Nearly deterministic loading of a single cesium atom in a magneto-optical trap and in a microscopic optical tweezer by feedback control

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ABSTRACT

Based on feedback control techniques and our realization of nearly complete transferring cold cesium (Cs) atoms from a magneto-optical trap (MOT) to a far-off-resonance microscopic optical tweezer, we investigated the possibility for nearly deterministic loading of a single Cs atom in a MOT and in a microscopic optical tweezer. We combined feedback controls on the gradient of the MOT quadrupole magnetic field (QMF) and on blue-detuned light-assisted collisions (LAC) of confined cold atoms in the tweezer. Using active feedback on QMF of the MOT, we have achieved ~ 98% of probability of single atom loading in a MOT. In a microscopic optical tweezer, by combining the feedback controls on the QMF and the LAC, we finally achieved ~ 95.2% of probability of single atom loading in the tweezer. This two-path feedback control scheme may be extended to load a small-size 2D tweezer array with exact single atom trapped in each site simultaneously. This is very important and promising to implement an addressable multiple-qubit system for demonstrating quantum register and quantum processor.

Keywords: single atoms; magneto-optical trap; feedback control; quadrupole magnetic field; microscopic optical tweezer; light-assisted collisions; blue-detuned laser; single-atom loading

1. INTRODUCTION

Controlled neutral atoms are one of the ideal candidates for deterministic demonstration of the processing of quantum information, quantum computation, quantum simulation, and quantum metrology ^{1~4}. Deterministic preparation and detection of single neutral atoms is necessary for all these applications. High efficient loading of a single atom into a magneto-optical trap (MOT) and a microscopic optical tweezer is the first step for many experiments in the fields of quantum optics, cold atom collisions, precision measurements, and quantum information processing devices.

In recent years, several groups have developed a few approaches for trapping and manipulating a single or a few neutral atoms based on a MOT ^{5,6} or a microscopic optical tweezer ^{7~10}. The first observation of a single neutral atom in a MOT was reported by Hu and Kimble in 1994 ⁵. By controlling the feedback on parameters of MOT to reduce the randomness of loading, Hilland *et al* ¹¹, Yoon *et al* ¹², and Förster *et al* ¹³ have realized a nearly deterministic source of neutral atoms with high occupation probabilities of desired-number atoms in a MOT. An available approach for single atom loading of a microscopic optical tweezer is the exploitation of a "collisional blockade mechanism". Schlosser *et al* ^{14, 15} have shown that, when the optical dipole trap volume is small enough, a "collisional blockade mechanism" that locks the atom number either zero or one in a microscopic optical dipole trap. In experiments they employed a red-detuned laser to induce the cold collision between trapped atoms, the single atom loading probability has been limited to ~ 50% ¹⁴. Grünzweig *et al* ¹⁶ have shown that, a blue-detuned laser can induce light-assisted collisions (LAC) between pairs of atoms, such that only one of the atoms is ejected from the microscopic optical tweezer. They showed that blue-detuned LAC could increase single atom loading probability up to 91% in a microscopic optical tweezer ¹⁷.

In order to demonstrated nearly deterministic loading of a single cesium (Cs) atom in a MOT and in a microscopic optical tweezer, we first implemented a large-magnetic gradient MOT, then produced a microscopic optical tweezer at 1064 nm by tightly focusing a 1064nm laser beam with a waist ~ $2.3 \mu m^{18-20}$. We then loaded this microscopic optical tweezer from the Cs MOT. The time dynamics of the atom number N in a microscopic optical tweezer can be described by a differential equation, $\dot{N} = R_L - L_{1atom} - 2L_{2atoms}$ (where R_L is the loading rate, L_{1atom} is the one-atom loss rate per

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atom due to background gas collisions, and L_{2atoms} is the two-body decay rate). For the purpose of active controlling the number of atoms, the loading rate R_L is controlled by changing the gradient of quadrupole magnetic field (QMF) of the MOT. In this way we can feedback control the QMF of the MOT to improve the probability of single atom loading in a MOT. Combining feedback controls on the QMF of the MOT and the blue-detuned LAC of cold atoms confined in the microscopic optical tweezer, we can achieve a high probability for single atom loading in the tweezer. Finally we have achieved ~ 98% of probability of single atom loading in a MOT and ~ 95.2% of probability of single atom loading in the tweezer.

2. EXPERIMENTAL SETUP

A sketch of the experimental setup is shown in Fig. 1(a) and have been detailed introduction in ^{18–20}. The microscopic optical tweezer beam was provided by a homemade laser-diode-pumped Nd:YVO₄ single-frequency 1064-nm laser with micrometer size ($w_0 = 2.3 \mu m$). With ~47mW beam power, the resulting trap depth is $U_0 = k \times 1.5 mK$ ($k = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant). The QMF was produced by a pair of water-cooled anti-Helmholtz coils. The supplied 20A current that yielded a field gradient of ~ 30 mT/cm (~ 300 Gauss/cm) in the axial direction of the coils could be switched off completely using our feedback control system. This high QMF efficiently reduces the loading rate of the MOT to about 1 atom/s, and Fig. 1(b) shows a typical time traces of fluorescence signal from a few-atom in MOT, with the help of high field gradient (240 Gauss/cm) we can load single or very few atoms in a MOT.



Fig. 1 (a) Schematic diagram of the single-atom MOT and the tightly-focused single-beam far-off-resonance microscopic optical tweezer. A glass cell is evacuated by vacuum pump system (not shown) to keep the pressure at about 1×10^{-10} Torr while Cs atoms are released from a Cs ampoule. A single-photon counting module (SPCM) is used to detect the collected fluorescence photons scattered by the trapped atoms. The microscopic optical tweezer is provided by strongly focusing ($1/e^2$ radius of beam waist $w_0 = 2.3 \mu m$) a laser-diode pumped Nd:YVO₄ single-frequency 1064-nm laser beam. Feedback is used to control the gradient of QMF of the MOT. (b) Typical fluorescence photon counting signal of the atoms trapped in a MOT. The discrete steps indicate the capture or loss of atoms.

We detect trapped single atoms by collecting its fluorescence signal and we use the kind of aspheric lens (N.A. = 0.29) to collect the fluorescence photons at 852 nm. Since the fluorescence emitted from single atoms is weak, we detect it by single-photon-counting module (SPCM) with a quantum efficiency of η_{APD} = 48% at λ = 852 nm. The exact number of atoms in the trap was measured from the fluorescence light emitted by the MOT. For the SPCM, the total detection efficiency is 0.6%. A typical counting rate for a single atom in a MOT is about 500~1500 counts in a time bin of 50 ms. The SPCM signal was separated into two parts. A fraction of the signal is sent to a multi-channel-scaler (MCS) card to record the number of atoms in the trap. The remaining signal is directed onto a pulse counter which can record the number of electrical pulses in certain time.

3. THE PRINCIPLES OF EXPERIMENTS

In this section, we first introduce the conventional approach for rate measurements and the characteristics of loading and loss events in MOT, and then we discuss the LAC in microscopic optical tweezer through red-detuned laser or a blue-detuned laser.

For a MOT, the time dynamics of the atom number N can be described by a model differential equation, is governed

by ^{6,20}: $dN / dt = R_L - \gamma N - \beta N(N-1) / V$, where N is the number of atoms, R_L is the loading rate, γ is the one-atom loss rate per atom due to background gas collisions, and β is the two-body decay rate. Trapping a single atom requires the minimization of the average number of atoms; several strategies exist to reduce R_L . Firstly, this can be achieved by reducing the cesium partial pressure in the vacuum chamber. Secondly, the capture cross section of the MOT, σ , and thus R_L , drastically decrease with increasing QMF ⁶: $R_L \propto (dB/dZ)^{-14/3}$. So that people can realize single atom loading in the large-magnetic gradient MOT. Also people can adopt feedback control on the QMF of the MOT based on near real time atom counting to increase the probability of desired atom number in the MOT ¹².



Fig. 2 (a) In the case of exiting the red-detuned laser in the microscopic optical tweezer, the atom pair is excited to an attractive potential, thus both atoms are accelerating to approach for collision to form a bind molecule or to gain much more energy for each atom, and finally both of atoms escaped from the tweezer. This red-detuned LAC leads to a maximal 50% of probability of single atom loading in the tweezer, depending on whether the initial atom number is even or odd. In the case of the blue-detuned laser in the microscopic optical tweezer, the atom pair is more like excited to an repulsive potential with the help of the appropriate parameters, the trap depth U, the saturation parameter s = I/Is, and the frequency detuning δ , thus both atoms are accelerating to approach first than away from. The gained energy for each atom can be properly controlled, and in each collision event only one of two atoms escaped from the tweezer. This blue-detuned LAC leads to only one atom remain in the tweezer finally, so that it can help to improve the probability of single atom loading. (b) Relevant hyperfine levels of Cs atoms involved in the experiment. The excitation laser's detuning δ is measured with respect to the free-space transition (take no ac Sark shifts of atom levels into count).

After we demonstrate an efficient single atom loading in the MOT, the next steps is transferring the atoms from the MOT into the microscopic optical tweezer, and then we adopt feedback control to improve the probability of single atom loading in the tweezer. We discuss the loading and the LAC induced by a red-detuned laser and a blue-detuned laser between cold atoms confined in a microscopic optical tweezer. In the case of the red-detuned laser, the atom pair is excited to an attractive potential leading to the atoms forming a molecule or gaining a large amount of kinetic energy. In each case, both atoms are lost, leading to a maximal 50% of single atom probability in the tweezer, depending on whether the initial atom number is even or odd. In the case of the blue-detuned laser, during the blue detuned laser excitation, two atoms form a loosely bound pair with one atom in the S state and the other in the P state and interact through the long-range dipole-dipole attractive potential $V(r) = -C_3/r^3$ (here, r is the interatomic distance, $C_3 = 3\hbar\Gamma/4k^3$,

 $\Gamma/2\pi = 5.2$ MHz is the natural linewidth and $\lambda = 852$ nm is the wavelength of the S-P transition)^{21, 22}, as represented in Fig. 2. If the kinetic energy acquired by an atom pair before it radiates back to the ground state exceeds the optical dipole trap depth U, it escapes from the trap, thus leading to the loss of two atoms. This interaction-induced loss mechanism, whether one or the other mechanism is dominant depends on the parameters of the experiment, namely, the trap depth U, the saturation parameter s = I/I_s (I is the laser intensity and I_s = 1.12 mW/cm²), and the frequency detuning δ of the excitation light with respect to the single-atom transition in free space [see Fig. 2(b)]. After such a collision, the atoms will have only enough energy for one of them to escape from the trap. The final result is only single atoms in a microscopic optical tweezer, which doesn't depend on whether the initial atom number is even or odd.

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

For the purpose of efficient and deterministic loading of single atoms in a microscopic optical tweezer, we first implemented a large-magnetic gradient Cs MOT and then transfer the atoms from the MOT into the microscopic optical tweezer. The loading rate in a MOT is controlled by changing the QMF gradient, trapping laser intensity and the size and the detuning of the trapping beams.

4.1 High-probability loading of single atom in a MOT by feedback control of QMF

In this experiment, under the condition of low magnetic field (180 Gauss/cm) with the laser intensity fixed at $I_{eff} = 20I_s$ and the detuning of $\Delta = -1.5\Gamma$, we can catch less than 10 atoms in the trap and with high magnetic field (240 Gauss/cm), we can obtain single atom, as shown in Fig. 3. Fig. 3(a) shows the historical statistics of typical fluorescence photon counting of atomic clouds by a SPCM in a time bin of 50 ms. The loading and loss events of atoms in a MOT is stochastic.

In order to efficient and deterministic loading of single atoms in a MOT, we implemented a real-time field-gradient feedback technique. The number of atoms measured via the fluorescence signal is used as a trigger signal for changing the QMF. At first, atoms are loaded into the trap at a low QMF of 120 Gauss/cm with a short waiting time and thus with a relatively high loading rate. After single atoms are loaded in a MOT, the experiment computer sends out a TTL pulse to tell the QMF which is rapidly increased to a high value of 240 Gauss/cm in order to reduce the loading rate almost to zero. At this high QMF, few atoms can be loaded into the trap in long time scale and only loss events occur. As soon as the atom escapes from the trap and thus the fluorescence falls below a certain level corresponding to the given number of atoms, the field gradient is rapidly decreased to a low value at which new atoms can be loaded into the trap quickly.

With this active feedback on QMF, we have demonstrated the real-time feedback controlling of single atoms and realized a deterministic atom source for single atoms as shown in Fig. 3(b). The observed single-atom probability is up to $\sim 98\%$ with the average trap lifetime of ~ 82 seconds.



Fig. 3 Occurrence of the fluorescence photon counts of trapped atoms in the MOT. The solid line shows a multi-peak Possionian fitting to the data. (a) Without the feedback control of QMF of the MOT, the results shows random loading of atoms in the MOT, and average atom number is less than one. Typical probability of single atom loading in the MOT is $\sim 35\%$. (b) With the feedback control of QMF of the MOT, the probability of single atom loading in the MOT is increased from $\sim 35\%$ to $\sim 98\%$. The randomness of atom loading in the MOT is supressed significantly.

4.2 Nearly deterministic loading of a single atom in a microscopic optical tweezer

After high loading efficiency of single atoms in a MOT, we then loaded this trap from a MOT surrounding the region of the microscopic optical tweezer. Atoms enter the microscopic optical tweezer randomly, are trapped thanks to the cooling effect of the MOT beams, and are expelled from the trap due mainly to inelastic two-body collisions assisted by the near-resonant light of the cooling beams and collisions with the residual background gas in the chamber (one-body losses). Fig4 show here that a single atom in a MOT can be transferred into microscopic optical tweezer with $\sim 99\%$ efficiency. After optimization, the trapped single atom can be transferred back and forth between the two traps many times. During the transmission, in order to decrease the probability of zero atom, we reduce the QMF, thus increase the probability of more than one atom, then we apply a tailored laser light pulse to induce light assisted-collisions which allows for controlling the energy released in inelastic collision, resulting in one atom in a microscopic optical tweezer.

Fig. 5(a) shows a probability of single atoms in a microscopic optical tweezer is 50% with the help of red-detuned laser. The atoms in the micron-scale microscopic optical tweezer formed due to the attractive potential, so that if the microscopic optical tweezer was originally an odd number of atoms, is remaining in the final microscopic optical

tweezer an atom, if an even number of atoms, and ultimately 0 atom remaining. A single atom probability in a microscopic optical tweezer is ~ 50%, depending on whether the initial atom number is even or odd. Fig. 6(b) shows a probability of single atom in a microscopic optical tweezer in case of the blue-detuned laser. Using the blue-detuned LAC, we achieve this by employing a blue-detuned laser to control the energy released, such that only one of the atoms is ejected from the microscopic optical tweezer in each inelastic collision. We induce the LAC of atoms in the $6S_{1/2}$ (Fg = 4) ground state using light that is blue detuned from the $6S_{1/2}$ (Fg = 4) – $6P_{3/2}$ (Fe = 5) hyperfine transition, With our choice of light intensity (I=20Is) and detuning ($\delta = +30$ MHz), such that we can limit the amount of energy released. This scheme requires careful balance of the amount of energy released in collisions and the trap depth of microscopic optical tweezer. The experimental results show that a scheme using LAC could obtain a 78% loading efficiency of single Cs atoms in the microscopic optical tweezer.



Fig. 4 Typical fluorescence photon counts of trapped atoms during the period of transferring atoms back and forth between the MOT and in the microscopic optical tweezer. The overlap time is 25ms when switching the MOT to the microscopic optical tweezer and back. The C_0 , C_1 and C_2 are typical fluorescence photon counting levels in the time bin of 50ms, which are corresponding to the cases of no atom, one atom and two atoms traped in the MOT and in the microscopic optical tweezer. The sudden drops of fluorescence photon counts signals to the level of ~ 350 counts per 50ms are due to no MOT light in the 1064nm tweezer. As soon as switching the microscopic optical tweezer to the MOT, the photon counting level cleary indicated the atom number. The transfer efficiency of a single atom between the MOT and the microscopic optical tweezer is nearly 100%.



Fig. 5 (a) Without the blue-detuned laser, typical occurrence of the fluorescence photon counts of trapped atoms shows $\sim 50\%$ of a single atom probability in the microscopic optical tweezer. (b) With the blue-detuned LAC (detuning is + 30 MHz relative to Fg=4 – Fe=5 cycling transition), typical occurrence of the fluorescence photon counts of trapped atoms shows $\sim 78\%$ of the outcomes for a single atom loading in the microscopic optical tweezer by appropriately choosing the blue-detuned laser's intensity. The solid lines in (a) and (b) show multi-peak Possionian fitting to the data. Due to the very small dipole-trap volume and the 'collisional blockade' mechanism the 2-atom peak is never seeing in the microscopic optical tweezer.

With the help of feedback on QMF, we can observe high probability of single atoms in a microscopic optical tweezer. During transmission one atom into the microscopic optical tweezer, in order to decrease the zero atom probability, we make the MOT working under large loading rate. But the large loading rate will increase the probability of more than single atoms, so that when transferring two atoms into the microscopic optical tweezer, the atoms in a microscopic optical tweezer undergo the LAC due to the presence of the blue-detuned light and in most cases one of two

atoms is expelled from the tweezer. Combine with feedback on QMF and controlled LAC, finally we achieved $\sim 95.2\%$ of single atom probability in the microscopic optical tweezer, as shown in Fig. 6.



Fig. 6 Combining the feedback controls of QMF of the MOT and of the controlled blue-detuned LAC, typical occurrence of the fluorescence photon counts of trapped atoms shows finally $\sim 95.2\%$ of a single atom probability in the microscopic optical tweezer. It is clearly improved compared with the case of only using the controlled blue-detuned LAC. Due to the very small dipole-trap volume and the 'collisional blockade' mechanism the 2-atom peak is never seeing in the microscopic optical tweezer.

5. CONCLUSION

In conclusion, we observed high efficient and deterministic loading of single Cs atoms in a MOT with a real-time field-gradient feedback technique. This is the first important step to study single atoms in a microscopic optical tweezer. After this, we transfer of the atoms from the MOT into the microscopic optical tweezer. During the feedback on QMF, there is probability that more than one atom are transferred into the microscopic optical tweezer. We use the technique of LAC to decrease the probability of more than one atom. Using the feedback on QMF and controlled LAC, we finally realize a probability of 95.2% single atoms in a microscopic optical tweezer. We also implement two different techniques for loading single Cs atoms into a microscopic optical tweezer.

Next, we plan to simulation of the two-dimensional arrays of microscopic microscopic optical tweezer. Using the technique of real-time field-gradient feedback and the LAC induced by near resonance blue-detuned laser, which may be used to present high efficient and deterministic loading of a single atom for many singly occupied microscopic microscopic optical tweezer.

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