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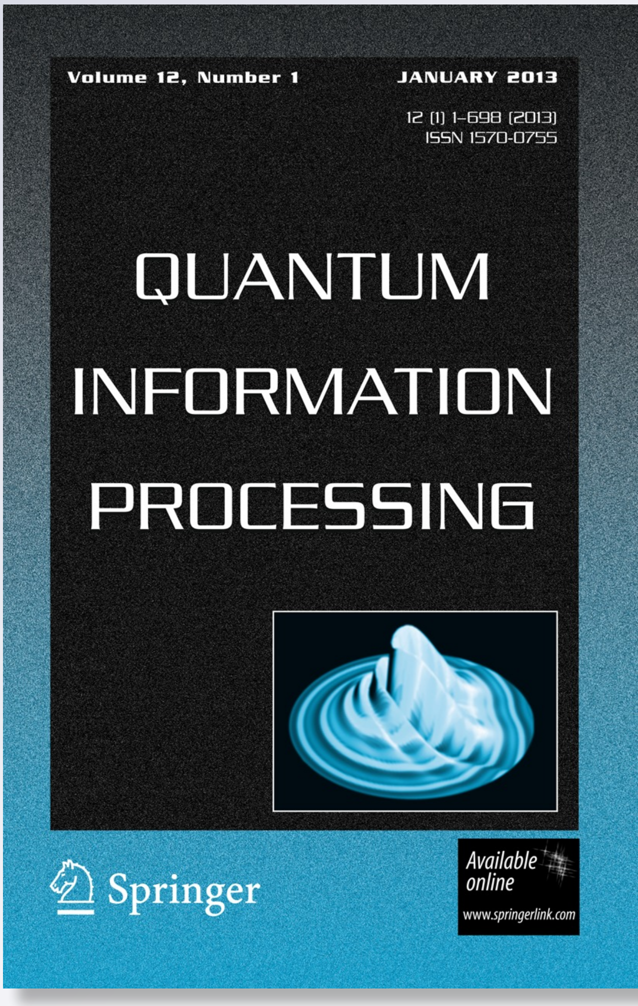

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Robust atomic entanglement in two coupled cavities via virtual excitations and quantum Zeno dynamics

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Abstract A novel proposal for the robust generation of atomic entanglement in two coupled cavities is proposed, for the first time via virtually excitation and quantum Zeno dynamics. Throughout the procedure, both cavity modes and atoms are only virtually excited, making the system robust against atomic and photonic decays. The influence of the atom-photon decay and the imperfection of the initial atom state on the prepared-state fidelity is also analyzed, which shows that the present scheme is feasible based on current technologies. At last, the proposal is generalized for the preparation of two atomic ensembles.

Keywords Quantum entanglement · Virtual excitation · Quantum Zeno dynamics

1 Introduction

Quantum entanglement, an important resource for testing quantum nonlocality and implementing quantum information science (QIS) [1,2], has been the focus in quantum physics since the famous EPR paradox in 1935 [3]. In the last twenty years, many researchers devoted themselves to studying quantum entanglement with various physical systems. Cavity quantum electrodynamics (CQED), studying the interaction

R.-C. Yang · G. Li · T.-C. Zhang (✉)
State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan 030006, People's Republic of China
e-mail: tczhang@sxu.edu.cn

R.-C. Yang
School of Physics and Opto-Electronics Technology, Fujian Normal University, Fuzhou 350007, People's Republic of China
e-mail: rcyang@fjnu.edu.cn

between the atom and the quantized field in a confined space, is being increasingly recognized as a promising technology to realize small quantum optical devices and QIS [4–7]. However, in previous schemes, most of them are based on atoms interacting with single cavities, which may be very difficult when many atoms should be individually addressed. Recently, there is a great interest in coupled-cavity arrays, for which some potential technologies have also been demonstrated experimentally [8,9]. Based on this technology, several quantum-state-preparation and distributed quantum computing proposals have been presented [10–13].

Though there has been much progress in coupled-cavity-arrays system, there is still a big challenge to build a large-scale architecture for quantum information processing in reality because of dissipation sources, such as atomic spontaneous emissions and photon losses [13,14]. To solve these problems, various kinds of methods are advanced to store the information in the ground states of Λ-type atoms due to their long-life storing time. At the same time, most of them show that the off-resonance method can be used to make the excited states of atoms and photons virtually excited [15–17] in order to reduce the influence of decoherence.

In addition, the quantum Zeno effect (QZE), first understood by Neumann [18] and named by Misra and Sudarshan [19], shows that the transition between quantum states can be slowed down and even inhibited by frequent measurement. Since the phenomenon was demonstrated in experiment [20], it has attracted much attention. Many novel experimental tests with various physical systems have shown that the QZE can be used to protect quantum states against decoherence [21–23]. It can not only hinder the transition process, but also induct the evolution of the system from the initial state, leaving the system in the "Zeno subspace" defined by the measurement [24,25]. This was called "quantum Zeno dynamics (QZD)" by Facchi and Pascazio in 2002 [26]. They have shown that QZD can be achieved via continuous coupling between the system and an external system instead of discontinuous measurements, according to von Neumann's projection postulate. Based on QZD, there are some schemes proposed for the implementation of controlled logic gates and the preparation of quantum entangled states [27–30].

In this article, combining virtual excitation (VE) and QZD, to our knowledge it is the first time, we present a novel scheme for the robust generation of atomic entanglement. We first consider the case where only two atoms are separately trapped in a cavity and the two cavities are coupled to each other directly. The two cavities and all atoms are virtually excited through QZD. Similar to the previous schemes via QZD, here we have used weak laser to resonantly drive one atomic transition with another transition resonantly coupled to cavities. But quite differently, the excited states for all atoms and photons in our proposal are virtually excited, which makes the scheme robust against atomic spontaneous emission and cavity decays simultaneously. After that, we generalize the scheme for generating the entanglement of two atomic ensembles.

This paper is organized as follows. In Sect. 1, we briefly introduce QZD. The scheme for the generation of two Λ-type atoms via VE and QZD is presented in Sect. 3. In Sect. 4 we discuss the feasibility and the decay influence for state preparation based on the current experimental status. The proposal is generalized for the generation of two atomic ensembles in Sect. 5. and a brief summary is given in Sect. 6.

2 QZD

Consider a system with total Hamiltonian H consisting of two parts: the system's Hamiltonian under observation H_{obs} and the interaction Hamiltonian H_{meas} between the system and the apparatus, i.e. [25,26]

$$H = H_{obs} + H_{meas}(K), \tag{1}$$

where K is a set of coupling coefficients between the system and the apparatus. The time-evolution operator for the whole system is $U(t) = \exp(-iHt)$. In the "infinitely strong measurement" limit $K \to \infty$ [25,26], the evolution operator $U(t)$ is proved to be diagonal with respect to H_{meas}, and has the form of [25,26]

$$U(t) = \exp\left[-it\left(H_{diag} + \sum_n \eta_n P_n\right)\right], \tag{2}$$

where

$$H_{diag} = \sum_n P_n H_{obs} P_n \tag{3}$$

with P_n being the orthogonal projection onto the invariant Zeno subspaces H_{P_n}, and the eigenspace of H_{meas} belonging to the eigenvalue $\eta_n (\eta_m \neq \eta_n$ when $m \neq n)$, i.e. $H_{meas} P_n = \eta_n P_n$. In other words, the Hamiltonian for the whole system, in the "infinitely strong measurement" limit, equals [26]

$$H_{eff} = \sum_n (P_n H_{obs} P_n + \eta_n P_n). \tag{4}$$

From the above effective Hamiltonian, it is easily deduced that the system state will remain in the same Zeno subspace as that of its initial state, i.e. $|\psi(t)\rangle \in H_{p_n} \ \forall |\psi(0)\rangle \in H_{p_n}$. Furthermore, if the Zeno subspace H_{p_n} is a dark subspace of H_{meas}, i.e. $\eta_n = 0$, the effective Hamiltonian will reduce to

$$H_{eff} = P_n H_{obs} P_n. \tag{5}$$

3 Atomic entanglement generation for two Λ-type atoms via VE and QZD

Now let us describe our scheme in detail. Suppose that there are two Λ-type atoms trapped in cavity 1 (C_1) and cavity 2 (C_2), respectively. The two single-mode cavities are directly coupled with each other [11] and each atom has three levels: one excited state $|e\rangle$ and two ground states $|g_1\rangle$ and $|g_2\rangle$. The $|g_1\rangle \to |e\rangle$ transition for each atom is resonantly driven by a classical laser, while the transition $|g_2\rangle \to |e\rangle$ is resonantly coupled to the cavity mode. The atomic transition structure and the schematic diagram are shown in Fig. 1. Here we assume that the $|g_1\rangle \to |g_2\rangle$ transition is

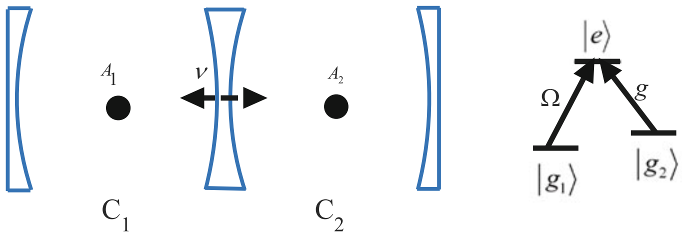

Fig. 1 (Color online) Schematic diagram and atomic transition configuration for the two-atom entanglement generation

dipole-forbidden. In the dipole and rotating-wave approximation, the Hamiltonian describing the whole system, in the interaction picture, reads

$$H_{ac} = \sum_{k=1}^{2} \left(\Omega_k |e\rangle_k \langle g_1| + g_k |e\rangle_k \langle g_2| a_k\right) + \nu a_1^{+} a_2 + H.c., \tag{6}$$

where the subscript described the k-th atom (cavity). a $\left(a^{+}\right)$ is the annihilation (creation) operator of the cavity mode. Ω, g and ν represent the Rabi frequency of the driving laser, atom-cavity coupling coefficient and hopping rate between the two cavities, respectively. $H.c.$ stands for Hermitian conjugate.

Suppose that the first (second) atom is initially prepared in the ground state $|g_1\rangle$ ($|g_2\rangle$) and the two cavities both in the vacuum state $|0\rangle$, leading the initial system state to be

$$|S(0)\rangle = |g_1\rangle_1 |g_2\rangle_2 |0\rangle_{C_1} |0\rangle_{C_2} = |g_1 g_2\rangle |00\rangle. \tag{7}$$

To reach the aim, we can first cooled the two atoms to the same ground state (such as $|g_1\rangle$) by optical pumping technique in cold atomic physics [31], and then the second atom are transferred to the other ground state $|g_2\rangle$ by Raman two-photon transition and Larmor spin precession induced by magnetic field pulses with high fidelity [32]. In this case the reduced Hamiltonian in the closing subspace

$$\begin{aligned} \{ &|\xi_1\rangle = |g_1 g_2\rangle |00\rangle, |\xi_2\rangle = |e g_2\rangle |00\rangle, |\xi_3\rangle = |g_2 g_2\rangle |10\rangle, \\ &|\xi_4\rangle = |g_2 g_2\rangle |01\rangle, |\xi_5\rangle = |g_2 e\rangle |00\rangle, |\xi_6\rangle = |g_2 g_1\rangle\, 00\rangle \}, \end{aligned} \tag{8}$$

will be

$$H'_{ac} = \Omega_1 |\xi_1\rangle\langle\xi_2| + g_1 |\xi_2\rangle\langle\xi_3| + \nu |\xi_3\rangle\langle\xi_4| + g_2 |\xi_4\rangle\langle\xi_5| + \Omega_2 |\xi_5\rangle\langle\xi_6| + H.c.. \tag{9}$$

In order to solve the system's evolution, let us introduce four orthogonal vectors

$$|D_1\rangle = \frac{1}{N_1}\left(g_1 |\xi_1\rangle - \Omega_1 |\xi_2\rangle\right), |D_2\rangle = \frac{1}{N_2}\left(g_2 |\xi_6\rangle - \Omega_2 |\xi_5\rangle\right), \tag{10a}$$

$$|B_1\rangle = \frac{1}{N_1}\left(\Omega_1|\xi_1\rangle + g_1|\xi_5\rangle\right), |B_2\rangle = \frac{1}{N_2}\left(\Omega_2|\xi_6\rangle + g_2|\xi_5\rangle\right), \tag{10b}$$

with $N_1 = \sqrt{\Omega_1^2 + g_1^2}$ and $N_2 = \sqrt{\Omega_2^2 + g_2^2}$. Then, the reduced Hamiltonian will be rewritten in the rotating basis $\{|D_1\rangle, |D_2\rangle, |B_1\rangle, |B_2\rangle, |\xi_2\rangle, |\xi_5\rangle\}$ as

$$H'_{ac} = H'_{ac-1} + H'_{ac-2}, \tag{11a}$$

$$H'_{ac-1} = \frac{\nu\Omega_1\Omega_2}{N_1N_2}|D_1\rangle\langle D_2| + H.c., \tag{11b}$$

$$\begin{aligned} H'_{ac-2} = {} & N_1|B_1\rangle\langle\xi_2| + N_2|B_2\rangle\langle\xi_5| + \frac{\nu g_1 g_2}{N_1N_2}|B_1\rangle\langle B_2| \\ & -\frac{\nu\Omega_1 g_2}{N_1N_2}|D_1\rangle\langle B_2| - \frac{\nu g_1\Omega_2}{N_1N_2}|D_2\rangle\langle B_1| + H.c. \end{aligned} \tag{11c}$$

If the Rabi frequencies Ω_1 and Ω_2 are adjusted to satisfy the relations

$$\Omega_1 << g_1, \;\; \Omega_2 << g_2, \tag{12a}$$

and

$$\Omega_1\Omega_2\nu << \min\left(N_1^2N_2, N_1N_2^2\right), \tag{12b}$$

then $\frac{\nu\Omega_1\Omega_2}{N_1N_2} << N_1, N_2, \frac{\nu g_1 g_2}{N_1N_2}, \frac{\nu\Omega_1 g_2}{N_1N_2}, \frac{\nu g_1\Omega_2}{N_1N_2}$, implying that the Hamiltonian H'_{ac} described in the Eq. (11) fulfills the "infinitely strong measurement" limit [26]. In this case, we can calculate the invariant Zeno subspaces for the subspace $\{|D_1\rangle, |D_2\rangle, |B_1\rangle, |B_2\rangle, |\xi_2\rangle, |\xi_5\rangle\}$ as

$$H_{P_1} = \left\{\left|\eta_1^1\right\rangle, \left|\eta_1^2\right\rangle\right\}, H_{P_2} = \{|\eta_2\rangle\}, H_{P_3} = \{|\eta_3\rangle\}, H_{P_4} = \{|\eta_4\rangle\}, H_{P_5} = \{|\eta_5\rangle\} \tag{13}$$

with corresponding eigenvalues $\eta_1 = 0$, $\eta_{2,3} = \pm\sqrt{\frac{\left(p^2-\sqrt{p^4-4q^2}\right)}{2}}$ and $\eta_{4,5} = \pm\sqrt{\frac{\left(p^2+\sqrt{p^4-4q^2}\right)}{2}}$ where

$$p^2 = N_1^2 + N_2^2 + \frac{\nu^2 g_1^2 g_2^2}{N_1^2N_2^2} + \frac{\nu^2\Omega_1^2 g_2^2}{N_1^2N_2^2} + \frac{\nu^2 g_1^2\Omega_2^2}{N_1^2N_2^2} \tag{14a}$$

$$q^2 = \left(N_1^2 + \frac{\nu^2 g_1^2\Omega_2^2}{N_1^2N_2^2}\right)\left(N_2^2 + \frac{\nu^2\Omega_1^2 g_2^2}{N_1^2N_2^2}\right), \tag{14b}$$

and

$$\left|\eta_1^1\right\rangle = \frac{1}{N_3}\left(N_1^2 N_2 \left|D_1\right\rangle - \nu\Omega_1 g_2 \left|\xi_5\right\rangle\right), \left|\eta_1^2\right\rangle = \frac{1}{N_4}\left(N_1^2 N_2 \left|D_2\right\rangle - \nu\Omega_2 g_1 \left|\xi_2\right\rangle\right), \tag{15}$$

with $N_3 = \sqrt{N_1^2 N_2^4 + \nu^2\Omega_1^2 g_2^2}$ and $N_4 = \sqrt{N_1^4 N_2^2 + \nu^2\Omega_2^2 g_1^2}$. It is noted that we do not give the expressions of $|\eta_m\rangle$ $(m = 2, 3, 4, 5)$ because they hardly have any effect on the system's evolution as long as the initial system state belongs to H_{P_1}.

Now, let us turn to Eqs. (15) and (10). It can easily be verified that

$$\left|\eta_0^1\right\rangle \approx \left|D_1\right\rangle \approx \left|\xi_1\right\rangle, \left|\eta_0^2\right\rangle \approx \left|D_2\right\rangle \approx \left|\xi_2\right\rangle \tag{16}$$

under the condition (12) and

$$\Omega_1 \nu g_2 << N_1 N_2^2, \Omega_2 \nu g_1 << N_1^2 N_2. \tag{17}$$

In other words, the initial system state $|S(0)\rangle = |\xi_1\rangle$ belongs to invariant Zeno subspace $H_{P_1} \approx \{|\xi_1\rangle, |\xi_2\rangle\}$. Then the effective Hamiltonian for the system is equal to

$$\begin{aligned} H_{ac-eff} &= P_1 H'_{ac-1} P_1 + \eta_1 P_1 \\ &\approx \frac{\Omega_1\Omega_2\nu}{N_1 N_2}\left(|\xi_1\rangle\langle\xi_1| + |\xi_6\rangle\langle\xi_6|\right)|\xi_1\rangle\langle\xi_6|\left(|\xi_1\rangle\langle\xi_1| + |\xi_6\rangle\langle\xi_6|\right) \\ &= \frac{\Omega_1\Omega_2\nu}{N_1 N_2}|\xi_1\rangle\langle\xi_6| + H.c. \;=\; \frac{\Omega_1\Omega_2\nu}{N_1 N_2}|g_1 g_2\rangle\langle g_2 g_1| \otimes |00\rangle\langle 00| + H.c. \end{aligned} \tag{18}$$

It should be noted that all the atomic (photonic) excited states have been eliminated in the effective Hamiltonian, meaning that the present system may be robust against atomic (photonic) decays.

Under the action of the effective Hamiltonian (18), the system state, at the interaction time t, is

$$|S(t)\rangle = \exp\left(-iH_{ac-eff}t\right)|S(0)\rangle \;= \left(\cos\omega t\,|g_1 g_2\rangle - i\sin\omega t\,|g_2 g_1\rangle\right)|00\rangle \tag{19}$$

with $\omega = \frac{\Omega_1\Omega_2\nu}{N_1 N_2}$. If the interaction time is chosen to be $t = \tau = \pi/4\omega$, then a maximally entangled state between these two atoms will be generated, i.e.

$$|S(\pi/2\omega)\rangle = \frac{1}{\sqrt{2}}\left(|g_1 g_2\rangle - i\,|g_2 g_1\rangle\right) \tag{20}$$

where the vacuum states of cavities have be eliminated. So far, we have completed the procedure of entanglement generation between two atoms trapped in two coupled cavities.

4 Discussion

The feasibility of the proposal is discussed in this section. Let us first consider the experimental conditions. In our scheme, two coupled optical cavities are needed. This can be achieved either by using the adjacent cavities [8] or optical fiber connected cavities [33] with acceptable parameters based on the present experiments discussed below. The three-level Λ-type atoms could be chosen, for example, the cesium atoms (^{133}Cs) with the excited state $\left|6P_{3/2}, F=4\right\rangle$ and two ground states $\left|6S_{1/2}, F=3\right\rangle$ and $\left|6S_{1/2}, F=4\right\rangle$, respectively [34].

We now consider the influence of atom and photon decays and imperfection of the initial atomic states on entanglement preparation. The whole system described by Eq. (7) will be governed by the master equation

$$\begin{aligned}\dot{\rho} = & -i\left[H_n, \rho\right] - \frac{\kappa_1}{2}\left(a_1^{+}a_1\rho - 2a_1\rho a_1^{+} + \rho a_1^{+}a_1\right)\\ & -\frac{\kappa_2}{2}\left(a_2^{+}a_2\rho - 2a_2\rho a_2^{+} + \rho a_2^{+}a_2\right)\\ & -\sum_{j=g_1,g_2}\sum_{m=1}^{2}\frac{\gamma_m^{ej}}{2}\left(\sigma_{ee}^{m}\rho - 2\sigma_{je}^{m}\rho\sigma_{ej}^{m} + \rho\sigma_{ee}^{m}\right),\end{aligned} \tag{21}$$

where $\sigma_{kl}^{m} = |k\rangle_m\langle l|\,(k,l=e,g_1,g_2)\,,\kappa_j\,(j=1,2)$ is the decay rate of the j-th cavity mode and γ_m^{ej} represents the branching ratio of the m-th atom decay from $|e\rangle_m$ to $|j\rangle_m$. For the sake of simplicity, we let $\kappa_1=\kappa_2=\kappa$, $\gamma_m^{eg_1}=\gamma_m^{eg_2}=\gamma/2$, $g_1=g_2=g$, $\Omega_1=\Omega_2=\Omega$ and suppose the two atoms are initially prepared in the state

$$\begin{aligned}\rho_a = & \left(p_1|g_1\rangle_1\langle g_1| + \sqrt{1-p_1^2}\,|g_2\rangle_1\langle g_2|\right)\\ & \otimes\left(p_2|g_2\rangle_2\langle g_2| + \sqrt{1-p_2^2}\,|g_1\rangle_2\langle g_1|\right)\end{aligned} \tag{22}$$

with p_1 (p_2) being the purity of the atom 1 (2) prepared in the $|g_1\rangle$ ($|g_2\rangle$), where we have supposed that there is no other atomic states except $|g_1\rangle$ and $|g_2\rangle$ initially. In order to solve the master equation, we assume that the system evolves in the subspace

$$\begin{aligned}&\{|\xi_1\rangle=|g_1g_1\rangle|00\rangle, |\xi_2\rangle=|eg_1\rangle|00\rangle, |\xi_3\rangle=|g_1e\rangle|00\rangle, |\xi_4\rangle=|ee\rangle|00\rangle, |\xi_5\rangle=|g_2g_1\rangle|10\rangle,\\ &|\xi_6\rangle=|g_1g_2\rangle|01\rangle, |\xi_7\rangle=|g_2e\rangle|10\rangle, |\xi_8\rangle=|eg_2\rangle|01\rangle, |\xi_9\rangle=|g_2g_1\rangle|01\rangle, |\xi_{10}\rangle=|g_1g_2\rangle|10\rangle,\\ &|\xi_{11}\rangle=|g_2e\rangle|01\rangle, |\xi_{12}\rangle=|eg_2\rangle|10\rangle, |\xi_{13}\rangle=|g_2g_2\rangle|02\rangle, |\xi_{14}\rangle=|g_2g_2\rangle|20\rangle, |\xi_{15}\rangle=|g_2g_2\rangle|11\rangle,\\ &|\xi_{16}\rangle=|g_1g_2\rangle|00\rangle, |\xi_{17}\rangle=|eg_2\rangle|00\rangle, |\xi_{18}\rangle=|g_2g_2\rangle|10\rangle, |\xi_{19}\rangle=|g_2g_2\rangle|01\rangle, |\xi_{20}\rangle=|g_2e\rangle|00\rangle,\\ &|\xi_{21}\rangle=|g_2g_1\rangle|00\rangle, |\xi_{22}\rangle=|g_2g_2\rangle|00\rangle\}.\end{aligned} \tag{23}$$

Then we use the quantum optics toolbox described in Ref. [35] to obtain the numerical solution. Here, we have defined the fidelity of the prepared state ρ' as: $F\left(\rho'\right) = Tr\left(\sqrt{\rho^{1/2}\rho'\rho^{1/2}}\right)$, where ρ is the idea entangled state of two atoms

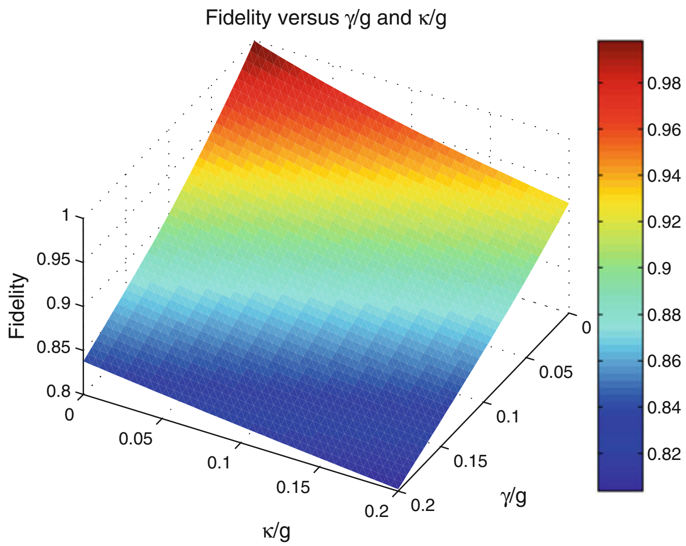

Fig. 2 (Color online) The fidelity as functions of the atomic and photonic decay, where we have chosen $g_1 = g_2 = \nu = g$ and $\Omega_1 = \Omega_2 = \Omega = 0.1\,g$

(i.e. $\rho = |S(\pi/2\omega)\rangle\langle S(\pi/2\omega)|$, where $|S(\pi/2\omega)\rangle$ being described in Eq. (20)) and ρ' is the prepared state, or output state which is in general a mixed state in real experiment [36]. Here the fidelity refers to state an overlap between the ideal entangled state $|S(\pi/2\omega)\rangle$ of two atoms and the output state ρ', i.e. $F = \langle S(\pi/2\omega)|\,\rho'\,|S(\pi/2\omega)\rangle$. For clarity, the fidelity of generated states versus κ/g and γ/g is shown in Fig. 2 with the initially perfect atom state (i.e. $p_1 = p_2 = 1$), where the hopping rate is chosen to be $\nu = g$. The illustration shows that the prepared-state fidelity decreases slowly as the increase of κ and γ and the decreasing rate induced by atomic spontaneous emission is slightly smaller than that induced by photon loss. For example, the fidelity F is about 95.5 % (90.4 %) when only atom (photon) decay is considered in the case of $\gamma/g = 0.1$ ($\kappa/g = 0.1$) and $F \approx 87.8\,\%$ when both are considered. Even when $\kappa/g = \gamma/g = 0.2$, the fidelity still reaches $F \approx 80.2\,\%$. We then take the imperfect atom state into account. For simplicity, we assume $p_1 = p_2 = p$ and $\kappa/g = \gamma/g = 0.1$. The fidelity of the prepared state F versus the probability p is illustrated in Fig. 3. Based on the illustration, we find out that the prepared-state fidelity increases along with the increase of the purity before $p = 96\,\%$ and then decreases and its maximum value is 94.8 %, meaning that appropriate imperfection of the initial state may increase the fidelity of the prepared entanglement.

Before ending the discussion, we estimate the fidelity and the required time for the generation of entanglement based on the current experimental situations. The limit of experimental parameters for Cs atoms trapped in Fabry-Perot cavity is about $(g_1, \kappa, \gamma) = 2\pi \times (110, 2.6, 14.2)$ MHz [37]. We also choose $g_1 = g_2 = \nu = g$, $\Omega_1 = \Omega_2 = \Omega = 0.1g$ and suppose that the fidelity of the two initial atom states are both 96 %, i.e. $p = 96\,\%$ [32] (These parameters are also chosen in the following passage). The fidelity and the required time for entanglement preparation is $F \approx 98.3\,\%$

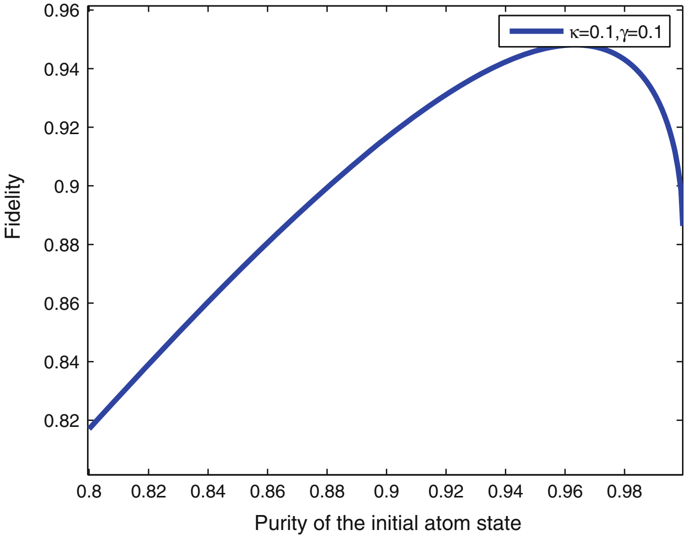

Fig. 3 (Color online) The fidelity versus the purity of the initial atom state, where we have chosen $g_1 = g_2 = \nu = g$, $\Omega_1 = \Omega_2 = \Omega = 0.1\,g$, $\kappa = \gamma = 0.1\,g$, and $p_1 = p_2 = p$

and $\tau \approx 0.13\,\mu$s, respectively. The toroidal-microcavity [38] coupled to the nanofibre is more feasible based on our scheme [39,40]. It has been demonstrated at Caltech that toroidal cavities on a chip can be fabricated using a combination of lithography and a silica reflow process, and a toroidal-microcavity with a diameters of a few micrometers was obtained. The atoms are located on a sub-wavelength scale near the resonator's surface and the system has the CQED parameters of $(g, \kappa, \gamma) = 2\pi \times (450, 1.7, 2.6)$ MHz (κ and γ are obtained from critical photon number and critical atom number) [37]. Based on this system, we conclude that the fidelity and the required time for our system are $F \approx 97.8\,\%$ and $\tau \approx 0.37\,\mu$s, respectively. In recent experiment, they have realized the CQED parameters of $(g_{\mathrm{max}}, \kappa, \gamma) = 2\pi \times (100, 21, 2.6)$ MHz with single caesium atoms coupled to a high-Q microtoroidal cavity [41], which implies the fidelity and the required time of $F \approx 86.2\,\%$ and $\tau \approx 0.11\,\mu$s for our system, respectively. If the CQED parameters $(g, \kappa, \gamma) = 2\pi \times (23.9, 2.6, 2.6)$ MHz in our lab are chosen [42,43], then $F \approx 94.3\%$ and $\tau \approx 2.9\,\mu$s. Therefore, our scheme is feasible based on the current technologies.

5 Generalization for the entanglement of two atomic ensembles

In this section, we show that our proposal can be easily generalized to entangle two atomic ensembles in two coupled cavities. Each ensemble possesses n_1 and n_2 Λ-type atoms and is trapped in one of the two coupled cavities similar to that described in Sect. 3. The atomic transition is also shown in Fig. 1. In this case, the Hamiltonian for the total system, in the interaction picture, can also be expressed as

$$H_I = \sum_{k=1}^{2}\sum_{l=1}^{n_k} \left(\Omega_{kl} \left|e\right\rangle_{kl} \left\langle g_1\right| + g_{kl} \left|e\right\rangle_{kl} \left\langle g_2\right| a_k\right) + \nu a_1^{+} a_2 + H.c. \tag{24}$$

Initially, the first ensembles is prepared in the Dicke state $|1, 0, n_1 - 1\rangle_1$, where

$$|1, 0, n_1 - 1\rangle_1 = \frac{1}{\sqrt{n_1}} \sum_{k=1}^{n_1} |g_2 g_2 \ldots g_1 \ldots g_2\rangle_{12\ldots k \ldots n_1}, \tag{25}$$

meaning that there is only one atom in the ground state $|g_1\rangle$, no atoms in the excited state $|e\rangle$, and $n_1 - 1$ atoms in the ground state $|g_2\rangle$. While the second ensemble is initially cooled to $|0, 0, n_2\rangle_2$ and the two cavities both in the vacuum state $|0\rangle$. Thus the initial system state is

$$|S(0)\rangle = |1, 0, n_1 - 1\rangle_1 |0, 0, n_2\rangle_2 |0\rangle_{C_1} |0\rangle_{C_2} = |1, 0, n_1 - 1\rangle |0, 0, n_2\rangle |00\rangle. \tag{26}$$

In the closed subspace

$$\begin{aligned} \left|\xi_1'\right\rangle &= |1, 0, n_1 - 1\rangle |0, 0, n_2\rangle |00\rangle, \left|\xi_2'\right\rangle = |0, 1, n_1 - 1\rangle |0, 0, n_2\rangle |00\rangle, \\ \left|\xi_3'\right\rangle &= |0, 0, n_1\rangle |0, 0, n_2\rangle |10\rangle, \left|\xi_4'\right\rangle = |0, 0, n_1\rangle |0, 0, n_2\rangle |01\rangle, \\ \left|\xi_5'\right\rangle &= |0, 0, n_1\rangle |0, 1, n_2 - 1\rangle |00\rangle, \left|\xi_6'\right\rangle = |0, 0, n_1\rangle |1, 0, n_2 - 1\rangle |00\rangle, \end{aligned} \tag{27}$$

the Hamiltonian in Eq. (24) can also be written in the form described in Eq. (9) with collective Rabi frequency $\Omega_k = \sqrt{\sum_{l=1}^{n_k} \Omega_{kl}^2 / n_k}$ and collective atom-cavity coupling rate $g_k = \sqrt{\sum_{l=1}^{n_k} g_{kl}^2}$. Thus, we can entangle these two atomic ensembles via VE and QZD as long the collective Rabi frequencies and collective atom-coupling rates satisfy the conditions described in Eqs. (12) and (17).

6 Conclusions

In summary, we have presented a scheme for robust atomic entanglement generation with three-level atoms trapped in two coupled cavities. Different from the previous schemes, this scheme is based on virtual excitations and QZD. The transition between one ground state and excited state is driven by weak lasers while another atomic transition is resonantly coupled to cavities. In addition, the excited states and cavities are all virtually excited in the whole procedure, which makes the scheme robust against atomic and cavity decays. We have analyzed the influence of spontaneous emission, photon losses and the imperfection of the initial atom state on the performance of the entanglement state preparation which shows that our proposal is quite feasible based on current technologies.

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