Atomic $N00N$ state generation in distant cavities by virtual excitations*

Yang Rong-Can(杨榕灿)[a)b)†], Li Gang(李 刚)[a)], Li Jie(李 杰)[a)], and Zhang Tian-Cai(张天才)[a)‡]

[a)] *State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan* 030006, *China*

[b)] *School of Physics and Opto-Electronics Technology, Fujian Normal University, Fuzhou* 350007, *China*

(Received 26 October 2010; revised manuscript received 6 January 2011)

A general scheme of generating $N00N$ states of virtually-excited $2N$ atoms is proposed. The two cavities are fibre-connected with N atoms in each cavity. Although we focus on the case of $N = 2$, the system can be extended to a few atoms with $N > 2$. It is found that all $2N$ atoms can be entangled in the form of $N00N$ states if the atoms in the first cavity are initially in the excited states and atoms in the second cavity are all in the ground states. The feasibility of the scheme is carefully discussed, it shows that the $N00N$ state with a few atoms can be generated with good fidelity and the scheme is feasible in experiment.

Keywords: $N00N$ states, optical cavity, optical fibre, virtual excitations

PACS: 03.65.Ud, 03.67.Mn **DOI:** 10.1088/1674-1056/20/6/060302

1. Introduction

Quantum entanglement, first noted by Einstein *et al.*,[1] plays a central role in quantum physics and now it is an important resource for quantum information science (QIS).[2] It is known that there are various kinds of entangled states,[3] such as GHZ states,[4] W states,[5,6] cluster states,[7,8] $N00N$ states,[9,10] and so on. Recently, much attention has been paid to the $N00N$ states in the form of[9,10]

$$|N00N\rangle = \frac{1}{\sqrt{2}}\left(\mathrm{e}^{\mathrm{i}\phi}\,|N0\rangle + |0N\rangle\right). \tag{1}$$

This $N00N$ state has a wide variety of prospective applications either in the QIS or the quantum metrology.[11] For example, it can be used to push the accuracy of phase measurement to the Heisenberg limit of $1/N$.[9−13] Several research groups have devoted themselves to the preparation of $N00N$ states.[14−19] However, most proposals are based on the linear optical components, leading to a very low success probability as the number of particles increases. Some people suggested using the optical nonlinear process for $N00N$ state generation. Yet, most schemes are difficult to carry out experimentally so far.

In addition, the cavity-quantum-electrodynamics (CQED) system, known as an effective system to study the interaction between light and atoms in a confined space, is a suitable candidate for demonstrating quantum information processing and quantum state engineering.[20,21] Many schemes have been proposed based on single cavity. However, atoms are unsuitable to act as flying qubits so that it is difficult to generate quantum entangled states between distant multiple atoms.[22,23] As the good flying qubits, photons can be used as the intermediary to entangle the atoms. Supposing each pair of atom and photon is initially entangled in one cavity and then the leaking photons are combined by virtue of linear optical components (such as beam splitters),[24−27] which eventually gives rise to the two-atom entanglement. Although this method can be used to entangle two atoms, it is still very difficult to generate the entanglement of distant multiple atoms.

In 2006, Serafini *et al.* proposed a scheme for distributed quantum computation in two cavities cou-

*Project supported in part by the National Natural Science Foundation of China (Grant Nos. 10974125 and 60821004), the State Basic Key Research Program of China (Grant No. 2006CB921102), and the Science Foundation of the Educational Committee of Fujian Province, China (Grant No. JA09041).

†E-mail: rcyang@fjnu.edu.cn

‡Corresponding author. E-mail: tczhang@sxu.edu.cn

pled by an optical fibre.[28] The scheme was generalized to multi-atoms system with one excited state or two excited states for the implementation of quantum logic gate[29−31] and quantum information processing via adiabatic passage.[32−34] In these schemes, the information transmission is carried out by photons, which is sensitive to photons' decay. Recently, Zheng suggested an alternative scheme for implementing the two-atom quantum phase gate for two distant cavities without any excitations of photons.[35]

In this paper we generalize the system to multiple atoms and suggest a scheme for the generation of the $N00N$ states in this paper. The system contains N atoms trapped in the first cavity while other N atoms in the second cavity. We first pay more attention to whether we can obtain the effective Hamiltonian only for the atomic system. It shows that the photonic fields are only virtually excited in the large detuning. We then focus on the case of $N = 2$ and discuss the preparation of $N00N$ state for four atoms with the photonic field initially in the vacuum state. It shows that the probability of each cavity having one excitation can be always suppressed very strongly when $\varDelta$ is a little larger than ν and an $N00N$ state of four atoms can be generated with relative phase being always close to $3\pi/2$ or $\pi/2$ alternately along with the evolution. We have taken into account the decays of atoms, cavities and the fibres.

We also discuss the case with more atoms and numerical simulation shows that the time for obtaining the ideal $N00N$ states increases exponentially along with the number of atoms. It shows that the scheme proposed here is feasible for generating robust $N00N$ states with a few atoms.

2. Model of $2N$ atoms distributed equally in two cavities coupled by an optical fibre

We first consider two identical cavities (labeled by indices 1 and 2) coupled by a single-transverse-mode optical fibre and each cavity contains N two-level atoms with excited state $|e\rangle$ and ground state $|g\rangle$. The configuration is shown in Fig. 1. The ground state and the excited state can be chosen to be, for example, the levels $\left|5\mathrm{S}_{1/2}, F=1\right\rangle$ and $\left|5\mathrm{P}_{3/2}, F=2\right\rangle$ of the ^{87}Rb atom, respectively.[36] We suppose that the total $2N$ atoms are identical and uncoupled with each other and furthermore only one fibre mode essentially interacts with the cavity modes.[28,35] In the short fibre limit, the total Hamiltonian of the system, in the rotating-wave approximation, is given by[28]

$$H = \omega_b b^\dagger b + \sum_{j=1}^{2}\left[\frac{\omega_0}{2}\sum_{k=1}^{N}\sigma_{z,jk} + \omega_c a_j^\dagger a_j + \left(g\sum_{k=1}^{N}\sigma_{jk}^{-}a_j^\dagger + \nu b a_j^\dagger + \text{H.c.}\right)\right], \tag{2}$$

where $b(a_j)$ is an annihilating operator for the fibre mode (the j-th cavity mode) with frequency $\omega_b(\omega_c)$, $\sigma_{z,jk} = |e\rangle_{jk}\langle e| - |g\rangle_{jk}\langle g|$ the Pauli operators of the k-th atom in the j-th cavity with frequency ω_0, $\sigma_{jk}^{-} = |g\rangle_{jk}\langle e|$ the lowering operator of the corresponding atom, $\nu(g)$ represents the cavity–fibre (atom–cavity) coupling coefficient and H.c. stands for Hermitian conjugate. Here we have assumed that all the atoms have the same coupling rate. In the frame of

$$H_0 = \omega_b b^\dagger b + \omega_c\sum_{j=1}^{2}\left(\frac{1}{2}\sum_{k=1}^{N}\sigma_{z,jk} + a_j^\dagger a_j\right),$$

the total Hamiltonian H becomes

$$H_{\mathrm{I}} = \frac{\varDelta}{2}\sum_{j=1}^{2}\sum_{k=1}^{N}\sigma_{z,jk} + \sum_{j=1}^{2}\left(g\sum_{k=1}^{N}\sigma_{jk}^{-}a_j^\dagger + \nu b a_j^\dagger + \text{H.c.}\right), \tag{3}$$

where $\varDelta = \omega_0 - \omega_c$ is the detuning of the atomic transition. The detuning between the fibre and the cavity has been set to zero, i.e. $\delta = \omega_c - \omega_b = 0$.

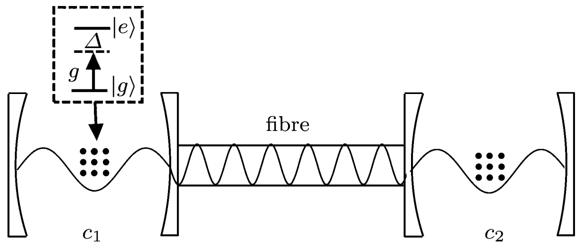

Fig. 1. Schematic diagram of $2N$ atoms distributed equally in two cavities coupled by an optical fibre and the transition of each atom.

In order to solve the evolution of the atom–cavity–fibre system, we use the approach described by Serafini *et al.*[28] and Zheng,[37] and similarly introduce three bosonic operators:

$$c_0 = \frac{1}{\sqrt{2}}\left(a_1 - a_2\right),$$

$$c_1 = \frac{1}{2}\left(a_1 + a_2 + \sqrt{2}b\right),$$

and

$$c_2 = \frac{1}{2}\left(a_1 + a_2 - \sqrt{2}b\right),$$

as well as three collective atomic operators:

$$A_{jz} = \sum_{k=1}^{N} \sigma_{z,jk},$$

$$A_j^+ = \sum_{k=1}^{N} \sigma_{jk}^+,$$

and

$$A_j^- = \sum_{k=1}^{N} \sigma_{jk}^-.$$

We can rewrite the Hamiltonian in Eq. (3) as

$$H_{\rm I} = H_0' + V, \tag{4a}$$

where

$$H_0' = \frac{\varDelta}{2}\sum_{j=1}^{2}\sum_{k=1}^{N}\sigma_{z,jk} + \sqrt{2}\nu\left(c_1^\dagger c_1 - c_2^\dagger c_2\right), \tag{4b}$$

and

$$V = \frac{g}{2}\big[A_1^+\big(c_1 + c_2 + \sqrt{2}c_0\big) + A_2^\dagger\big(c_1 + c_2 - \sqrt{2}c_0\big)\big] + \text{ H.c.} \tag{4c}$$

With respect to H_0', the interaction Hamiltonian is now

$$\begin{aligned} H_{\rm I}' = \frac{g}{2}\Big[&A_1^+\Big(c_1\,{\rm e}^{{\rm i}\left(\varDelta-\sqrt{2}v\right)t} + c_2\,{\rm e}^{{\rm i}\left(\varDelta+\sqrt{2}v\right)t} \\ &+ \sqrt{2}c_0\,{\rm e}^{{\rm i}\varDelta t}\Big) + A_2^+\Big(c_1\,{\rm e}^{{\rm i}\left(\varDelta-\sqrt{2}v\right)t} \\ &+ c_2\,{\rm e}^{{\rm i}\left(\varDelta+\sqrt{2}v\right)t} - \sqrt{2}c_0\,{\rm e}^{{\rm i}\varDelta t}\Big)\Big] + \text{H.c.} \end{aligned} \tag{5}$$

In order to obtain the state of the system under the action of this time-dependent interaction Hamiltonian, we adopt the time-averaging method to obtain a time-independent effective Hamiltonian $H_{\rm eff} = -{\rm i}H_{\rm I}'(t)\int H_{\rm I}'(t')\,{\rm d}t'$ for the interaction Hamiltonian.[38] If we further consider the large-detuning condition

$$\varDelta - \sqrt{2}\nu, \quad \sqrt{2}\nu \gg g, \tag{6}$$

we can neglect the high-frequency oscillating terms and the effective Hamiltonian $H_{\rm eff}$ will be

$$\begin{aligned} H_{\rm eff} = \frac{g^2}{4}\Bigg[&\frac{1}{(\varDelta-\sqrt{2}v)}\big[\big(A_1^+ + A_2^+\big)\big(A_1^- + A_2^-\big)c_1c_1^\dagger \\ &- \big(A_1^- + A_2^-\big)\big(A_1^+ + A_2^+\big)c_1^\dagger c_1\big] \\ &+ \frac{1}{(\varDelta+\sqrt{2}v)}\big[\big(A_1^+ + A_2^+\big)\big(A_1^- + A_2^-\big)c_2c_2^\dagger \\ &- \big(A_1^- + A_2^-\big)\big(A_1^+ + A_2^+\big)c_2^\dagger c_2\big] \\ &+ \frac{2}{\varDelta}\big[\big(A_1^+ - A_2^+\big)\big(A_1^- - A_2^-\big)c_0c_0^\dagger \\ &- \big(A_1^- - A_2^-\big)\big(A_1^+ - A_2^+\big)c_0^\dagger c_0\big]\Bigg]. \end{aligned} \tag{7}$$

It shows that there is no energy exchange between atoms and bosonic fields.

Supposing that the cavities and fibre modes are initially in the vacuum states, meaning that there are no populations in these three bosonic fields, we find that the effective Hamiltonian reduces to

$$\begin{aligned} H_{\rm eff} &= \frac{g^2}{4}\Bigg[\left(\frac{1}{\varDelta-\sqrt{2}v} + \frac{1}{\varDelta+\sqrt{2}v}\right) \\ &\quad\times\big(A_1^+ + A_2^+\big)\big(A_1^- + A_2^-\big) \\ &\quad+ \frac{2}{\varDelta}\big(A_1^+ - A_2^+\big)\big(A_1^- - A_2^-\big)\Bigg] \\ &= \lambda_1\big(A_1^+A_1^- + A_2^+A_2^-\big) \\ &\quad+ \lambda_2\big(A_1^+A_2^- + A_1^-A_2^+\big), \end{aligned} \tag{8}$$

with

$$\lambda_1 = \frac{g^2}{4}\left(\frac{1}{\varDelta-\sqrt{2}\nu} + \frac{1}{\varDelta+\sqrt{2}\nu} + \frac{2}{\varDelta}\right), \tag{9a}$$

$$\lambda_2 = \frac{g^2}{4}\left(\frac{1}{\varDelta-\sqrt{2}\nu} + \frac{1}{\varDelta+\sqrt{2}\nu} - \frac{2}{\varDelta}\right). \tag{9b}$$

In the above effective Hamiltonian Eq. (8), the first two terms describe the Stark shift for the states $|e\rangle_{jk}$ and $|g\rangle_{jk}$ and the Raman coupling among atoms in the cavity 1 and cavity 2. While the third and fourth terms represent the Raman coupling between atoms in cavity 1 and cavity 2, respectively.

3. Generation of the four-atom $N00N$ states

In this section, based on the Hamiltonian described in Eq. (8), we first focus on the generation of a four-atom $N00N$ state (i.e. $N=2$). We assume that two cavities and fibre fields are initially in the vacuum states and both atoms in cavity 1 are initially prepared in the excited states $|ee\rangle$, while another two atoms in cavity 2 are in the ground states $|gg\rangle$. So the initial state of the atomic system is

$$|\psi(0)\rangle = |ee\rangle_{11,12}\,|gg\rangle_{21,22} = |2\rangle_1\,|0\rangle_2 = |20\rangle \tag{10}$$

with

$$\begin{cases} |0\rangle_j = |g\rangle_{j1}\,|g\rangle_{j2}\,, \\ |1\rangle_j = \dfrac{1}{\sqrt{2}}\left(|e\rangle_{j1}\,|g\rangle_{j2} + |g\rangle_{j1}\,|e\rangle_{j2}\right), \\ |2\rangle_j = |e\rangle_{j1}\,|e\rangle_{j2}\,. \end{cases} \tag{11}$$

Due to the conservation of total excitons $N = \sum_{j,k=1}^{2} \sigma_{z,jk}$, the state of the system will only evolve in the subspace $\{|20\rangle, |11\rangle, |02\rangle\}$. In this case, the effective Hamiltonian described in Eq. (8) reduces to

$$H_{\text{eff}} = \begin{pmatrix} 2\lambda_1 & 2\lambda_2 & 0 \\ 2\lambda_2 & 4\lambda_1 & 2\lambda_2 \\ 0 & 2\lambda_2 & 2\lambda_1 \end{pmatrix}. \tag{12}$$

The state of the whole system at time t is thus

$$\begin{aligned} |\psi(t)\rangle &= \exp(-\mathrm{i}tH_{\text{eff}})\,|\psi(0)\rangle \\ &= \sum_{m=0}^{2} c_m(t)\,|2-m, m\rangle \end{aligned} \tag{13}$$

with

$$c_0 = \frac{1}{2}\,\mathrm{e}^{-\mathrm{i}2\lambda_1 t} + \frac{\mathrm{e}^{-\mathrm{i}3\lambda_1 t}}{2}\left[\cos\left(t\sqrt{\lambda_1^2 + 8\lambda_2^2}\right) + \mathrm{i}\,\frac{1}{\sqrt{1 + 8/(\Delta^2/\nu^2 - 1)^2}}\sin\left(t\sqrt{\lambda_1^2 + 8\lambda_2^2}\right)\right], \tag{14a}$$

$$c_1 = \frac{\mathrm{i}2\,\mathrm{e}^{-3\lambda_1 t}}{\sqrt{(\Delta^2/\nu^2 - 1)^2 + 8}}\sin\left(t\sqrt{\lambda_1^2 + 8\lambda_2^2}\right), \tag{14b}$$

$$c_2 = -\frac{1}{2}\,\mathrm{e}^{-\mathrm{i}2\lambda_1 t} + \frac{\mathrm{e}^{-\mathrm{i}3\lambda_1 t}}{2}\left[\cos\left(t\sqrt{\lambda_1^2 + 8\lambda_2^2}\right) + \mathrm{i}\,\frac{1}{\sqrt{1 + 8/(\Delta^2/\nu^2 - 1)^2}}\sin\left(t\sqrt{\lambda_1^2 + 8\lambda_2^2}\right)\right], \tag{14c}$$

where we have used the relations of Eq. (9).

According to the results given by Eqs. (13) and (14), we can easily conclude that the probability of the state $|11\rangle$ ($|c_1|^2$) decreases dramatically as the Δ/ν increases. When $\Delta/\nu \to \infty$, $|c_1|^2 \to 0$. For instance, if $\Delta/\nu \geq \sqrt{\sqrt{32}+1} \approx 2.58$, then $|c_1|^2$ is always smaller than 0.1, while if $\Delta/\nu \geq \sqrt{\sqrt{92}+1} \approx 3.25$, then $|c_1|^2 \leq 0.04$. However, if $\Delta/\nu \to \infty$, the probability of the state $|02\rangle$ also goes to zero, i.e. $|c_2|^2 \to 0$. In order to show the relations between the probabilities and the ratio Δ/ν clearly, the time-dependent probabilities ($|c_0|^2$, $|c_1|^2$ and $|c_2|^2$) versus g^2t/ν and Δ/ν are shown in Fig. 2, where the conditions described in Eq. (6) have been considered, which comprise of two parts: $\sqrt{2} < \Delta/\nu < 2\sqrt{2}$, $\nu \gg g/\left(\Delta/\nu - \sqrt{2}\right)$ and $\Delta/\nu \geq 2\sqrt{2}$, $\nu \gg g/\sqrt{2}$. From the figures, it clearly shows that $|c_1|^2$ is always suppressed to less than $4/9 \approx 0.44$ and it decreases much more quickly than $|c_2|^2$ along with the increase of Δ/ν. Thus, if we can choose Δ/ν appropriately to suppress $|11\rangle$ to a small value (such as $|c_1|^2 \leq 0.1$), the system is almost oscillating between the two states $|20\rangle$ and $|02\rangle$, which is the "two levels" of $N00N$ states of the four atoms. In addition, the results also show that the interaction time required for the excitation transferring from $|20\rangle$ to the other two states $|11\rangle$ and $|02\rangle$ increases dramatically along with the increase of Δ/ν and ν. The above analysis tells us that in order to generate the $N00N$ states with high fidelity and suppress the probability of $|11\rangle$, we need to choose large Δ/ν and small ν as possible as we can. Consider this we choose two general situations: $\nu = 5\sqrt{2}g$, $\Delta/\nu = 2\sqrt{2}$ and $\nu = 10g/\left(2 - \sqrt{2}\right)$, $\Delta/\nu = 2$, corresponding to 0.07 and 0.23, respectively, for the largest value of $|c_1|^2$ to reveal the feasible preparation of the $N00N$ states.

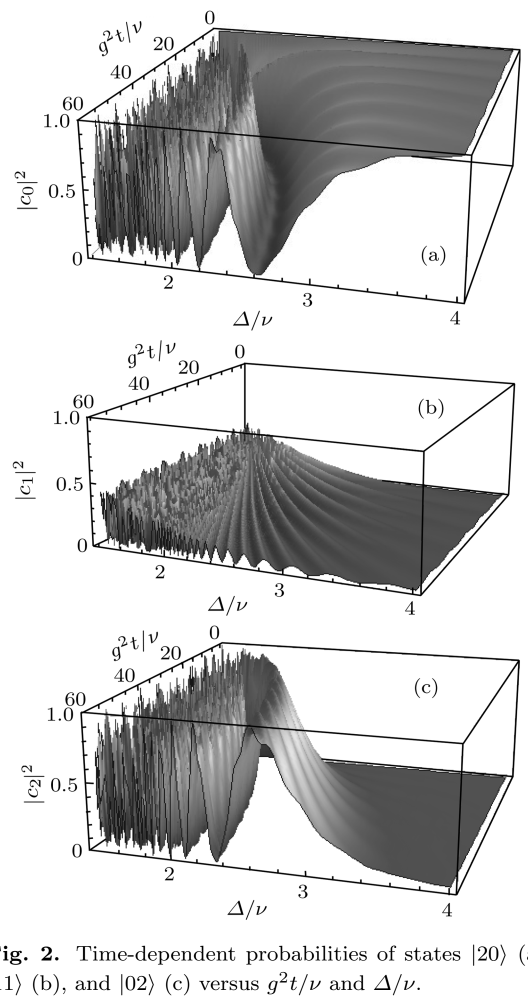

Fig. 2. Time-dependent probabilities of states $|20\rangle$ (a), $|11\rangle$ (b), and $|02\rangle$ (c) versus g^2t/ν and Δ/ν.

Before determination of the four-atom $N00N$ state in the form of Eq. (1), we have to figure out the relative phase of $|20\rangle$ and $|02\rangle$, i.e. $\phi = \mathrm{Arg}\,(c_0 - c_1)$, which is plotted in Fig. 3. It is novel that the argument of ϕ shows bistable values and in most of the time it is approximately equal to either $3\pi/2$ or $\pi/2$ alternately. In addition, we also find that the period gT of being each stable argument is proportional to Δ/ν, where T is the duration time.

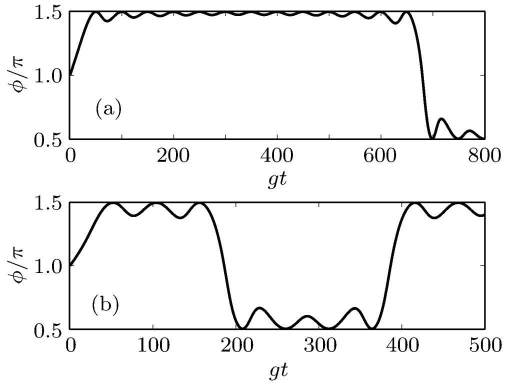

Fig. 3. Evolution of the relative phase of $|20\rangle$ and $|02\rangle$ as a function gt, where we have chosen (a) $\nu = 5\sqrt{2}g$, $\Delta/\nu = 2\sqrt{2}$ and (b) $\nu = 10g/\left(2-\sqrt{2}\right)$, $\Delta/\nu = 2$.

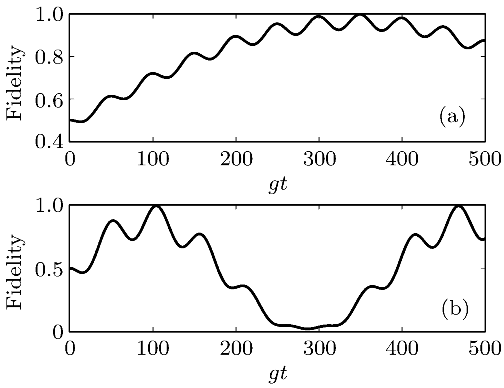

Fig. 4. The fidelity F versus gt, where we have chosen (a) $\nu = 5\sqrt{2}g$, $\Delta/\nu = 2\sqrt{2}$ and (b) $\nu = 10g/\left(2-\sqrt{2}\right)$, $\Delta/\nu = 2$.

In order to study the evolution of the amplitudes of $|20\rangle$ and $|02\rangle$ and the quality of the generated entangled states, we use the fidelity $F = |_4\langle N00N|\psi(t)\rangle|^2$ as a measure of qualifying the $N00N$ state, where the four-atom ideal $N00N$ state is chosen with a relative phase $3\pi/2$, i.e.

$$|N00N\rangle_4 = \frac{1}{\sqrt{2}}\left(-\mathrm{i}\,|20\rangle + |02\rangle\right). \tag{15}$$

The evolution of the fidelity is shown in Fig. 4. It tells us that the four-atom $N00N$ states with high fidelity can be prepared at $g\tau \approx 345.9$ for $\nu = 5\sqrt{2}g$, $\Delta/\nu = 2\sqrt{2}$ and $g\tau \approx 147.2$ for $\nu = 10g/\left(2-\sqrt{2}\right)$, $\Delta/\nu = 2$, where τ is the interaction time. It should be noted that the fidelity $F \approx 0$ in Fig. 4(b) just means that we have prepared another orthogonal $N00N$ state with the relative phase $\pi/2$ $(-3\pi/2)$, instead of $3\pi/2$ $(-\pi/2)$.

4. Discussions

Let us first analyse how the decays of system affect the $N00N$ state preparation for $N = 2$. The decay rate of the atom is represented by $\varGamma$. Without loss of generality, we suppose that the decays of cavity modes and fibre mode are the same and are both denoted as κ. The Hamiltonian described in Eq. (3) is replaced by a non-Hermitian Hamiltonian[39,40]

$$H'_{\mathrm{eff}} = H_{\mathrm{I}} - \mathrm{i}\frac{\varGamma}{2}\sum_{j=1}^{2}\sum_{k=1}^{2}|e\rangle_{jk}\langle e| - \mathrm{i}\frac{\kappa}{2}\left(\sum_{j=1}^{2}a_j^{\dagger}a_j + b^{\dagger}b\right). \tag{16}$$

The evolution of the relative phase and the fidelity of the generated $N00N$ state can be solved numerically and the results are depicted in Figs. 5 and 6, respectively. Here we have chosen $\varGamma = \kappa \approx 10^{-3}$ g. It clearly shows that the relative phase of the generated $N00N$ state is almost insensitive to the atomic decay. However, the fidelity decreases dramatically as the increase of Δ/ν (see Fig. 6). For example, the fidelity of the $N00N$ states is about 0.6 and 0.8 for $\nu = 5\sqrt{2}g$, $\Delta/\nu = 2\sqrt{2}$ and $\nu = 10g/\left(2-\sqrt{2}\right)$, $\Delta/\nu = 2$, respectively.

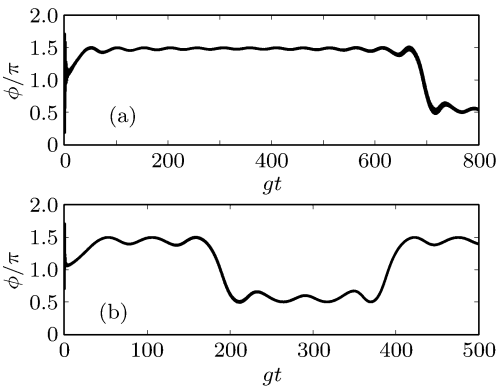

Fig. 5. Evolution of relative phase of $|20\rangle$ and $|02\rangle$ versus gt when considering the atomic decay, where we have chosen $\varGamma \approx 10^{-3}g$, (a) $\nu = 5\sqrt{2}g$, $\Delta/\nu = 2\sqrt{2}$ and (b) $\nu = 10g/\left(2-\sqrt{2}\right)$, $\Delta/\nu = 2$.

We now discuss the feasibility of the scheme. According to Ref. [27], the cavity–fibre rate is defined by $\nu \approx \sqrt{4\pi\tilde{\nu}c/l}$ with $\tilde{\nu}$ being the decay rate of the cavities' fields into a continuum of fibre modes and l the length of the fibre. In order to obtain $\sqrt{2}\nu \gg g$, the system should satisfy $\tilde{\nu}/l \gg g^2/8\pi c$. Consider a toroidal-microcavity system, the largest value for g is $2\pi \times 2.5$ GHz.[41] If we choose $l \approx 0.1$ m, then we have $\tilde{\nu} \gg g^2/80\pi c = 2\pi \times 0.13$ GHz. This can be achieved in some present physical systems[42] and thus our scheme is feasible based on current technologies.

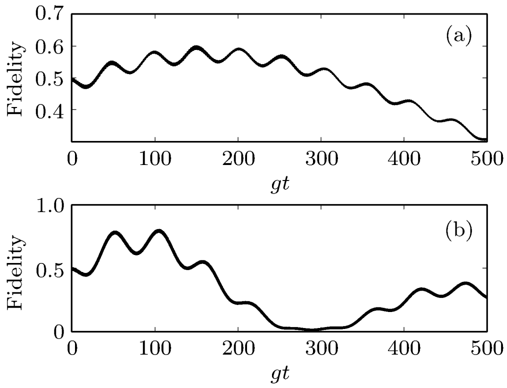

Fig. 6. The fidelity F versus gt when considering the atomic decay. We have chosen $\Gamma \approx 10^{-3}g$, (a) $\nu = 5\sqrt{2}g$, $\Delta/\nu = 2\sqrt{2}$ and (b) $\nu = 10g/\left(2-\sqrt{2}\right)$, $\Delta/\nu = 2$.

5. Generalization of the $N00N$ states to a few atoms

In this section, we generalize our scheme for the preparation of $N00N$ states to more than four atoms, i.e. $N > 2$. Similar to Section 3, we assume that the state for N atoms in the first cavity is prepared initially in $|ee\cdots e\rangle_{11,12,\ldots,1N}$, while the state for the other N atoms in the second cavity is $|gg\cdots g\rangle_{21,22,\ldots,2N}$. So, the initial state of the system is

$$\begin{aligned} |\psi(0)\rangle &= |ee\cdots e\rangle_{11,12,\ldots,1N}\,|gg\cdots g\rangle_{21,22,\ldots,2N} \\ &= |N\rangle\,|0\rangle \\ &= |N,0\rangle\,, \end{aligned} \tag{17}$$

where we have used the symmetric Dicke state $|s\rangle = \left(C_N^s\right)^{-1/2}\sum_{s=1}^{N} P_s\left(|e_1,e_2,\ldots e_s,g_{s+1},\ldots,g_N\rangle\right)$ with s atoms in the excited state $|e\rangle$ and $N-s$ atoms in the ground state to represent the atomic state in each cavity. Here $\{P_s\}$ denotes the set of all distinct permutations of the qubits. Under the action of the effective Hamiltonian described in Eq. (8), the initial system will evolve in the subspace $\{|N,0\rangle\,,|N-1,1\rangle\,,\ldots,|0,N\rangle\}$. By applying the collective raising (lowering) operator A^+ (A^-) to the symmetric Dicke state $|s\rangle$, we can obtain

$$\begin{aligned} A^+\,|s\rangle &= \sqrt{(s+1)\,(N-s)}\,|s+1\rangle\,, \\ A^-\,|s\rangle &= \sqrt{s\,(N-s+1)}\,|s-1\rangle\,, \end{aligned} \tag{18}$$

and

$$A^+A^-\,|s\rangle = s\,(N-s+1)\,|s\rangle\,. \tag{19}$$

Combining Eqs. (8), (18) and (19), one can easily obtain each element of the matrix for $H_{\rm eff}$ in the subspace $\{|N,0\rangle\,,|N-1,1\rangle\,,\ldots,|0,N\rangle\}$, which are

$$\begin{aligned} &\langle j,N-j|\,H_{\rm eff}\,|j,N-j\rangle \\ &= (j+1)\,(N-j)+j\,(N-j+1)\,, \\ &\quad j = N,N-1,\ldots,0, \end{aligned} \tag{20a}$$

$$\begin{aligned} &\langle k,N-k|\,H_{\rm eff}\,|k-1,N-k+1\rangle \\ &= k\,(N-k+1)\,\lambda_2, \end{aligned} \tag{20b}$$

$$\begin{aligned} &\langle k-1,N-k+1|\,H_{\rm eff}\,|k,N-k\rangle \\ &= k\,(N-k+1)\,\lambda_2, \quad k = N,N-1,\ldots,1. \end{aligned} \tag{20c}$$

All the other elements of the matrix are equal to zero. In order to show the required time for generating the $N00N$ state with total $2N$ atoms, we have to use the numerical simulation. The time for generating the ideal $N00N$ states with $2N$ atoms as the function of the atom number N are shown in Fig. 7. Here we have chosen (a) $\nu = 5\sqrt{2}g$, $\Delta/\nu = 2\sqrt{2}$ and (b) $\nu = 10g/\left(2-\sqrt{2}\right)$, $\Delta/\nu = 2$. The results show that the required interaction time increases exponentially along with the atomic number N. This means that it is very hard to prepare the $N00N$ states with a large number of atoms. However it is still possible to prepare the $N00N$ state with a few atoms. One may reduce the ratio Δ/ν to increase the number of atoms a little bit more but the preparation of $N00N$ states with a large number of atoms is still a big challenge.

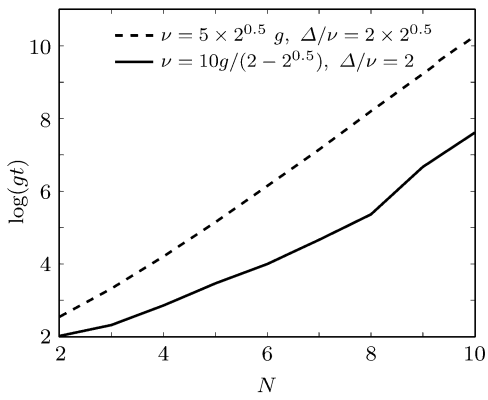

Fig. 7. The dimension-less value gt as the function of atomic number N, where we have chosen $\nu = 5\sqrt{2}g$, $\Delta/\nu = 2\sqrt{2}$ (dash line) and $\nu = 10g/\left(2-\sqrt{2}\right)$, $\Delta/\nu = 2$ (solid line).

6. Conclusion

In conclusion, we have investigated the model of $2N$ atoms distributed in two cavities coupled by an optical fibre. It shows that the cavities and fibre modes are only virtually excited in the large detuning $\Delta - \sqrt{2}\nu$, $\sqrt{2}\nu \gg g$. For $N = 2$, the generation of four-atom $N00N$ states is discussed in detail. The four-atom $N00N$ state can be generated with high fidelity for initially two atoms in the first cavity at two excitations and another two atoms in the ground state and the photonic fields in the vacuum states. We prove that the probability of each atom being in one cavity $|11\rangle$ can be strongly suppressed when Δ/ν is large enough. It is interesting that the relative phase for the generated $N00N$ states is locked to a bistable state, either close to $3\pi/2$ or $\pi/2$. The influence of the decays of system on the fidelity of the generated $N00N$ states is discussed and it shows that the preparation of $N00N$ state is still feasible. We also generalize the system to more atoms and numerical simulation shows that the time for obtaining the ideal $N00N$ states increases exponentially along with the number of atoms, which implies that it is possible to obtain the $N00N$ states with a few atoms, but preparing $N00N$ states with large number of atoms is still a challenge. Yet, the scheme proposed here is feasible for generating $N00N$ states with a few atoms.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[2] Liu J, Wang Q, Kuang L M and Zeng H S 2010 *Chin. Phys.* B **19** 030313

[3] Dür W, Cirac J I and Tarrach R 1999 *Phys. Rev. Lett.* **83** 3562

[4] Ashfaq H K, Rameez-ul-Islam and Farhan S 2010 *Chin. Phys.* B **19** 040309

[5] Zheng S B 2006 *Phys. Rev.* A **74** 054303

[6] Fang M F and Kang G D 2008 *Chin. Phys.* B **17** 3729

[7] Briegel H J and Raussendorf R 2001 *Phys. Rev. Lett.* **86** 910

[8] Yang R C, Li H C, Lin X and Huang Z P 2007 *Chin. Phys.* **16** 2219

[9] Bollinger J J, Itano W M, Wineland D J and Heinzen D J 1996 *Phys. Rev.* A **54** R4649

[10] Lee H, Kok P, Cerf N J and Dowling J P 2002 *Phys. Rev.* A **65** 030101(R)

[11] Mitchell M W, Lundeen J S and Steinberg A M 2004 *Nature* **429** 161

[12] Jones J A, Karlen S D, Fitzsimons J, Ardavan A, Benjamin S C, Briggs G A D and Morton J J L 2009 *Science* **324** 1166

[13] Witelli C, Spagnolo N, Sciarrino F and De Martini F 2009 *J. Opt. Soc. Am.* B **26** 892

[14] Gerry C C and Campos R A 2001 *Phys. Rev.* A **64** 063814

[15] Walther P, Pan J W, Aspelmeyer M, Ursin R, Gasparoni S and Zeilinger A 2004 *Nature* **429** 158

[16] Sun F W, Ou Z Y and Guo G C 2006 *Phys. Rev.* A **73** 023808

[17] Kapale K T and Dowling J P 2007 *Phys. Rev. Lett.* **99** 053602

[18] D'Angelo M, Garuccio A and Tamma V 2008 *Phys. Rev.* A **77** 063826

[19] Chen Y A, Bao X H, Yuan Z S, Chen S, Zhao B and Pan J W 2010 *Phys. Rev. Lett.* **104** 043601

[20] Bose S, Knight P L, Plenio M B and Vedral V 1999 *Phys. Rev. Lett.* **83** 5158

[21] Yang R C, Li H C, Chen M X and Lin X 2006 *Chin. Phys.* **15** 2315

[22] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392

[23] Osnaghi S, Bertet P, Auffeves A, Maioli P, Raimond J M and Haroche S 2001 *Phys. Rev. Lett.* **87** 037902

[24] Blinov B B, Moehring D L, Duan L M and Monroe C 2004 *Nature* **428** 153

[25] Parkins A S and Kimble H J 2000 *Phys. Rev.* A **61** 052104

[26] Mermin N D 1990 *Phys. Rev. Lett.* **65** 1838

[27] Lin G W, Zou X B, Lin X M and Guo G C 2009 *Phys. Rev.* A **79** 042332

[28] Serafini A, Mancini S and Bose S 2006 *Phys. Rev. Lett.* **96** 010503

[29] Yin Z Q and Li F L 2007 *Phys. Rev.* A **75** 012324

[30] Yang Z B, Wu H Z, Su W J and Zheng S B 2009 *Phys. Rev.* A **80** 012305

[31] Nohama F K and Roversi J A 2008 *J. Phys.* B: *At. Mol. Opt. Phys.* **41** 045503

[32] Chen L B, Ye M Y, Lin G W, Du Q H and Lin X M 2007 *Phys. Rev.* A **76** 062304

[33] Zhou Y L, Wang Y M, Liang M and Li C Z 2009 *Phys. Rev.* A **79** 044304

[34] Yang Z B, Ye S Y, Serafini A and Zheng S B *J. Phys.* B: *At. Mol. Opt. Phys.* **43** 085506

[35] Zheng S B 2009 *Appl. Phys. Lett.* **94** 154101

[36] Wilk T, Gaëtan A, Evellin C, Wolters J, Miroshnychenko Y, Grangier P and Browaeys A 2010 *Phys. Rev. Lett.* **104** 010502

[37] Zheng S B 2010 *Chin. Phys.* B **19** 064204

[38] James D F V 2000 *Fortschr. Phys.* **48** 823

[39] Zheng S B 2005 *Phys. Rev.* A **71** 062335

[40] Aguiar Pinto A C and Thomaz M T 2003 *J. Phys.* A: *Math. Gen.* **36** 7461

[41] Lin G W, Zou X B, Lin X M and Guo G C 2009 *Appl. Phys. Lett.* **95** 224102

[42] Hartmann M J, Brandão F G S L and Plenio M B 2006 *Nature Phys.* **2** 849