

Optical control of a magnetic Feshbach resonance in an ultracold Fermi gas

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We use laser light near resonant with a molecular bound-to-bound transition to control a magnetic Feshbach resonance in ultracold Fermi gases of ^{40}K atoms. The spectrum of excited molecular states is measured by applying a laser field that couples the ground Feshbach molecular state to electronically excited molecular states. Nine strong bound-to-bound resonances are observed below the $^2P_{1/2} + ^2S_{1/2}$ threshold. We use radio-frequency spectroscopy to characterize the laser-dressed bound state near a specific bound-to-bound resonance and show clearly the shift of the magnetic Feshbach resonance using light. The demonstrated technology could be used to modify interatomic interactions with high spatial and temporal resolutions in the crossover regime from a Bose-Einstein condensate to a Bardeen-Cooper-Schrieffer superfluid.

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The ability to control the strength of interatomic interactions has led to revolutionary progress in the field of ultracold atomic gases [1]. Magnetic-field-induced Feshbach resonance is one such powerful tool and has been used widely in atomic gases of alkali atoms to understand strong correlation of quantum many-body systems [1]. An alternative technique for tuning interatomic interactions is optical Feshbach resonance (OFR) [2,3], in which free atom pairs are coupled to an excited molecular state by laser field near a photoassociation resonance. OFR is particularly useful for controlling interatomic interactions in atomic gases of alkaline-earth-metal atoms [4–7], because of the lack of magnetic structure in the ground state of these atoms. It also offers more flexible control on the interaction strength with high spatial and temporal resolutions, since the laser intensity can be varied on short length and time scales. However, OFR often suffers from rapid loss of atoms due to the light-induced inelastic collisions between atoms. Recently, optical lasers in combination with magnetic Feshbach resonances have been developed to modify the interatomic interaction in Bose gas [8,9] atoms and have been shown to reduce the loss rate by an order of magnitude compared with the ordinary OFR in ^{87}Rb [8,10]. In this Rapid Communication, we demonstrate the realization of such a laser-controlled magnetic Feshbach resonance in a strongly interacting Fermi gas of ^{40}K atoms.

Strongly interacting atomic Fermi gas is a clean and easily controllable system with rich and intriguing physical properties [11,12]. It provides a new platform for solving some challenging problems in condensed matter physics and for quantum simulating novel exotic quantum states of matter, such as the high-temperature superconductivity and BEC-BCS crossover [13]. Magnetic Feshbach resonance has already been shown to play a key role in exploring strongly interacting Fermi gases with balanced or imbalanced spin populations, leading to the creation of molecules [14–17], realization of fermionic superfluidity [18–22], and discovery of fermionic

universality [23–27]. The additional independent control of interatomic interactions with laser light, as demonstrated in this work, would give rise to the possibility of revealing new quantum phases of atomic Fermi gases.

To characterize a strongly interacting atomic Fermi gas, radio-frequency (rf) spectroscopy is a valuable experimental tool. It has been used extensively to determine the s -wave scattering length near a Feshbach resonance by directly measuring the rf shift induced by mean-field interactions [28], to demonstrate many-body effects and quantum unitarity [29], and to probe the occupied spectral function of single-particle states and the energy dispersion through BEC-BCS crossover [30].

Here we experimentally investigate magnetic Feshbach resonance in combination with laser light and characterize the laser-modified bound state by using rf spectroscopy. Feshbach molecules are created in a ^{40}K atomic Fermi gas consisting of an equal mixture of atoms in the $|9/2, -9/2\rangle \equiv |F = 9/2, m_F = -9/2\rangle$ and $|9/2, -7/2\rangle$ hyperfine states, by ramping a magnetic bias field from above the broad s -wave Feshbach resonance at $B_0 = 202.20 \pm 0.02$ G [31] to certain values below the resonance. We measure the spectrum of the excited molecular state by applying a laser field near resonant with transitions between the ground Feshbach molecular state and the electronically excited molecular states, and show that there are nine strong bound-to-bound transitions. At a specific bound-to-bound transition, we demonstrate that the magnetic Feshbach resonance is notably shifted by a detuned laser light, as evidenced by the magnetic field and frequency detuning dependence of the laser-dressed bound state in rf spectroscopy. Our measurements are in good agreement with a simple theoretical model.

The experimental procedure starts with a degenerate Fermi gas of about $N \simeq 2 \times 10^6$ ^{40}K atoms in the $|9/2, 9/2\rangle$ internal state, which has been evaporatively cooled to $T/T_F \simeq 0.3$ with bosonic ^{87}Rb atoms in the $|F = 2, m_F = 2\rangle$ state inside a crossed optical trap [32–36]. Here T is the temperature and T_F is the Fermi temperature defined by $T_F = E_F/k_B = (6N)^{1/3} \hbar \bar{\omega} / k_B$ with a geometric mean trapping frequency $\bar{\omega}$. A 780-nm laser pulse of 0.03 ms is used to remove all

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the ^{87}Rb atoms in the mixture without heating ^{40}K atoms. Subsequently, fermionic atoms are transferred into the lowest state $|9/2, -9/2\rangle$ via a rapid adiabatic passage induced by a rf field with duration of 80 ms at $B \simeq 4$ G. To prepare a Fermi gas with equal spin population in the $|9/2, -9/2\rangle$ and $|9/2, -7/2\rangle$ states, a homogeneous magnetic bias field along the z axis (gravity direction) produced by two coils (operating in the Helmholtz configuration) is raised to about $B \simeq 219.4$ G and then a rf ramp around 47.45 MHz is applied for 50 ms. These two hyperfine states form the incoming state $|o_\uparrow\rangle \otimes |o_\downarrow\rangle = |9/2, -9/2\rangle \otimes |9/2, -7/2\rangle$ in the open channel for a pair of atoms, as shown in Fig. 1(a). We use a magnetically controlled Feshbach resonance at

$B_0 = 202.20 \pm 0.02$ G to convert a pair of atoms into an extremely weakly bound molecule. With an adiabatic magnetic field sweep across the resonance, molecules are prepared in the electronic ground dimer state $|g\rangle$ in the closed channel.

To optically control the magnetic Feshbach resonance, a laser beam is derived from a Ti:sapphire laser using an acousto-optical modulator in the single-pass configuration, which allows precise control of the laser intensity and duration time of the pulse. The laser is implemented as a traveling wave with a $1/e^2$ radii of $200 \mu\text{m}$ and its linear polarization is perpendicular to the magnetic bias field of Feshbach resonance. We apply the laser pulse in a rectangular temporal

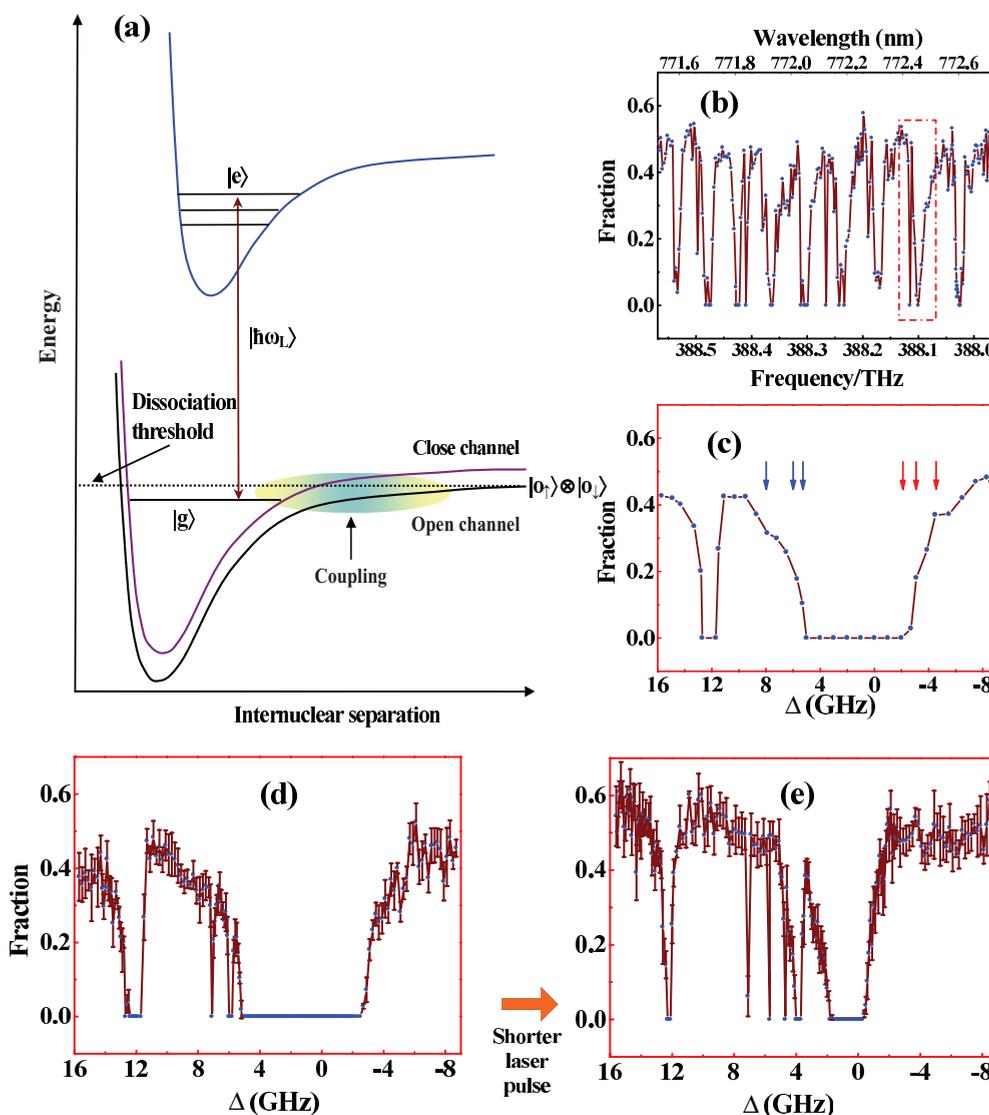


FIG. 1. (Color online) Energy level diagram and excited-state spectroscopy. (a) Schematic diagram of the energy levels. A magnetic Feshbach resonance couples atoms in the incoming open channel $|o_\uparrow\rangle \otimes |o_\downarrow\rangle$ to a bound dimer state $|g\rangle$ in a different potential (the closed channel). A laser with frequency ω_L , near resonant with a bound-to-bound transition, transfers molecules in $|g\rangle$ to one of the electronically excited dimer states $|e\rangle$. (b) Bound-to-bound spectroscopy for $^{40}\text{K}_2$ molecules below the $^2P_{1/2} + ^2S_{1/2}$ threshold. (c) Enlarged view of the eighth strong resonance labeled in (b), plotted as a function of the detuning $\Delta = (2\pi\hbar)(\omega_L - \omega_{eg})$, where $\omega_{eg} \simeq 388\,103.7$ GHz is the resonance frequency of the eighth bound-to-bound transition. Arrows indicate the detunings used in Fig. 3(a). (d) Bound-to-bound spectroscopy of (c) with fine frequency resolution. (e) The spectroscopy with a shorter laser pulse duration (1.5 ms) with the same laser power, which shows multisubstructure at the vibrational level.

shape with intensity I up to 15 ms. Near resonant with one of the $|g\rangle - |e\rangle$ bound-to-bound transitions, the laser induces loss in the population of Feshbach molecules due to the excitation to $|e\rangle$ and subsequent fast spontaneous radiative decay to unobserved states. In order to determine the number of remaining molecules in the trap, after turning off the laser, a Gaussian-shaped rf pulse with duration about 400 μs is applied to dissociate the remaining molecules into free atoms in the state $|9/2, -9/2\rangle \otimes |9/2, -5/2\rangle$. After the rf pulse, we abruptly turn off the optical trap and magnetic field, and let the atoms ballistically expand for 12 ms in a magnetic field gradient applied along the \hat{z} axis and then take absorption image along the \hat{y} direction. The atoms in different hyperfine states N_σ ($\sigma = |-7/2\rangle, |-5/2\rangle \dots$) are spatially separated and analyzed, from which we determine the fraction $N_{-5/2}/(N_{-5/2} + N_{-7/2})$ for different laser frequencies to obtain the spectrum of the excited molecular states.

Figure 1(b) reports the bound-to-bound spectroscopy for excited $^{40}\text{K}_2$ molecules below the $^2P_{1/2} + ^2S_{1/2}$ threshold at the magnetic field $B = 201.60$ G. Here the bound molecules are illuminated with the laser intensity of $I = 50$ mW and the pulse duration time of 15 ms. The frequency of the rf pulse used to dissociate molecules is fixed to a value that is about 20 kHz larger than the Zeeman splitting between the hyperfine states $|9/2, -7/2\rangle$ and $|9/2, -5/2\rangle$, which corresponds to the transition from the bound molecules to the free atom state $|9/2, -9/2\rangle \otimes |9/2, -5/2\rangle$. The spectrum in Fig. 1(b) covers a wavelength range from 771.5 to 772.7 nm with coarse frequency resolution about 3 GHz. We find nine strong bound-to-bound loss resonances with an average frequency spacing of ~ 64 GHz. A specific loss resonance at $\omega_L \simeq 388\,103.7$ GHz is highlighted in Fig. 1(c) with an enlarged view. We have extended the frequency scan from 770.5 to 780.0 nm, but cannot detect any further strong bound-to-bound resonances. According to the photoassociation data for $^{39}\text{K}_2$ [37], there are three long-range attractive potentials (labeled by the quantum numbers 0_u^+ , 0_g^- , and 1_g based on the Hund's case (c) selection rules [37]) adiabatically connected to the $^2P_{1/2} + ^2S_{1/2}$ threshold, as a result of spin-orbit interaction [38]. The observed resonances should be related to the bound states of these three potentials. In principle, each observed bound state would display rich substructures induced by vibration, rotation, hyperfine interaction, and Zeeman interaction of molecules. As shown in Figs. 1(d) and 1(e), we are able to resolve some of these substructures by increasing the laser frequency resolution or reducing the pulse duration. Figure 1(e) reports the spectroscopy near the $\omega_L \simeq 388\,103.7$ GHz resonance with the pulse duration of 1.5 ms and the frequency resolution of 100 MHz. Multisubstructures at the vibrational level can be clearly identified. A careful parametrization of these fine structures will be addressed in the future, as it requires much better precision of the frequency calibration.

Now we use the above bound-to-bound spectroscopic information to shift the magnetic Feshbach resonance. We focus on the strong resonance at $\omega_{eg} \simeq 388\,103.7$ GHz. To keep the rate for incoherent processes as low as possible, we increase the detuning and intensity of the laser light. For a large detuning, the excited molecular state is essentially empty and may therefore be adiabatically eliminated. The resulting dressed ground molecular state acquires an ac Stark shift

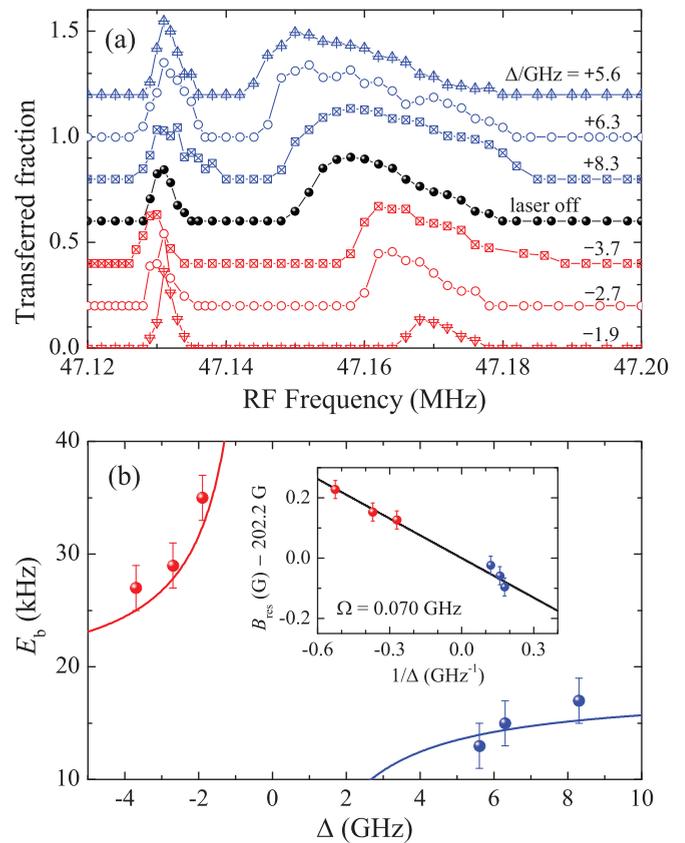


FIG. 2. (Color online) rf spectroscopy and binding energy of the laser-dressed bound state near the Feshbach resonance $B_0 = 202.20 \pm 0.02$ G. With the light off, Feshbach molecules are created below the resonance at $B = 201.60$ G, corresponding to an s -wave scattering length $a_s \simeq 2216a_0$, where a_0 is the Bohr radius. The dimensionless interaction parameter of the Fermi cloud is $1/(k_F a_s) \simeq 0.62$. (a) The rf spectroscopy at different detunings, offset vertically for clarity. (b) The binding energy as a function of the detuning. The solid lines are the theoretical predictions from a simple theory as outlined in the Supplemental Material [39]. The inset shows the resonance position of the shifted Feshbach resonance as a function of the inverse detuning.

(see the Supplemental Material for details [39]),

$$\delta = \frac{\Omega^2}{4(\Delta + i\gamma/2)} \simeq \frac{\Omega^2}{4\Delta} - \left(\frac{\Omega^2}{4\Delta^2} \right) \frac{i\gamma}{2}, \quad (1)$$

where Ω is the Rabi frequency of laser beam, $\Delta = (2\pi\hbar)(\omega_L - \omega_{eg})$ is the detuning, and $\gamma \sim 2\pi \times 6$ MHz stands for the fast spontaneous radiative decay of the excited molecular state [10]. Our measurements are performed under the condition $\Omega \ll \Delta \sim (2\pi\hbar) \times 1$ GHz, so that the effective decay rate $\gamma_{\text{eff}} \equiv (\gamma\Omega^2/8\Delta^2) \sim 2\pi \times 1$ kHz and therefore the atomic loss should be greatly suppressed. Such a suppression was also observed in the recent experiment for bosonic ^{87}Rb atoms [10], where a large detuning was used.

Our experimental sequence is almost the same as before, except that the light intensity is ramped within 15 ms to a final power of ~ 85 mW. A rf field is then applied to dissociate molecules into the free-atom state. After the rf pulse, the optical trap, magnetic field, and laser light are abruptly turned off. The relative spin population of the final

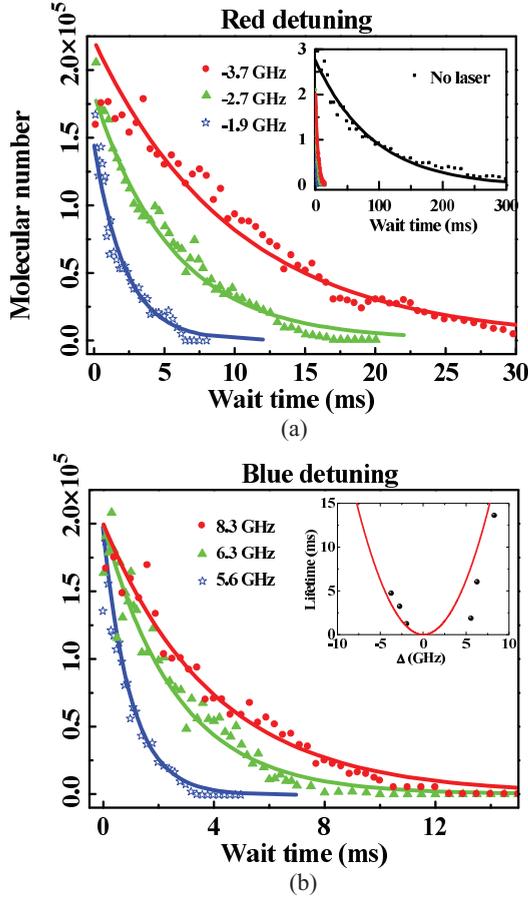


FIG. 3. (Color online) Decay of the number of ground-state bound molecules at different laser detunings: red detuning (a) and blue detuning (b). The inset in (a) shows the decay without laser. The inset in (b) reports the lifetime vs detuning, from which we extract $\gamma \simeq 2\pi \times 6$ MHz by using a parabolic fit.

state $|9/2, -5/2\rangle$, the fraction $N_{-5/2}/(N_{-5/2} + N_{-7/2})$, is then measured as a function of the rf frequency. Figure 2(a) shows the rf spectroscopy at different detunings below the Feshbach resonance (at the magnetic field $B = 201.60$ G). The corresponding binding energy E_b of the ground-state dressed molecules, extracted following the procedure in the Supplemental Material [39], is given in Fig. 2(b). Compared with the data in the absence of laser light, the red and blue detunings of the light tend to increase and decrease the binding energy, respectively. The smaller $|\Delta|$, the larger the change in the binding energy. Figures 3(a) and 3(b) report the population of the ground-state bound molecules as a function of the duration time of the laser pulse for different laser detunings used in Fig. 2(a). With the light on, the loss rate of ground-state molecules increases rapidly with decreasing the absolute value of detuning. The observed lifetime is typically about several milliseconds, in agreement with the estimated effective decay rate $\gamma_{\text{eff}} \sim 2\pi \times 1$ kHz. It is encouraging that the longest lifetime in our measurements (at the maximum blue detuning) can reach ~ 10 ms, which is only one order of magnitude lower than that without laser. Thus, our optically controllable Fermi system turns out to be very stable.

We also measure the rf spectroscopy and determine the binding energy at different magnetic fields while fixing the

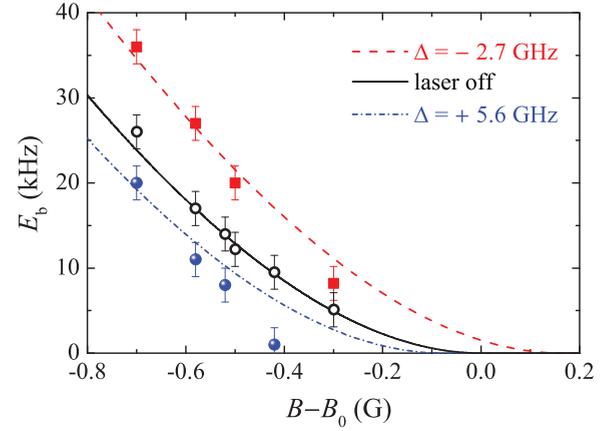


FIG. 4. (Color online) Binding energy of the laser-dressed bound state at different detunings. The data at the red and blue detunings (symbols), $\Delta = (2\pi\hbar) \times (-2.7)$ GHz and $\Delta = (2\pi\hbar) \times (+5.6)$ GHz, have been compared with the two-body predictions (lines). In the numerical calculation, we take $\Omega = (2\pi\hbar) \times 0.070$ GHz.

laser detuning, as reported in Fig. 4. In this scenario, it is clear that, to prepare a Fermi cloud with the same interatomic interaction (i.e., to have the same binding energy in Fig. 4), one can either tune the magnetic field without the light, or tune the laser detuning at a given B field. The latter manner is ideally suited to explore the nonequilibrium dynamics or equilibrium novel states of matter of strongly interacting Fermi gases, as the laser light can be varied over short time and length scales. In Fig. 5, we demonstrate the high temporal resolution of the optically controlled Feshbach resonance. When the laser light is abruptly turned off just before the rf field is applied, the rf spectroscopy is roughly the same as that without the light, in sharp contrast with the result for which the laser is always on. This indicates that the optical control of the Feshbach resonance is almost instantaneous and is effective within a time scale less than $400 \mu\text{s}$ (i.e., the duration of the rf pulse).

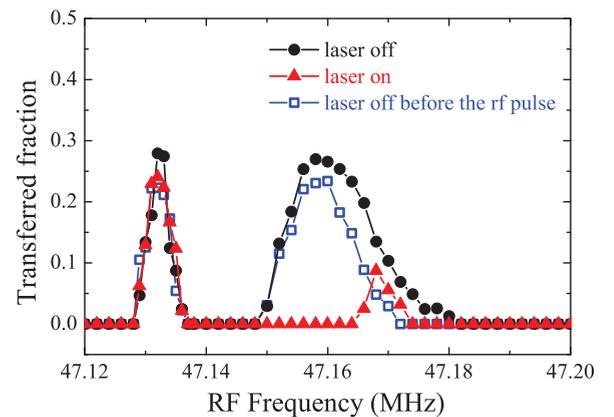


FIG. 5. (Color online) Demonstration of the high temporal resolution of the laser-dressed bound state at $B_0 = 201.60$ G. Solid circles: rf spectroscopy with the laser off. Solid triangles: rf spectroscopy with the laser on. Empty squares: rf spectroscopy with the laser light abruptly turned off just before the switch-on of the rf field. The laser detuning is $\Delta = (2\pi\hbar) \times (-1.9)$ GHz.

In conclusion, we have demonstrated the technology of controlling the magnetic Feshbach resonance using laser light in ultracold atomic Fermi gases. The spectrum of electronically excited molecular states of $^{40}\text{K}_2$ below the $^2P_{1/2} + ^2S_{1/2}$ threshold has been measured by applying a laser field near resonant with the bound-to-bound transitions between the ground and excited molecular states. Nine strong bound-to-bound resonances have been identified below the $^2P_{1/2} + ^2S_{1/2}$ threshold, each of which can be easily used to control the magnetic Feshbach resonance of ^{40}K atoms. We have characterized the laser-induced shift of the Feshbach resonance in the large detuning regime and the stability of the system, using the standard tool of rf spectroscopy, and have understood

the data within a simple two-body theory. The optical tunability of interatomic interactions, as demonstrated in this work, paves a new way to explore the fascinating quantum many-body system of strongly interacting Fermi gases.

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