

Bright two-color tripartite entanglement with second harmonic generation

Shuqin Zhai, Rongguo Yang, Kui Liu, Hailong Zhang, Junxiang Zhang, and Jiangrui Gao*

State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan 030006, P. R. China
jrgao@sxu.edu.cn

Abstract: The bright two-color tripartite entanglement is investigated in the process of type II second harmonic generation (SHG) operating above threshold. The two pump fields and the second harmonic field are proved to be entangled, and the dependence of the entanglement degree on pump parameter σ and normalized frequency Ω is also analyzed.

© 2009 Optical Society of America

OCIS codes: (270.0270) Quantum optics; (270.6570) Squeezed states; (190.2620) Harmonic generation

References and links

1. A. K. Ekert, "Quantum cryptography based on Bell's theorem," *Phys. Rev. Lett.* **67**(6), 661–663 (1991).
2. J. Jing, J. Zhang, Y. Yan, F. Zhao, C. Xie, and K. Peng, "Experimental demonstration of tripartite entanglement and controlled dense coding for continuous variables," *Phys. Rev. Lett.* **90**(16), 167903 (2003).
3. S. L. Braunstein, and P. van Loock, "Multipartite entanglement for continuous variables: A quantum teleportation network," *Phys. Rev. Lett.* **84**(15), 3482–3485 (2000).
4. D. Deutsch, and R. Jozsa, "Rapid solution of problems by quantum computation," *Proc. R. Soc. Lond. A* **439**(1907), 553–558 (1992).
5. A. S. Villar, M. Martinelli, C. Fabre, and P. Nussenzveig, "Direct production of tripartite pump-signal-idler entanglement in the above-threshold optical parametric oscillator," *Phys. Rev. Lett.* **97**(14), 140504 (2006).
6. K. N. Cassemiro, A. S. Villar, P. Valente, M. Martinelli, and P. Nussenzveig, "Experimental observation of three-color optical quantum correlations," *Opt. Lett.* **32**(6), 695–697 (2007).
7. K. N. Cassemiro, A. S. Villar, M. Martinelli, and P. Nussenzveig, "The quest for three-color entanglement: experimental investigation of new multipartite quantum correlations," *Opt. Express* **15**(26), 18236–18246 (2007).
8. N. B. Grosse, W. P. Bowen, K. McKenzie, and P. K. Lam, "Harmonic entanglement with second-order nonlinearity," *Phys. Rev. Lett.* **96**(6), 063601 (2006).
9. N. B. Grosse, S. Assad, M. Mehmet, R. Schnabel, T. Symul, and P. K. Lam, "Observation of entanglement between two light beams spanning an octave in optical frequency," *Phys. Rev. Lett.* **100**(24), 243601 (2008).
10. S. Zhai, R. Yang, D. Fan, J. Guo, K. Liu, J. Zhang, and J. Gao, "Tripartite entanglement from the cavity with second-order harmonic generation," *Phys. Rev. A* **78**(1), 014302 (2008).
11. L. Longchambon, N. Treps, T. Coudreau, J. Laurat, and C. Fabre, "Experimental evidence of spontaneous symmetry breaking in intracavity type II second-harmonic generation with triple resonance," *Opt. Lett.* **30**(3), 284–286 (2005).
12. M. J. Collett, and C. W. Gardiner, "Squeezing of intracavity and traveling-wave light fields produced in parametric amplification," *Phys. Rev. A* **30**(3), 1386–1391 (1984).
13. P. van Loock, and A. Furusawa, "Detecting genuine multipartite continuous-variable entanglement," *Phys. Rev. A* **67**(5), 052315 (2003).
14. K. Schneider, and S. Schiller, "Multiple conversion and optical limiting in a subharmonic-pumped parametric oscillator," *Opt. Lett.* **22**(6), 363–365 (1997).

Entanglement between more than two parts plays a central role in the process of quantum information, such as quantum cryptography [1], controlled dense coding [2], teleportation [3], and quantum computation [4]. And the multipartite entanglement states with different frequencies will be more important since it would facilitate many quantum information protocols of interspecies quantum teleportation, disparate nodes in a quantum information network.

The generation of continuous variable (CV) multicolored entangled beams can be realized with nonlinear process. It was reported that the tripartite entanglement among signal, idler and

pump modes was existent when a nondegenerate optical parametric oscillator (OPO) was operated above threshold [5]. And subsequently the quantum correlation experiments based on this scheme were demonstrated [6,7]. Apart from the parametric down-conversion, the second harmonic generation (SHG) can also be used to yield the multicolored entanglement. The bipartite entanglement with different wavelength between the fundamental and second-harmonic fields was theoretically investigated and experimentally demonstrated [8,9]. Thereafter, we suggested the generation of the tripartite entanglement of the fundamental and second-harmonic fields with two frequencies through the type-II SHG below threshold [10]. In this paper, we discuss the quantum characteristics of the two reflected fundamental modes and the generated second harmonic mode in the triple resonant type-II SHG system operating above threshold. In this case, second-harmonic generation and parametric downconversion occur simultaneously. The tripartite quadrature entanglement between three modes are analyzed in terms of experiment-related parameters such as normalized pump parametric, cavity losses and normalized frequency. Under appropriate conditions, triply resonant type-II SHG system can be also exploited as a bright continuous variable tripartite entanglement resource, but the quantum correlation is different between above and below threshold.

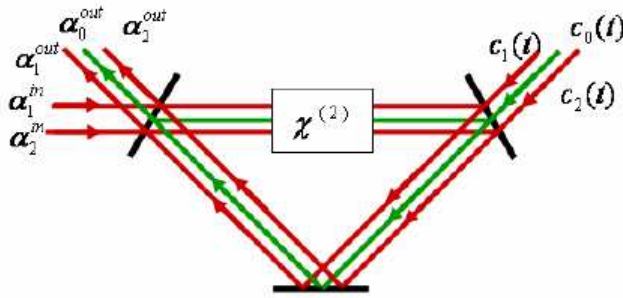


Fig. 1. The sketch of SHG

The system under analysis is shown in Fig. 1. It consists of a type II phase-matched crystal $\chi^{(2)}$ placed inside a one-side cavity. Two sub-harmonic pump modes α_1 and α_2 are incident upon the optical medium. Their frequencies are degenerate ($\omega_1 = \omega_2$), and the polarizations are orthogonal for the phase-matching condition. Then the harmonic mode α_0 is generated when the energy-matching condition $\omega_1 + \omega_2 = \omega_0$ is satisfied. All of the sub-harmonic modes and harmonic mode resonate in the cavity simultaneously.

Assuming perfect phase matching, the semi-classical dynamic Eqs. can be expressed as

$$\tau \dot{\hat{a}}_0(t) = -\gamma_0 \hat{a}_0(t) - \chi \hat{a}_1(t) \hat{a}_2(t) + \sqrt{2\gamma_{b0}} \hat{a}_0^{in}(t) e^{i\phi_0} + \sqrt{2\gamma_{c0}} \hat{c}_0(t)$$

$$\begin{aligned}\tau \dot{\hat{a}}_1(t) &= -\gamma_1 \hat{a}_1(t) + \chi \hat{a}_2^+(t) \hat{a}_0(t) + \sqrt{2\gamma_{b1}} \hat{a}_1^{in}(t) e^{i\phi_1} + \sqrt{2\gamma_{c1}} \hat{c}_1(t) \\ \tau \dot{\hat{a}}_2(t) &= -\gamma_2 \hat{a}_2(t) + \chi \hat{a}_1^+(t) \hat{a}_0(t) + \sqrt{2\gamma_{b2}} \hat{a}_2^{in}(t) e^{i\phi_2} + \sqrt{2\gamma_{c2}} \hat{c}_2(t)\end{aligned}\quad (1)$$

Here \hat{a}_0 , \hat{a}_1 , and \hat{a}_2 are the annihilation operators of harmonic and two sub-harmonic pump fields inside the cavity, respectively. \hat{a}_i^{in} ($i = 0, 1, 2$) denotes the input amplitude operators of the three fields. The roundtrip time τ in the cavity is assumed to be same for all the three fields. χ is the effective nonlinear coupling parameter, which is proportional to the second order susceptibility of the crystal. The total loss parameter for each mode is $\gamma_i = \gamma_{bi} + \gamma_{ci}$, ($i = 0, 1, 2$), where γ_{bi} is related to the amplitude reflection coefficients r_i and amplitude transmission coefficients t_i of the coupling mirror of the optical cavity by the formula: $r_i = 1 - \gamma_{bi}$, $t_i = \sqrt{2\gamma_{bi}}$, and γ_{ci} represents extra intracavity loss parameter. $\hat{c}_i(t)$ is the vacuum noise term corresponding to intracavity loss.

In general case, assuming that the two pump modes and harmonic mode have zero initial phase $e^{i\phi_0} = e^{i\phi_1} = e^{i\phi_2} = 1$, the two pumping modes have the same positive real amplitude β , the cavity transmission factor and the extra-losses for the two pump fields are assumed to be the same $\gamma_1 = \gamma_2 = \gamma$, $\gamma_{b1} = \gamma_{b2} = \gamma_b$, $\gamma_{c1} = \gamma_{c2} = \gamma_c$.

Stationary mean field solutions α_0 , α_1 , and α_2 can be obtained by setting $\dot{\hat{a}}_0$, $\dot{\hat{a}}_1$ and $\dot{\hat{a}}_2$ to be zero:

$$-\gamma_0 \alpha_0 - \chi \alpha_1 \alpha_2 = 0 \quad (2a)$$

$$-\gamma \alpha_1 + \chi \alpha_2^* \alpha_0 + \sqrt{2\gamma_b} \beta = 0 \quad (2b)$$

$$-\gamma \alpha_2 + \chi \alpha_1^* \alpha_0 + \sqrt{2\gamma_b} \beta = 0 \quad (2c)$$

Equations (2b) and (2c) show that both α_1 and α_2 are real numbers. The pumping threshold β^{th} and pump parameter σ can be expressed as

$$\beta^{th} = \sqrt{\frac{2\gamma^3 \gamma_0}{\chi^2 \gamma_b}} \quad (3a)$$

$$\sigma = \frac{\beta}{\beta^{th}} \quad (3b)$$

Using the input-output relations $\alpha_i^{in} + \alpha_i^{out} = \sqrt{2\gamma_b} \alpha_i$, the amplitude of output field above threshold ($\sigma \geq 1$) can be written as

$$\alpha_1^{out} = \sqrt{\frac{2\gamma\gamma_0}{\chi^2 \gamma_b}} \left(\sigma \gamma_b - \sigma \gamma - \gamma_b \sqrt{\sigma^2 - 1} \right) \quad (4a)$$

$$\alpha_2^{out} = \sqrt{\frac{2\gamma\gamma_0}{\chi^2 \gamma_b}} \left(\sigma \gamma_b - \sigma \gamma + \gamma_b \sqrt{\sigma^2 - 1} \right) \quad (4b)$$

$$\alpha_0^{out} = -\frac{\gamma\sqrt{2\gamma_0}}{\chi}. \quad (4c)$$

The output amplitude of two fundamental modes depending on pump parameter σ are shown in Fig. 2 at the condition of $\gamma > \gamma_b$ (solid line) and $\gamma = \gamma_b$ (dashed line) respectively. We also show the case below threshold ($\sigma \leq 1$) in Fig. 2 [10]. It can be seen obviously that when the cavity is operating below threshold, the two output fundamental modes are equal, but when it is above threshold, the two modes are different from each other because of spontaneous symmetry-breaking phenomenon [11], which will affect the entanglement in the system. The solid line in Fig. 2 is for the case of existing an internal cavity loss ($\gamma > \gamma_b$), the outputs of the two modes are out of phase with unbalanced values; For the idea case of $\gamma = \gamma_b$, the two modes have the same absolute values.

In the following section, entanglement between two pumping sub-harmonic modes and the harmonic mode of triple resonant SHG operating above threshold is considered.

The dynamics of the quantum fluctuations can be described by linearizing the classical Eqs. of motion around the stationary state. Setting

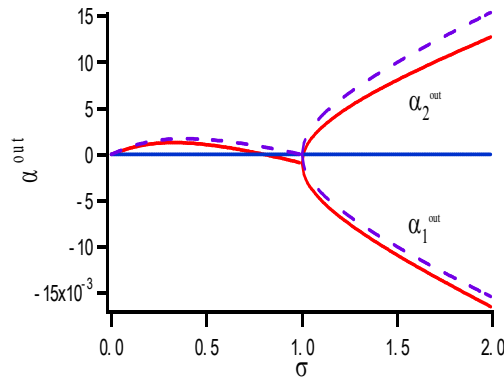


Fig. 2. The dependence of amplitude of the two output harmonic modes on pump parameter (σ). solid line: $\gamma = 0.02$, $\gamma_b = 0.018$, $\gamma_0 = 0.1$; dashed line: with $\gamma = \gamma_b = 0.02$, $\gamma_0 = 0.1$;

$$\hat{a}_i(t) = \alpha_i + \delta\hat{a}_i(t) (i = 0, 1, 2)$$

$$\hat{a}_0^{in}(t) = \delta\hat{a}_0^{in}(t) \quad (5)$$

$$\hat{a}_i^{in}(t) = \beta + \delta\hat{a}_i^{in}(t) (i = 1, 2),$$

using the definitions of the quadrature phase amplitudes $\hat{X} = \hat{a} + \hat{a}^+$, $\hat{Y} = (\hat{a} - \hat{a}^+)/i$, and the input-output relations of optical cavity for the fluctuations of quadrature-phase amplitude [12], $\delta\hat{X}_i^{out}(\omega) = \sqrt{2\gamma_{bi}}\delta\hat{X}_i(\omega) - \delta\hat{X}_i^{in}(\omega)$, the Fourier transformation of the output quadrature components of harmonic field and two pump fields above threshold can be solved with the Eqs. (1-5).

In order to discuss the bright tripartite entanglement among the two pumping sub-harmonic modes and harmonic mode of triple resonant SHG operating above threshold, we introduce the sufficient inseparability criterion for tripartite CV entanglement proposed by van Loock and Furusawa [13]:

$$S_1^{out} = \langle \delta^2(\hat{Y}_1^{out} - \hat{Y}_2^{out}) \rangle + \langle \delta^2(\hat{X}_1^{out} + \hat{X}_2^{out} - g_1\hat{X}_0^{out}) \rangle \leq 4$$

$$S_2^{out} = \langle \delta^2(\hat{Y}_0^{out} + \hat{Y}_1^{out}) \rangle + \langle \delta^2(\hat{X}_1^{out} + g_2 \hat{X}_2^{out} - \hat{X}_0^{out}) \rangle \leq 4 \quad (6)$$

$$S_3^{out} = \langle \delta^2(\hat{Y}_0^{out} + \hat{Y}_2^{out}) \rangle + \langle \delta^2(g_3 \hat{X}_1^{out} + \hat{X}_2^{out} - \hat{X}_0^{out}) \rangle \leq 4$$

the related correlation spectra in Eq. (6) of the total amplitude quadratures and relative phase quadratures of three mode proposed are calculated by:

$$\begin{aligned} \langle \delta^2(\hat{Y}_1^{out} - \hat{Y}_2^{out}) \rangle &= \langle [\delta\hat{Y}_1^{out}(\omega) - \delta\hat{Y}_2^{out}(\omega)][\delta\hat{Y}_1^{out}(\omega) - \delta\hat{Y}_2^{out}(\omega)]^+ \rangle \\ \langle \delta^2(\hat{Y}_0^{out} + \hat{Y}_1^{out}) \rangle &= \langle [\delta\hat{Y}_0^{out}(\omega) + \delta\hat{Y}_1^{out}(\omega)][\delta\hat{Y}_0^{out}(\omega) + \delta\hat{Y}_1^{out}(\omega)]^+ \rangle \\ \langle \delta^2(\hat{Y}_0^{out} + \hat{Y}_2^{out}) \rangle &= \langle [\delta\hat{Y}_0^{out}(\omega) + \delta\hat{Y}_2^{out}(\omega)][\delta\hat{Y}_0^{out}(\omega) + \delta\hat{Y}_2^{out}(\omega)]^+ \rangle \\ \langle \delta^2(\hat{X}_1^{out} + \hat{X}_2^{out} - g_1 \hat{X}_0^{out}) \rangle &= \\ &\quad \langle [\delta\hat{X}_1^{out}(\omega) + \delta\hat{X}_2^{out}(\omega) - g_1 \delta\hat{X}_0^{out}(\omega)] \\ &\quad \times [\delta\hat{X}_1^{out}(\omega) + \delta\hat{X}_2^{out}(\omega) - g_1 \delta\hat{X}_0^{out}(\omega)]^+ \rangle \\ \langle \delta^2(\hat{X}_1^{out} + g_2 \hat{X}_2^{out} - \hat{X}_0^{out}) \rangle &= \\ &\quad \langle [\delta\hat{X}_1^{out}(\omega) + g_2 \delta\hat{X}_2^{out}(\omega) - \delta\hat{X}_0^{out}(\omega)] \\ &\quad \times [\delta\hat{X}_1^{out}(\omega) + g_2 \delta\hat{X}_2^{out}(\omega) - \delta\hat{X}_0^{out}(\omega)]^+ \rangle \\ \langle \delta^2(g_3 \hat{X}_1^{out} + \hat{X}_2^{out} - \hat{X}_0^{out}) \rangle &= \\ &\quad \langle [g_3 \delta\hat{X}_1^{out}(\omega) + \delta\hat{X}_2^{out}(\omega) - \delta\hat{X}_0^{out}(\omega)] \\ &\quad \times [g_3 \delta\hat{X}_1^{out}(\omega) + \delta\hat{X}_2^{out}(\omega) - \delta\hat{X}_0^{out}(\omega)]^+ \rangle \end{aligned} \quad (7)$$

where g_1, g_2, g_3 are scaling factors. The correlation noise spectra S_1^{out} , S_2^{out} and S_3^{out} can be minimized by choosing the adjustable scaling factors. And in the calculation, the normalized quadrature phase noise correlation functions of $\langle \delta\hat{X}_{c_i}(\omega)\delta\hat{X}_{c_i}(-\omega) \rangle = 1$ and $\langle \delta\hat{Y}_{c_i}(\omega)\delta\hat{Y}_{c_i}(-\omega) \rangle = 1$ for the vacuum noise term $\hat{c}_i(t)$ are used. The satisfaction of any pair of the inequalities is sufficient for full inseparability of three-party entanglement. The smaller the values of the inequalities are, the larger the correlation degree will be obtained.

The correlation spectra of these three quantities with optimized choice of the three parameters g_i are plotted in Figs. 3 and 4., the dot lines are corresponding to standard quantum limit (SQL) 4. Figure 3 shows the minimum correlation spectra of S_1^{out} (solid curve), S_2^{out} (dashed curve) and S_3^{out} (dash-dotted curve) as a function of normalized pump parameter $\sigma = \beta/\beta^{th}$. As we discussed above, the outputs of the two fundamental modes become unbalanced because of symmetry-breaking, the correlation spectra of S_2^{out} and S_3^{out} are separated from threshold and become very different. One of them (S_2^{out}) is large than SQL 4 almost in all Fig. 3. It is different from below threshold [10]. In fact, the threshold ($\sigma = 1$) is an inflexion for all three criteria. When pump parameter is in between 1 to 1.2, all of the three inequalities are satisfied, when pump parameter is in between 1.2 and 4, two of the three inequalities are satisfied. Which means the tripartite entanglement between the

fundamental and harmonic modes are obtained in all region of Fig. 3. And meanwhile the dependences of the three correlations spectra on normalized frequency ($\Omega = \omega\tau / \gamma$) with a pump parameter $\sigma = 1.5$ are shown in Fig. 4. Two of the inequalities (S_1^{out} (solid curve) and S_3^{out} (dash-dotted curve)) are satisfied when the normalized frequency is below 1.5. It satisfy sufficient inseparability criterion for tripartite CV entanglement below normalized analysis frequency 1.5. The correlation noise spectrum S_3^{out} is lower than others, it means the quantum correlation between one of the fundamental modes and harmonic mode is stronger than others. Compared to the result below threshold, which quantum correlation between two fundamental modes is strong, the fundamental modes and harmonic mode presents strong coupling above threshold.

In summary, by means of semi-classical method, we analyzed the continuous-variable tripartite entanglement characteristic of the reflected fundamental fields and the second harmonic field in a triple resonant type-II SHG operating above threshold. The full inseparability of the three output modes as a function of normalized frequency and pump parameter are calculated. Under certain conditions, type-II SHG system operating above threshold can be also exploited as a bright two color tripartite entanglement resource. But the quantum correlation characters are very different because of spontaneous symmetry-breaking. With the advantage of simplicity of SHG process, the calculated results may be a useful reference for the generation and application of entanglement in quantum communication networks. Furthermore, with multiple conversion in a subharmonic-pumped parametric oscillator operating above threshold [14], it is possible to develop multicolor multiparty (more than 3) entanglement with some special condition. It will be more interesting.

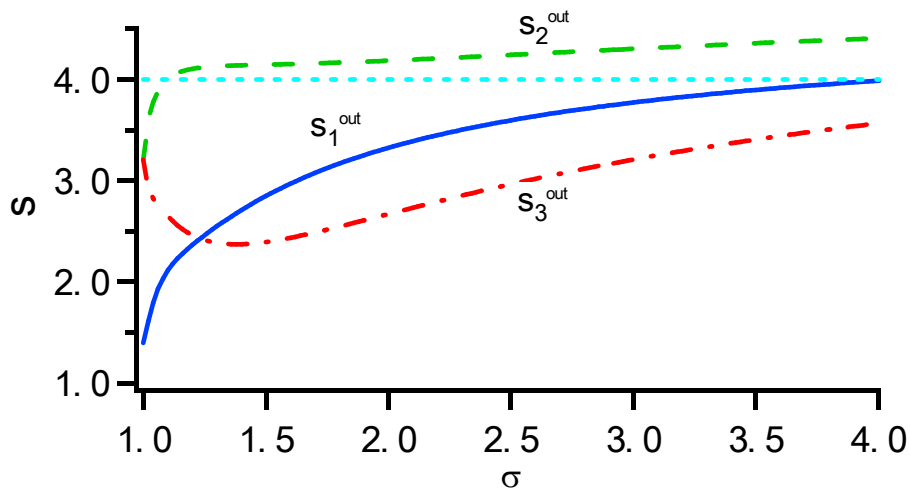


Fig. 3. The quantum correlation spectra S_1^{out} (solid curve), S_2^{out} (dashed curve) and S_3^{out} (dash-dotted curve) versus normalized pump parameter ($\sigma = \beta / \beta^{th}$) with $\gamma=0.03; \gamma_b=0.027; \gamma_0=0.1; \gamma_w=0.09; \Omega=0.2$.

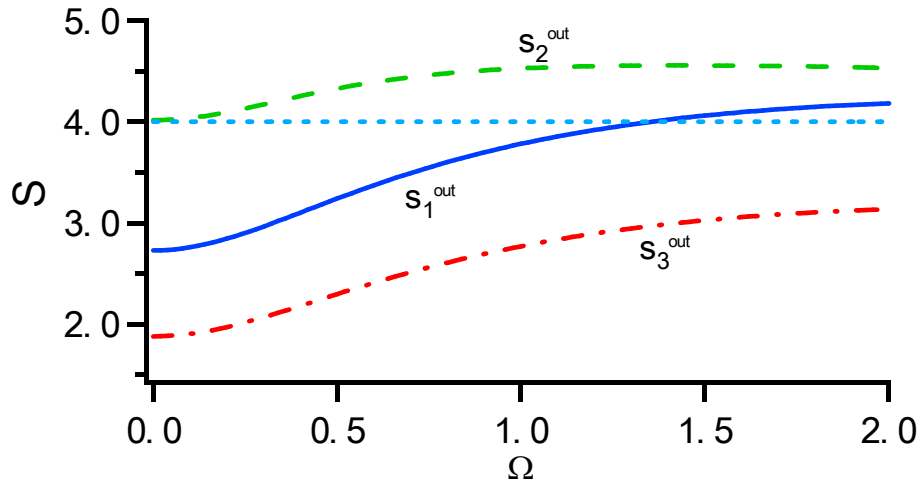


Fig. 4. The quantum correlation spectra S_1^{out} (solid curve), S_2^{out} (dashed curve) and S_3^{out} (dash-dotted curve) versus normalized frequency with $\gamma=0.03; \gamma_b=0.027; \gamma_o=0.1; \gamma_{b0}=0.09; \sigma=1.5$.

Acknowledgments

This work was supported in part by National Natural Science Foundation of China (No. 10774096, 60708010), NSFC Project for Excellent Research Team (Grant No.60821004), National Basic Research Program of China (2006CB921101) and the Research Fund for the Returned Abroad Scholars of Shanxi Province.