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Comparison of the Noise Properties of Squeezed Probe Light in Optically Thick and Thin Quantum Coherence Media for Weak and Strong Coupling Lights *

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The output amplitude noises of one squeezed probe light which is at resonance throughout different optical depths media in strong- and weak-coupling-field regimes are investigated theoretically. By comparing the output quantum noises for different Rabi frequencies of coupling field and also for different optical depths, it is found that the optimal squeezing preservation of the probe light occurs in an optically thin medium with strong-coupling-field, where we can obtain the output squeezing close to the input one at nonzero detection frequency.

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Quantum coherence of electromagnetically induced transparency (EIT) provides an elegant approach for slowing and storage of light without absorption and even localizing atoms.[1,2] In an EIT medium, the coupling field can reversibly map and retrieve the information between the weak probe field and the atoms which have long lived ground states.[3] Experiments involving squeezing storage through the EIT medium in hot atoms or magneto-optical trap (MOT) have been demonstrated.[4−7] Measuring the quantum noise of the retrieved states can verify whether the atomic memory is faithful. In Ref. [8], the authors concluded that both entanglement and squeezing of the probe field could be almost perfectly preserved in an EIT medium under the condition of sufficiently small ground state decoherence rate. The theoretical calculation also demonstrated the validity of the preservation and transfer of quantum state in an EIT medium by choosing suitable parameters.[9] Moreover, the survival and transfer of the squeezed vacuum in double EIT have also been investigated.[10] However, the Rabi frequencies of the strong coupling field in all those calculations[8−10] were considered to be larger than the decay rate of the optical transition of probe light, which was discussed from the view of quantum interference[11] in which the medium exhibits Autler–Townes (AT) splitting in the absorption line under the condition of strong coupling field. Also, EIT is a similar but distinct phenomenon which should be the consequence of destructive interference between two competing excitation pathways under the condition of weak coupling field.[12] If the Rabi frequency of the coupling field is greater than the polarization decay rate of the probe transition, which is named as the strong-coupling-field regime, the absorption spectrum of the probe light exhibits AT splitting with a transparency gap between two absorption peaks.[13] In the weak-coupling-regime where the Rabi frequency of the coupling field is less than the polarization decay rate, the destructive interference leads to the reduction of the probe absorption, which is shown as EIT. It was also pointed out that efficient coupling between atoms and light needs the atoms to have large optical depth, especially in quantum memory.[14,15] However in this Letter, the theoretical study of the quantum noise spectra of the outgoing probe light in optically thick and thin media shows that the optimum quantum preservation of squeezed light depends on not only the optical depth but also the coupling regime of EIT or AT splitting effect.

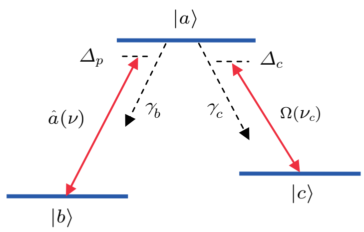

Fig. 1. Schematic of the Λ-type system.

Consider a closed three-level Λ-type system as shown in Fig. 1. A classical coupling field of frequency ν_c with Rabi frequency Ω and a weak quantum probe field of frequency ν interact with the transitions $|c\rangle \leftrightarrow |a\rangle$ and $|b\rangle \leftrightarrow |a\rangle$, respectively. The decays of the corresponding atomic dipole operators are γ_{ba} and γ_{ca}, respectively; γ_b and γ_c are the spontaneous decays for each transition. The corresponding one-

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photon detunings and two-photon detuning are given by $\Delta_p = \omega_{ab} - \nu$, $\Delta_c = \omega_{ac} - \nu_c$, and $\delta = \Delta_p - \Delta_c$.

The zeroth-order steady state solutions of the slowly varying atomic operators described in Ref. [8] are obtained as

$$\begin{aligned}\hat{\sigma}_{aa}^{(0)} &= \frac{-2\gamma_{ca}\gamma_{bc}|\Omega|^2}{-2\gamma_{bc}\gamma_a|\gamma_{ca}+i\Delta_c|^2 - 2\gamma_{ca}|\Omega|^2(\gamma_b+3\gamma_{bc})},\\ \hat{\sigma}_{bb}^{(0)} &= \frac{-\gamma_{bc}\gamma_a|\gamma_{ca}+i\Delta_c|^2 - 2\gamma_{ca}|\Omega|^2(\gamma_b+\gamma_{bc})}{-2\gamma_{bc}\gamma_a|\gamma_{ca}+i\Delta_c|^2 - 2\gamma_{ca}|\Omega|^2(\gamma_b+3\gamma_{bc})},\\ \hat{\sigma}_{ca}^{(0)} &= \frac{i\Omega - 2i\Omega\hat{\sigma}_{aa}^{(0)} - i\Omega\hat{\sigma}_{bb}^{(0)}}{\gamma_{ca}+i\Delta_c},\end{aligned} \tag{1}$$

with the dephasing rate for the two ground states γ_{bc}, and the total spontaneous decay of the excited state satisfying $\gamma_a = \gamma_b + \gamma_c$. It can be seen from Eq. (1) that for the case of weak coupling limit ($\Omega \le \gamma_{ba}$), the assumption that all the atoms are in the ground state $|b\rangle$ is not valid. We will consider a general case without taking the assumption in the following.

The evolution equation for the slowly varying annihilation operator of the quantum probe $\hat{a}$ and the first-order equations which determine the coherence $\hat{\sigma}_{ba}^{(1)}$ as a function of $\hat{a}$ are given by

$$\begin{aligned}&\Big(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\Big)\hat{a}(z,t) = ig^* N\hat{\sigma}_{ba}(z,t),\\ &\frac{\partial}{\partial t}\hat{\sigma}_{ba}^{(1)} = -(i\Delta_p+\gamma_{ba})\hat{\sigma}_{ba}^{(1)} + ig\hat{a}(\hat{\sigma}_{bb}^{(0)} - \hat{\sigma}_{aa}^{(0)})\\ &\qquad + i\Omega\hat{\sigma}_{bc}^{(1)} + \hat{F}_{ba},\\ &\frac{\partial}{\partial t}\hat{\sigma}_{bc}^{(1)} = -(i\delta+\gamma_{bc})\hat{\sigma}_{bc}^{(1)} - ig\hat{a}\hat{\sigma}_{ac}^{(0)} + i\Omega^*\hat{\sigma}_{ba}^{(1)} + \hat{F}_{bc},\end{aligned} \tag{2}$$

with $g = \wp_{ab}\sqrt{\nu/2\varepsilon_0 V\hbar}$ representing the atom-field coupling constant and $\wp_{ab}$ being the atomic dipole moment for the $|b\rangle \leftrightarrow |a\rangle$ transition. V is the interaction volume and N is the number of atoms. Taking Fourier transform of Eqs. (2) and solving them for $\hat{a}(z,\omega)$, then we obtain the output field at the exit of the cell which has the length L:

$$\begin{aligned}\hat{a}(L,\omega) = &\ e^{-\Lambda(\omega)L}\hat{a}(0,\omega) + \frac{g^* N}{c}\int_0^L e^{-\Lambda(\omega)(L-s)}\\ &\times \frac{i\gamma_2\hat{F}_{ba}(s,\omega) - \Omega\hat{F}_{bc}(s,\omega)}{\gamma_1\gamma_2+|\Omega|^2}ds,\end{aligned} \tag{3}$$

$$\Lambda(\omega) = \frac{|g|^2N}{c}\times\Big[\frac{\gamma_2\big(\hat{\sigma}_{bb}^{(0)} - \hat{\sigma}_{aa}^{(0)}\big)}{\gamma_1\gamma_2+|\Omega|^2} - i\frac{\Omega\hat{\sigma}_{ac}^{(0)}}{\gamma_1\gamma_2+|\Omega|^2}\Big] - \frac{i\omega}{c}, \tag{4}$$

where ω is the detection frequency and $\gamma_1 = \gamma_{ba} + i(\Delta_p - \omega)$, $\gamma_2 = \gamma_{bc} + i(\delta - \omega)$.

In terms of the definition of quadrature flux spectrum and the correlation functions of Langevin noise operators described in Refs. [8,16], one obtains the normalized quadrature amplitude spectrum of the output probe related to the input via the relation

$$S_X(L,\omega) = S_1(\omega) + S_2(\omega) + S_3(\omega), \tag{5}$$

$$\begin{aligned}S_1(\omega) = &\frac{S_X^{in}(\omega)}{4}(\exp\{-[\Lambda(\omega)+\Lambda(-\omega)]L\}\\ &+\exp\{-[\Lambda(\omega)+\Lambda^*(\omega)]L\}\\ &+\exp\{-[\Lambda^*(-\omega)+\Lambda(-\omega)]L\}\\ &+\exp\{-[\Lambda^*(-\omega)+\Lambda^*(\omega)]L\}),\end{aligned} \tag{6}$$

$$\begin{aligned}S_2(\omega) = &-\frac{S_Y^{in}(\omega)}{4}(\exp\{-[\Lambda(\omega)+\Lambda(-\omega)]L\}\\ &-\exp\{-[\Lambda(\omega)+\Lambda^*(\omega)]L\}\\ &-\exp\{-[\Lambda^*(-\omega)+\Lambda(-\omega)]L\}\\ &+\exp\{-[\Lambda^*(-\omega)+\Lambda^*(\omega)]L\}),\end{aligned} \tag{7}$$

$$\begin{aligned}S_3(\omega) = &\frac{|g|^2N}{c}\times\frac{1-\exp[-(\Lambda(\omega)+\Lambda^*(\omega))L]}{\Lambda(\omega)+\Lambda^*(\omega)}\\ &\times\Big\{\frac{|\gamma_2|^2[(\gamma_b+\gamma_a)\sigma_{aa}^{(0)} + \gamma_{bc}(1-\sigma_{aa}^{(0)} - 2\sigma_{bb}^{(0)})]}{|\gamma_1\gamma_2+|\Omega|^2|^2}\\ &+\frac{|\Omega|^2[\gamma_b\hat{\sigma}_{aa}^{(0)} + \gamma_{bc}(1-\sigma_{aa}^{(0)})]}{|\gamma_1\gamma_2+|\Omega|^2|^2}\Big\} + \frac{|g|^2N}{c}\\ &\times\frac{1-\exp[-(\Lambda(-\omega)+\Lambda^*(-\omega))L]}{\Lambda(-\omega)+\Lambda^*(-\omega)}\\ &\times\Big\{\Big[i\tilde{\gamma}_2^*\Omega\gamma_{bc}\hat{\sigma}_{ac}^{(0)} - i\tilde{\gamma}_2\Omega^*\gamma_{bc}\hat{\sigma}_{ca}^{(0)}\\ &+|\Omega|^2[\gamma_c\hat{\sigma}_{aa}^{(0)} + \gamma_{bc}(1-\sigma_{aa}^{(0)})]\Big]\\ &\cdot\Big[|\tilde{\gamma}_1\tilde{\gamma}_2+|\Omega|^2|^2\Big]^{-1}\Big\},\\ &\tilde{\gamma}_1 = \gamma_{ba} + i(\Delta_p+\omega),\\ &\tilde{\gamma}_2 = \gamma_{bc} + i(\delta+\omega).\end{aligned} \tag{8}$$

In our calculation and discussion, all the parameters are normalized to the decay rate γ_{ba} of the probe transition. In addition, we have set $\gamma_{ba} = \gamma_{ca}$ and $\Delta_c = 0$ for simplicity. As described in Refs. [14,17], the parameter $|g|^2NL/c$ represents the coupling strength between the light and atoms, and it is related to the optical depth (OD) of the system, which satisfies $|g|^2NL/c = \gamma_a \times OD/4$. For optically thin and thick media, $|g|^2NL/c$ takes different values.

Consider an input 3 dB squeezed vacuum state with $S_X^{in}(\omega) = 0.5$, $S_Y^{in}(\omega) = 2$, which is resonant to its corresponding transition satisfying $\Delta_p = 0$. Note that $S_X(L,\omega) = 1$ represents the shot noise level (SNL). Figure 2 shows the amplitude noise of the outgoing probe versus the Rabi frequency of the coupling field for different detection frequencies in optically thick medium with $|g|^2NL/c = 25$ (Fig. 2(a)) and optically thin medium with $|g|^2NL/c = 2$ (Fig. 2(b)). It is obviously seen that the amplitude noise decreases with the increase of the Rabi frequency of coupling field for any detection frequency after reaching maximum. Once the Rabi frequency of the coupling field becomes large enough, the amplitude noise $S_X(L,\omega)$ of the outgoing probe beam can always get close to the initial squeezing value no matter whether the medium is optically thick or thin, which indicates that the ideal situation of squeezing preservation occurs in the regime of strong-coupling-field. We can also see that the larger the detection frequency is, the higher the Rabi frequency of the coupling field is needed to obtain the ideal squeezing preservation. For the case of an optically thick medium with $|g|^2NL/c = 25$, the output noise decreases more slowly than that for the case of an optically thin medium with $|g|^2NL/c = 2$ when the Rabi frequency of coupling light increases. Thus, it can be concluded that the ideal squeezing preservation may need to be performed in an optically thin system in strong-coupling-field regime with a large enough Rabi frequency of the coupling field.

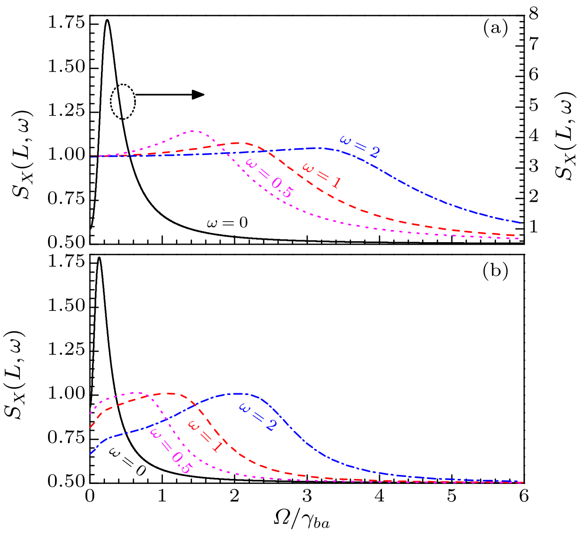

Fig. 2. The amplitude noise vs the Rabi frequency of the coupling field Ω for different detection frequencies. Solid lines: $\omega = 0$, dotted lines: $\omega = 0.5$, dashed lines: $\omega = 1$, dash-dotted lines: $\omega = 2$ with (a) $|g|^2NL/c = 25$, (b) $|g|^2NL/c = 2$. Other parameters are $\gamma_b = 1$, $\gamma_c = 1$, $\gamma_{bc} = 0.01$, $\Delta_p = \Delta_c = 0$, $S_X^{in}(\omega) = 0.5$, $S_Y^{in}(\omega) = 2$.

Figure 3(a) illustrates the noise spectra versus the detection frequency for different optical depths in strong-coupling-field regime with $\Omega = 3.6$. It can be seen that there appears a dip like an EIT window around $\omega = 0$ for each outgoing squeezing spectrum, the output noise turns out to be larger when the optical depth increases. As was discussed in Ref. [10], the transparency interval 2Ω between the two absorption peaks of the AT splitting in the strong-coupling-field regime can be viewed as the transparency width in which the squeezing of light can be preserved in the frequency domain. Outside this transparency window, the outgoing field fluctuations are absorbed by atoms,[9] and the noise from the atoms will contaminate the output light. Here we can see that, however, for an optically thick medium (e.g. dotted line), the squeezing of the output probe can not be well preserved even in the transparency window.

Although the optimal squeezing preservation occurs at the zero detection frequency, the relaxation oscillation of the laser at low frequency prevents us from obtaining the best squeezing at zero detection frequency in realistic measurement. However, we can obtain much smaller noise close to the input one in an optically thin medium, i.e. small enough OD at nonzero detection frequency as illustrated in Fig. 3. This result can also be demonstrated in terms of Eq. (19) in Ref. [8], Eq. (6) in Ref. [9] and Eq. (18) in Ref. [10]. In addition, we can find that the width which describes the squeezing preservation range decreases when the optical depth becomes larger. The result is consistent with the prediction of Ref. [9]. If we keep the Rabi frequency of the coupling field unchanged, the width of the squeezing window will decrease gradually with the increase of OD. From the above analyses, it can be concluded that it is more effective to manipulate the quantum state preservation at nonzero detection frequency in an optically thin medium than in optically thick one.

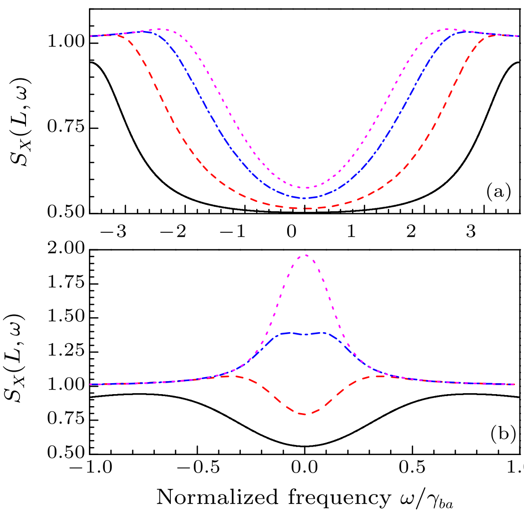

Fig. 3. The amplitude noise spectra vs the detection frequency ω for different optical depths in the strong-coupling-field regime with $\Omega = 3.6$ (a) and in the weak-coupling-field regime with $\Omega = 0.8$ (b). The solid, dashed, dash-dotted and dotted lines are the noises of $|g|^2NL/c = 1$, $|g|^2NL/c = 5$, $|g|^2NL/c = 15$, $|g|^2NL/c = 25$, respectively. The other parameters are the same as those in Fig. 2.

Figure 3(b) shows the similar results that the dependence of noise spectra on the detection frequency for different optical depths in weak-coupling-field regime. When $|g|^2NL/c$ increase from 1 to 5 (see the solid and dashed curves), the squeezing window becomes narrow. However, if we further increase $|g|^2NL/c$ (see the dash-dotted and dotted curves), the output noise will be higher than SNL.

From the above analyses, it is evident that the squeezing preservation in a quantum coherence medium can be well performed in a strong-coupling-field regime, and it is suggested that the experiments on storage and retrieval of quantum state should be investigated in an optically thin medium with large enough coupling field. However, as analyzed in Refs. [14,15], the optimal memory performance needs optical depth to be significantly greater than one. Thus, we can conclude that proper optical depth is necessary to satisfy the conditions of optimal storage efficiency and optimal quantum state preservation at the same time. In addition, Refs. [15,18] gave a possible reason for the background noise in the retrieved signal that the thermal photons emitted by the excited state contaminates the signal mode due to the repopulation of the ground states. While in our calculation, the excess noise originates not only from the Langevin atomic noise $S_3(\omega)$ resulted from the random decay process, but also from the phase-to-amplitude converted noise $S_2(\omega)$ due to the interaction between atoms and lights.

In conclusion, the theoretical calculation and discussion show that the squeezing of the probe light can be well kept in the strong-coupling-field regime for either optically thick or optically thin media. While in the weak-coupling-field regime, only an optically thin medium can be used to preserve the input squeezing. It is more efficient to store the quantum state in an optically thin medium for a strong coupling field instead of an optically thick medium.

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