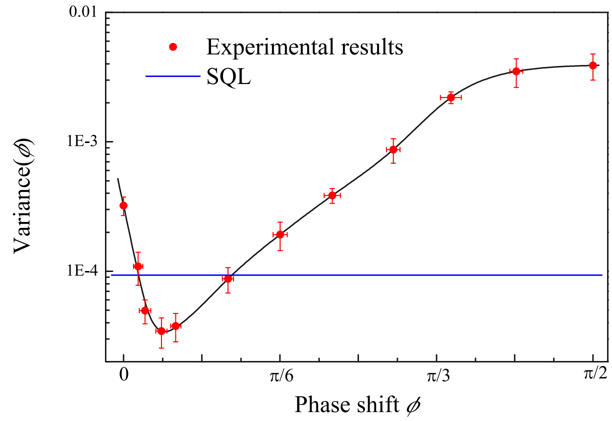

Fig. 3. Estimation variance results. Estimation variance versus twelve phase shifts with fixed number of homodyne samples. The blue curve and the red dots correspond to the SQL and the estimation results, respectively.

4 h and the continuous working time of the entire optical system can be significantly improved and it provides a reference for the practical of this quantum phase estimation scheme.

And the other part of fundemental wavelength from the laser is used as the injected signal of NOPA and the local oscillator (LO) beams of the balanced homodyne detectors (BHD). The NOPA consists of an α-cut KTP crystal and a concave mirror, which can realize type-II noncritical phase matching without walk-off effect [36]. The front face of the KTP is coated with the films of HR at 1080 nm while the transmittance is 18% for 540 nm, which serves as the input coupler. The end face of the crystal is cut to 1° along y–z plane of the crystal to satisfy the simultaneous resonance of the pump, signal and idler modes. The output coupler with a radius of curvature of 50 mm is coated with the transmittance 12.5% for 1080 nm and HR for 540 nm which is mounted on a piezoelectric transducer (PZT) to actively lock the cavity length of NOPA. The triple-resonance of pump, signal and idler modes in the NOPA is realized by adjusting the temperature of KTP around the phase-match point and moving the wedged KTP crystal transversally in the optical cavity. Nonlinear interactions can occur when the triple-resonance is realized and then the parametric down-conversion modes can be generated. The two output subharmonic fields from the NOPA operated at the parametric deamplification condition are an EPR entangled state with perpendicular polarization. The generated entangled state is coupled into two single-mode squeezed states with a half-wave plate at 22.5° and a polarization beam splitter (PBS) after the NOPA cavity. The obtained two single-mode squeezed states are orthogonal amplitude squeezed state and the orthogonal phase squeezed state respectively. This is also the advantage of the NOPA cavity, two types of nonclassical optical states can be generated by controlling the angle of the half-wave plate after the cavity. The EPR entangled state of light at 1080 nm can be generated when the angle of the half-wave plate is at 0°. By controlling the half-wave plate at 22.5°, two single-mode squeezed states of light with orthogonal polarizations can be obtained. We choose the quadrature amplitude squeezed state as the probe beam in our experiment. The noise power spectra of the generated squeezed state by BHD are shown in Figure 2. In actual experimental generation processing, there is some inevitably extra noise in antisqueezing component of the squeezed state due to the extra noise and phase fluctuation in its generation system. The noise suppression of one quadrature is measured as -5.99 dB relative to the SQL while the variance of the orthogonal quadrature is amplified by $+10.95$ dB.

The generated squeezed state undergoes an unknown phase shift ϕ and then combined with a strong local oscillator at a 50/50 beam splitter (BS) for homodyne detection to obtain the quadrature component associated with the phase shift ϕ. The true value of the phase can be infered by Bayesian inference from the homodyne measurements. The estimation variances measured at twelve different phase shifts in $[0, \pi/2]$ range are marked as circles in Figure 3. It is obvious that the estimation accuracy beyond the SQL can be realized in a phase interval near the optimal phase shift without any feedback control. The phase estimation accuracy is enhanced from 9.3 E-5 to 4.2 E-5 at the optimal phase with the squeezed state at a given photon number. We experimentally prove a simple and convenient scheme for quantum phase estimation of any unknown phase shift on a single-mode based on homodyne detection.

4 Conclusion

As an important non-classical light field, squeezed state is closely related to precision measurement due to its special characteristics that the noise of one quadrature is reduced relative to the SQL. We experimentally realize a simple and stable scheme of high precision phase estimation with squeezed state. The higher level of the squeezed state, the more accuracy of the phase estimation precision in a certain range. The squeezed state has been used in many situations such as phase precision estimation, displacement measurement, magnetic field measurement, time measurement and gravitational wave detection. With the development of science and technology, high precision measurements are more and more important and precision measurements with squeezed state will have a wider applications. In particular, the recent advances in the generation of high quality squeezed state and the development of the new squeezed source [37] will further promote the rapidly development of quantum precision measurement.

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Author contribution statement

X.J. conceived the original idea. X.J. and J.Y. designed the experimental scheme. J.Y., Y.Q. and J.Q. constructed and performed the experiment. X.J., J.Y. and Z.Y. wrote the paper.

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