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Discussion

Entanglement and nonclassicality evolution of the atom in a squeezed vacuum

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Abstract

As an important parameter, von Neumann entropy has been used to characterize the entanglement between atom and light field. We discussed the entanglement and nonclassicality evolution of an atom in a squeezed vacuum—a typical nonclassical field, and compare it with that of the coherent state. It shows that the atom-field entanglement in squeezed vacuum is much stronger and stabler than that in coherent state, whereas the nonclassicality of the light field depends on its initial status. This investigation is trying to find a new insight into the relation between entanglement of atom-field system and nonclassicality of light fields. The result shows that the entanglement between the atom and the field can be maintained well in the squeezed vacuum and this implies better control of atom and photon mutually.

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Quantum entanglement is a physical phenomenon emerging among the subsystems of a multi-system and it is now becoming a key word in the quantum information processing (QIP) [1]. Qualifying quantitatively the quantum entanglement is an important issue of describing the nonclassical properties of a quantum system. In principle, measuring one subsystem, the rest of the system will be affected. By using nonlinear optical processes, people have already generated some useful entangled states in optical domain, such as EPR states [2–4], multipartite entanglement [5,6]. They have been the workhorses of quantum information science based on pure optics. Another system of quantum entanglement is the atom–photon system. As the atom–photon interaction has been reached to the strong-coupling region by cavity quantum electrodynamics system [7], atom–cavity–photon becomes a good and reliable system for the generation of controllable quantum entanglement. The basic interaction between a single two-level atom and a single-mode quantized field was described by Jaynes–Cummings model (JCM) [8], and it has been discussed in detail from various points of view [9–13], and it is well known that the behaviors of the atom-field interaction is strongly related to their initial states. Once an atom and a light field are strongly entangled with each other, the atom can be fully controlled by photon. It is thus important to investigate the quantum entanglement of atom-field system. On the other hand, nonclassicality of light field also plays an important role in QIP and quantum measurement. The squeezed vacuum state, which can be produced in experiments and used for EPR states [14], is the typical nonclassical state. It has been found that the atom in nonclassical field shows some behaviors different from that in classical field [15–18]. To get a new insight into the relation between quantum entanglement of the atom-field system and nonclassicality of the light field, it is useful to investigate the atom-field entanglement under the nonclassical environment. Furuichi and Abdel-Aty discussed the

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entanglement in a squeezed two-level atom [19], and showed the entanglement is sensitive to the squeezing parameter, but they did not analyze its evolution systematically. There are several definitions of quantum entanglement [20], but for a bipartite pure system composed here of the atom and the field, all these definitions are equivalent [21]. The von Neumann entropy [22] has been widely used to characterize the quantum entanglement of a multipartite system [20,23,24]. It has been pointed out that the intrinsic decoherence will make the field entropy enhanced but no change with the atom entropy [24]. In this paper we investigate the evolution of the atom-field entanglement about an atom in a pure squeezed vacuum state and compare the results to that in the coherent states (CS). It is shown that the atom-field entanglement in squeezed vacuum can be maintained at maximum for a long time and easily to reach the maxima, which means that the squeezed vacuum field can improve the quantum entanglement. This also implies that one can easily and mutually control such a quantum system in SVS environment.

Let us consider the basic model of an atom interacts with a field. Practically a nearly pure single-mode squeezed state can be generated [25] and can be treated approximately as a single-mode. Under the rotating-wave (RW) approximation, the Hamiltonian can be written as [26]

$$H = \hbar\omega_a\sigma_z + \hbar\omega_L a^+a + \hbar g(\sigma_+ a + a^+\sigma_-), \tag{1}$$

where ω_a is the atomic transition frequency and g is the atom-field coupling constant, σ_z and $\sigma_\pm$ are the atomic inversion and transition operator, respectively. This Hamiltonian gives rise to the following time-involution operator in the interaction picture [27]:

$$U(t,0) = \begin{pmatrix} \cos\left(\sqrt{aa^+}gt\right) & -\mathrm{i}a\frac{\sin\left(\sqrt{a^+a}gt\right)}{\sqrt{a^+a}} \\ -\mathrm{i}a^+\frac{\sin\left(\sqrt{aa^+}gt\right)}{\sqrt{aa^+}} & \cos\left(\sqrt{a^+a}gt\right) \end{pmatrix}. \tag{2}$$

If the atom is initially in the excited state, the initial atom-field state can therefore be written as

$$|\psi_{\mathrm{af}}(0)\rangle = |\phi_{\mathrm{f}}\rangle \otimes |e\rangle. \tag{3}$$

The time-evolved atom-field state is then given by

$$|\psi_{\mathrm{af}}(t)\rangle = \widehat{C}|\phi_{\mathrm{f}}\rangle \otimes |e\rangle + \widehat{S}|\phi_{\mathrm{f}}\rangle \otimes |g\rangle, \tag{4}$$

where we have defined the operators

$$\widehat{C} = \cos\left(\sqrt{aa^+}gt\right), \quad \widehat{S} = -\mathrm{i}a^+\frac{\sin\left(\sqrt{aa^+}gt\right)}{\sqrt{aa^+}}. \tag{5}$$

After taking a trace over the atom coordinates, we get the reduced density operator of the field accordingly:

$$\rho_{\mathrm{f}}(t) = |c\rangle\langle c| + |s\rangle\langle s|, \tag{6}$$

where $|c\rangle = \widehat{C}|\phi_{\mathrm{f}}\rangle$ and $|s\rangle = \widehat{S}|\phi_{\mathrm{f}}\rangle$ are the states that operators $\widehat{C}$ and $\widehat{S}$ act on the initial state of the field, respectively.

The atom-field quantum entanglement can be discussed by using Von Neumann entropy which is defined as [23]

$$S = -\mathrm{Tr}(\rho\ln\rho), \tag{7}$$

where ρ is the density operator. For a pure state this entropy vanishes, $S = 0$; whereas for a statistical mixture the entropy is nonzero, $S \neq 0$. It was proved that the total entropy of a system remains constant when the system is governed by the time-dependent Schrödinger equation [9]. What we are interested is thus the partial entropy of a sub-system, such as the field or the atom. From the reduced density matrices of the atom and field we get the partial entropy [23,28]

$$S_{\mathrm{a}}(t) = -\mathrm{Tr}_{\mathrm{a}}[\rho_{\mathrm{a}}(t)\ln\rho_{\mathrm{a}}(t)], \quad S_{\mathrm{f}}(t) = -\mathrm{Tr}_{\mathrm{f}}[\rho_{\mathrm{f}}(t)\ln\rho_{\mathrm{f}}(t)]. \tag{8}$$

Unlike the entropy of complete system, the partial entropy varies with time. According to the following triangle inequality for two interacting sub-system demonstrated by Araki and Lieb [29]

$$|S(\rho_{\mathrm{A}}) - S(\rho_{\mathrm{B}})| \leqslant S \leqslant S(\rho_{\mathrm{A}}) + S(\rho_{\mathrm{B}}), \tag{9}$$

if the system is a pure quantum state at initial time, the partial entropies of the subsystems will be equal all the time after, which means if the atom and the field do not interact with each other at the beginning, their partial entropies will keep equal during the interaction process.

Let us consider an atom in a squeezed vacuum. As we have known, the density operator of squeezed vacuum state (SVS) $|\xi\rangle$ is

$$\rho_{\mathrm{f}}(0) = \sum_{n,n'} \frac{\sqrt{(2n)!(2n')!}(\tanh r)^{n+n'}}{\cosh r(n!2^n n'!2^{n'})}|2n\rangle\langle 2n'|, \tag{10}$$

where $\xi = r\exp(\mathrm{i}\theta)$, r and θ are the squeezing parameter and the squeezing angle, respectively. Obviously the density operator of the squeezed vacuum is not diagonalized and in order to diagonalize it for entropy investigation, its eigenvalues are needed [23]. Note that an eigenstate of (6) must be of the form:

$$|\psi\rangle = \psi_c|c\rangle + \psi_s|s\rangle. \tag{11}$$

Consider the action of ρ_{f} on $|\psi\rangle$:

$$\begin{aligned}\rho_{\mathrm{f}}|\psi\rangle &= (|c\rangle\langle c| + |s\rangle\langle s|)(\psi_c|c\rangle + \psi_s|s\rangle) \\ &= \left(\langle c|c\rangle + \frac{\psi_s}{\psi_c}\langle c|s\rangle\right)\psi_c|c\rangle + \left(\langle s|s\rangle + \frac{\psi_c}{\psi_s}\langle s|c\rangle\right)\psi_s|s\rangle.\end{aligned} \tag{12}$$

Consequently for $|\psi\rangle$ to be an eigenstate of ρ_{f} it must satisfy

$$\langle c|c\rangle + \frac{\psi_s}{\psi_c}\langle c|s\rangle = \langle s|s\rangle + \frac{\psi_c}{\psi_s}\langle s|c\rangle. \tag{13}$$

Write $\langle c|s\rangle = |\langle c|s\rangle|\exp(\mathrm{i}\phi)$ and suppose that

$$\psi_c = \bar{\psi}_c\exp(\mathrm{i}\phi/2), \quad \psi_s = \bar{\psi}_s\exp(-\mathrm{i}\phi/2), \tag{14}$$

then Eq. (13) becomes

$$|\langle c|s\rangle|(\psi_c^2 - \psi_s^2) = (\langle c|c\rangle - \langle s|s\rangle)\psi_c\psi_s. \tag{15}$$

For SVS, we can easily get

$$\langle c|c\rangle = \sum_n P_{2n}\cos^2\left(\sqrt{n+1}gt\right),$$
$$\langle s|s\rangle = \sum_n P_{2n}\sin^2\left(\sqrt{n+1}gt\right), \tag{16}$$
$$\langle c|s\rangle = 0,$$

where

$$P_{2n} = |\langle 2n|\xi\rangle|^2 = \frac{(\tanh r)^{2n}(2n)!}{\cosh r(n!2^n)^2}, \tag{17}$$

is the probability of finding $2n$ photon in the state and $P_{2n+1} = 0$ (n: integer).

Since $\psi_c\psi_s = 0$, the eigenvalues are

$$\pi_1 = \langle c|c\rangle, \quad \pi_2 = \langle s|s\rangle. \tag{18}$$

Then the partial entropy of the field is given by

$$S_f(t) = -\pi_1 \ln \pi_1 - \pi_2 \ln \pi_2. \tag{19}$$

There are several quantities to investigate the nonclassicalities of light fields, such as quadrature squeezing, sub-Poissonian photon statistics and the negativity of the Wigner function [30] of a state. Without loss of generality, here we consider the photon statistics and the quadrature fluctuation.

Sub-Poissonian photon statistics is a typical nonclassical effect. For a single mode light field, it can be expressed by the Mandel Q parameter:

$$Q = \frac{\langle a^+aa^+a\rangle - (\langle a^+a\rangle)^2}{\langle a^+a\rangle} - 1. \tag{20}$$

$Q = 0$, $Q > 0$ and $Q < 0$ correspond to the Poissonian, super-Poissonian and sub-Poissonian statistics of the light field. The quadrature squeezing means

$$(\Delta X_i)^2 < \frac{1}{4} \quad (i = 1 \text{ or } 2), \tag{21}$$

where X_1, X_2 are two quadrature components of the field which are defined as

$$X_1 = \frac{1}{2}(a + a^+), \quad X_2 = \frac{1}{2i}(a - a^+), \tag{22}$$

where $a(a^+)$ is the annihilation (creation) operator of the field. Generally, if the squeezing angle θ is nonzero, we can define the rotate quadrature components Y_1 and Y_2, which satisfy

$$Y_1 + iY_2 = (X_1 + iX_2)\exp(-i\theta/2), \tag{23}$$

and their fluctuations can be given by

$$(\Delta Y_1)^2 = \frac{1}{4}\langle a^2e^{-i\theta} + a^{+2}e^{-i\theta} + aa^+ + a^+a\rangle - \frac{1}{4}\left(ae^{-i\theta/2} + a^+e^{i\theta/2}\right)^2, \tag{24a}$$

$$(\Delta Y_2)^2 = \frac{1}{4}\langle aa^+ + a^+a - a^2e^{-i\theta} - a^{+2}e^{-i\theta}\rangle + \frac{1}{4}\left(ae^{-i\theta/2} + a^+e^{i\theta/2}\right)^2. \tag{24b}$$

If one of the components satisfies $(\Delta Y_i)^2 < \frac{1}{4}$ ($i = 1$ or 2), the field is quadrature squeezed.

Using the reduced density operator of the field expressed by Eq. (6) we can get the Mandel Q parameter and the quadrature fluctuations of the field.

Let us first consider the entanglement evolution. Fig. 1 shows the atom-field entanglement evolution in the squeezed vacuum (black line) and coherent state (grey line) with the mean photon number 16 (Fig. 1a) and 0.5 (Fig. 1b).

From Fig. 1a we can see that for the coherent state the partial entropy of the field reaches the maximum at the beginning and then gets back to its minimum gradually and after that it returns back and oscillates regularly during the "revival" period. In the "collapse" period, the system tends to return the initial pure state. As for SVS, the partial entropy of the field reaches the maximum quickly from the very beginning and then oscillates randomly around the maximum entanglement but it does not return the initial pure state anymore. This indicates that, compare to the case of the coherent state, the atom and the field in squeezed vacuum environment can easily maintain strong entanglement. The insets are the results in the range of $gt = \pi$, and it shows that to get the similar entanglement, the requirement for interaction coupling gt is much smaller in SVS than that in CS, which implies that SVS may be helpful to establish quantum entanglement. As we will see later, the nonclassicality of SVS is reduced or even disappeared in this process. Fig. 1b is the results when the mean photon number is 0.5. Obviously, lower mean photon number means lower intensity and weaker squeezing, and in this case maintaining the maximum entanglement becomes more difficult. But still it is more regular and has more chances to reach the maximum entanglement for SVS than that for CS.

In order to see the difference more clearly, we show the partial entropies versus mean photon number for different coupling in Fig. 2. It indicates that the atom-field entanglement in SVS is not so sensitive to the interaction coupling gt and it goes to the maximum very rapidly. For CS, the entanglement also varies with gt but the proper couplings are required in order to get the maximum entanglement.

To see how the field changes in the process, let us now consider properties of the light field after the interaction. Fig. 3 is the evolution of the Mandel Q parameter in the SVS and CS with the mean photon number 16 (Fig. 3a) and 0.5 (Fig. 3b). There shows the strong super-Poissonian statistics for SVS, and for CS it shows sub-Poissonian statistics occasionally, but usually the photon statistics is close to Poissonian. Lower photon excitation results in deeper sub-Poissonian statistics. It is easy to understand since SVS itself is a two-photon coherent state which appears strong super-Poissonian statistics usually.

With the same parameters as in Fig. 3, we show the results of the evolution of two quadrature fluctuations in Fig. 4. From Fig. 4a we can see that for the CS the squeezing will

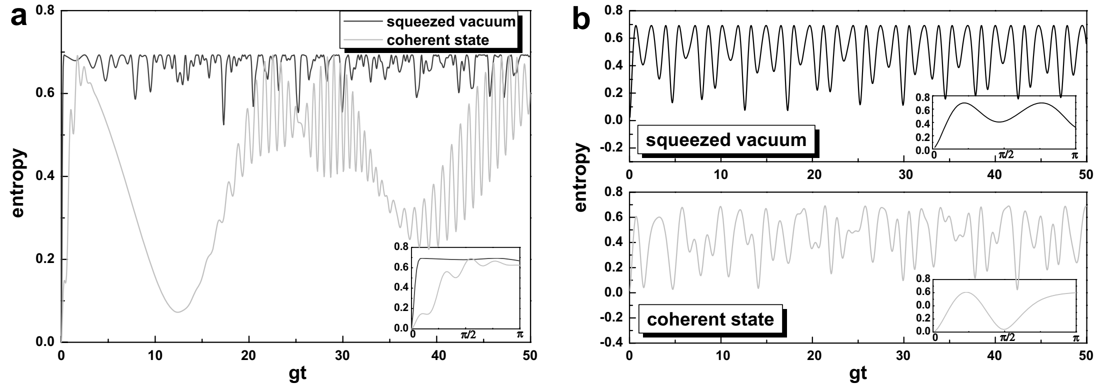

Fig. 1. Evolution of partial entropy for initial mean photon number 16 (a) and 0.5 (b) (Black: SVS case; Grey: CS case).

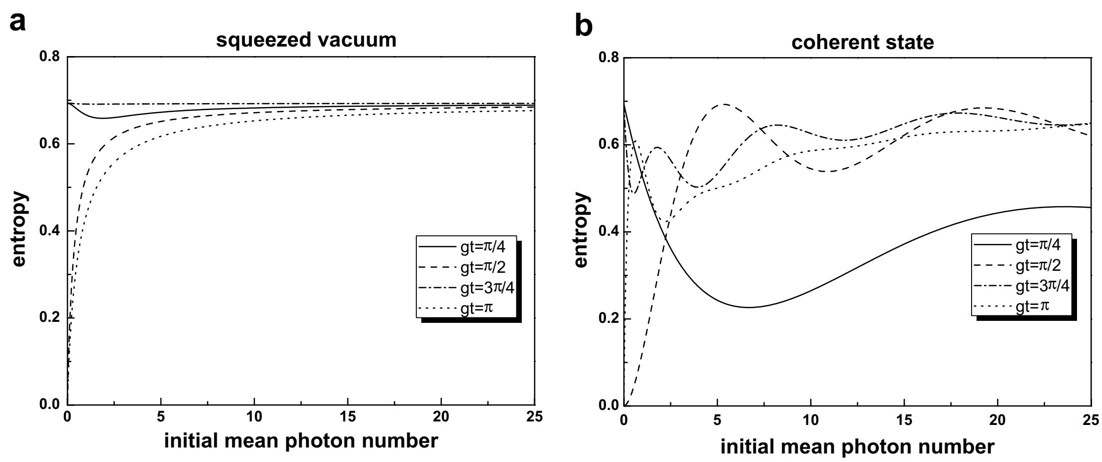

Fig. 2. Field partial entropy versus mean photon number with different coupling.

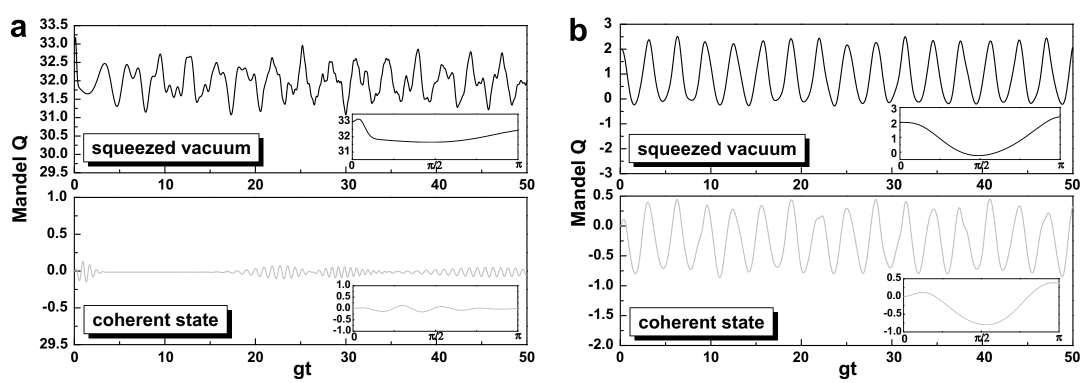

Fig. 3. Evolution of Mandel Q parameter for mean photon number 16 (a) and 0.5 (b).

exist only in the X_1 component. And for the SVS the initial squeezing in X_1 disappears quickly and will not reappear anymore. This indicates that when a SVS interacts with an atom, its initial nonclassicality (the squeezing) will lose whereas the quantum entanglement of the system will be enhanced. When the mean photon number is 0.5 (see Fig. 4b), the light field will show squeezing only in X_1 in the case of CS but asynchronous in X_1 and X_2 in the case of SVS. Such squeezing depends on its initial parameter when an atom is in a SVS: if the initial squeezing is strong, the field will show strong super-Poissonian statistics and also strong entanglement, and its initial squeezing will disappear very soon; if the initial squeezing is weak enough, the field will show weak sub-Poissonian statistics and also

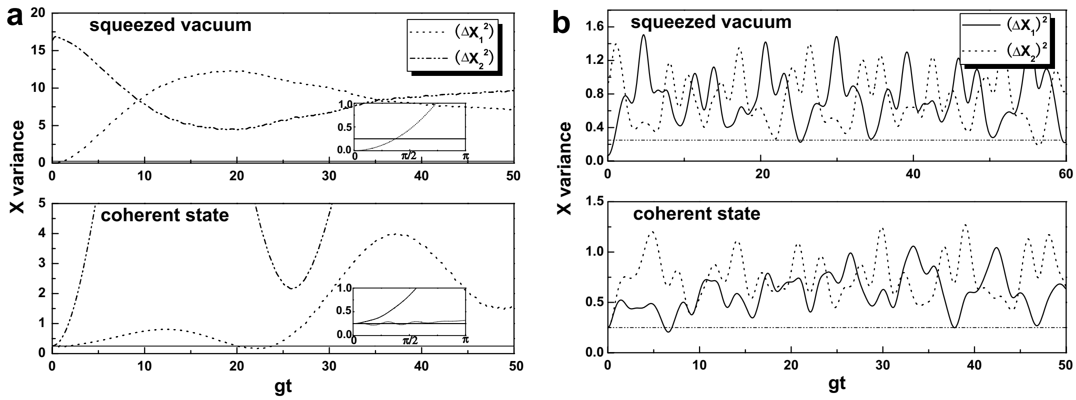

Fig. 4. Evolution of quadrature fluctuation for SVS and CS with mean photon number 16 (a) and 0.5 (b).

low entanglement, while its initial squeezing will be maintained. Again, the results imply that the SVS environment can improve the entanglement in such interaction process.

The different behaviors of the entanglement between the SVS and the CS are related to the nonclassicality of the states. Actually, the single mode SVS, also called two-photon coherent state [31] can be generated from the degenerate parametric oscillator (DPO), and it is the bunched entangled photon beam [32] which only consists twin photons. With a beamsplitter and one or two squeezed vacuum states one can produce entangled states [14,33]. Such nonclassical feature is characterized by the squeezing [34]. In the experiment situation, higher squeezing means higher entanglement of the degenerate EPR, and perfect squeezing means ideal EPR state for continuous variables. It has been shown that the entanglement can be transferred from field to atoms [35] and the results discussed here could be regarded as a kind of entanglement transfer from the bunched entangled photon pairs to the atom–photon system. As was pointed out, when considered the cavity decay and the atomic decay, due to the decoherence, the entanglement will be decreased [35,36]. We also investigated the corresponding result for the coherent squeezed state which includes coherent component and found that the entanglement is expected between the SVS and the CS. Again, better squeezing corresponds to better entanglement.

In conclusion, we have investigated the evolution of the atom-field entanglement and the field nonclassicality when an atom interacts with a SVS or a CS, including photon statistics and quadrature fluctuations. It is found that the atom-field entanglement in squeezed vacuum environment is much stronger than that in coherent environment under the same conditions. Strong and stable quantum entanglement can be obtained with the SVS. The possibility of keeping entanglement longer and intenser by the nonclassical SVS implies that one can use the nonclassical light field to realize better control of the atom–photon interaction processing.

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