Low-frequency phase measurement with high-frequency squeezing

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Abstract: Squeezed-state enhanced audio-frequency signal measurement is crucial for some special applications, such as gravitational wave detection. But generation of squeezed state of light at such frequency is more difficult than that at megahertz-frequency. In this paper we propose an experimental scheme to measure low-frequency phase signal with high-frequency squeezing. To utilize the high-frequency sidebands of the squeezed light, a two-frequency intense laser is applied in the interferometry instead of a single-frequency laser as usual. This technique is in the reach of modern quantum optics technology.

OCIS codes: (270.0270) Quantum optics; (270.6570) Squeezed states; (120.3180) Interferometry.

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Introduction

The concept of radiation field squeezing [1] has attracted much interest since its application to gravitational wave detection was proposed [2]. This sort of radiation field was first generated with a four-wave mixing process by Slusher et al [3] and improved with an optical parametric oscillator (OPO) by Wu et al [4]. It has been useful in various quantum-enhanced measurement schemes, such as squeezed-light magnetometry [5], displacement measurement [6], spectroscopy [7], polarization measurement [8], phase measurement [9], and so on. Squeezed states of light are also vital ingredients for continuous-variable quantum communication and quantum computation [10]. In some applications, such as gravitational wave detection, it is appealing to measure low-frequency signals with a squeezed light state [11]. Low-frequency squeezing has attracted much interest in recent years as terrestrial gravitational wave detectors (GW-detectors) are approaching their shot noise limit, which has been successfully overcome with squeezing at audio frequencies very recently [12,13]. This experiment further enhanced confidence and passion in applying squeezed state to large scale GW-detectors. Actually great progress of generating this kind of squeezing has been made in recent years [14–20]. It's found that coherently controlling the phases of the experimental set-up while not to introduce extra noise is one of the keys to generate low-frequency squeezing [16–20]. In 2007, an unprecedented experiment of generating squeezed vacuum states with a noise power 6.5 dB below vacuum noise within the entire detection bandwidth of ground-based GW-detectors (10 Hz - 10 kHz) was demonstrated by using a sophisticated control scheme [20]. An alternative way to conquer this difficulty is to use two-frequency laser and broadband squeezing at higher frequency, which has been primarily used in many quantum optics laboratories, to enhance the signal-to-noise ratio (SNR) of an interferometer for lower-frequency phase measurement. When the carriers with frequencies other than central frequency of squeezed state appear in the interferometer, high-frequency squeezing sidebands will come into play. J. Gea-Banacloche and G. Leuchs [21] theoretically showed that, in Michelson interferometers stabilized with phase modulation technique, broadband squeezing is needed. Almost at the same time, Yurke et al [22] proposed a squeezed-state enhanced two-frequency interferometer to perform sub-shot-noise measurement of low-frequency signals by reading the photocurrent at frequency $2\nu_s$ (frequency interval of the two-frequency laser), well away from the low-frequency technical noise. In this study, we read low-frequency signals directly and calculate the SNR. This technique can be straightforwardly extended to the other squeezing-enhanced measurement schemes mentioned above.

Theoretical model

We consider a squeezing-enhanced Mach-Zehnder interferometer as in Ref [9]. shown in Fig. 1. The main idea is that by transferring the frequency of the intense laser to the frequencies of the entangled sidebands of the squeezed field, the low-frequency signal originating from the sidebands close to the intense laser frequencies can be detected with enhanced SNR for the entangled squeezing sidebands. The frequency relations of the usual

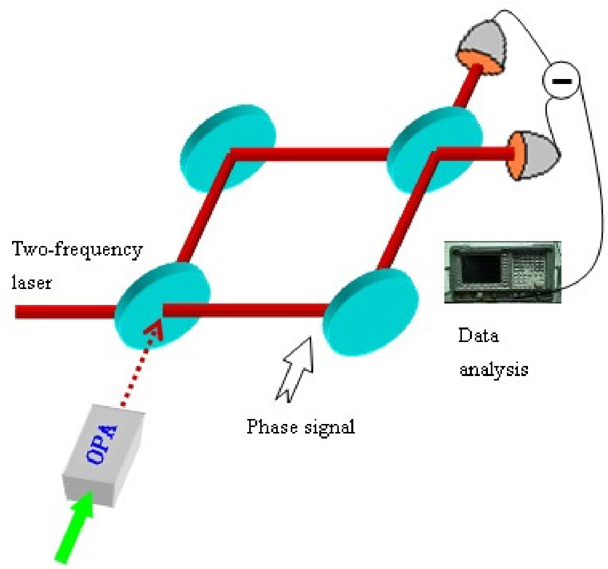

Fig. 1. Squeezed-state enhanced Mach-Zehnder interferometer. OPA refers to optical parametrical amplifier. BS1 and BS2 refer to 50% beam splitters. Frequencies of the two-frequency laser are $\omega_0+\Omega$ and $\omega_0-\Omega$. Dashed arrow refers to squeezing with center frequency ω_0 generated by OPA.

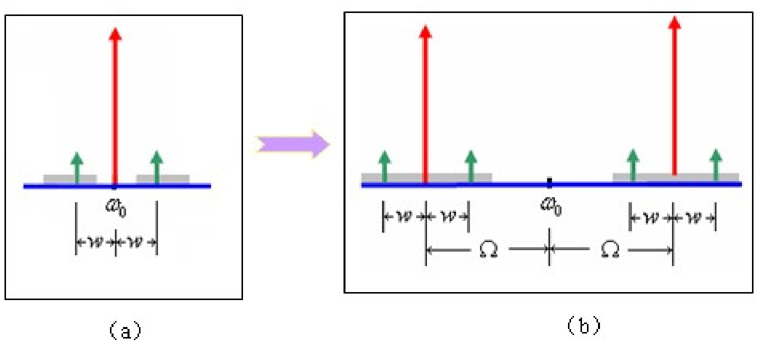

Fig. 2. Frequency relations of two-frequency intense laser, squeezing and signal sidebands for: (a) usual interferometer; (b) our scheme. w is the low frequency to be measured. The shadowed areas are entangled sidebands of the squeezed state. Long arrows are carrier frequencies of the intense laser and the short arrows are signal sidebands.

scheme and our scheme are shown in Fig. 2. Suppose the two-frequency laser and squeezed state have annihilation operators $\hat{A}=\hat{a}_+\exp\left[-i(\omega_0+\Omega)t\right]+\hat{a}_-\exp\left[-i(\omega_0-\Omega)t\right]$ and $\hat{B}=\hat{b}\exp\left(-i\omega_0 t\right)$ respectively, where ω_0 is the optical circular frequency and Ω is the circular frequency, at which squeezing occurs. With the mean-field approximation, the

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annihilation operators $\hat{a}_+$, $\hat{a}_-$ and $\hat{b}$ can be expressed as the sum of its mean field and fluctuation, i.e. $\hat{a}_i = \alpha_i + \delta\hat{a}_i \quad (i = +,-)$, and $\hat{b} = \delta\hat{b}$. Here, squeezing is supposed to be vacuum-squeezing, so that the corresponding mean field is zero. The mean amplitudes $\alpha_+ = \alpha_- = \alpha$ are real numbers by choosing a proper phase reference and are supposed to be equal. The average photon numbers in unit measurement time is $N = 2\alpha^2 = P/\hbar\omega_0$, where P is the optical power of the intense two-frequency laser. With the relative phase φ at the first 50% beam splitter BS1 and relative phase $\pi/2+\theta(t)$ at the second 50% beam splitter BS2 in the Mach-Zehnder interferometer, the annihilation operators of the various modes in Fig. 1 are related to each other via

$$\hat{C} = \frac{1}{\sqrt{2}}\left[\hat{A} + \hat{B}\exp\left(i\varphi\right)\right], \quad (1)$$

$$\hat{D} = \frac{1}{\sqrt{2}}\left[\hat{A} - \hat{B}\exp\left(i\varphi\right)\right], \quad (2)$$

and

$$\begin{aligned}\hat{E} &= \frac{1}{\sqrt{2}}\left[\hat{C} + \hat{D}\exp\left(i\pi/2 + i\theta(t)\right)\right] \\ &= \frac{1}{2}\left\{\left[1+\exp\left(i\pi/2+i\theta(t)\right)\right]\hat{A} + \left[1-\exp\left(i\pi/2+i\theta(t)\right)\right]\hat{B}e^{i\varphi}\right\},\end{aligned} \quad (3)$$

$$\begin{aligned}\hat{F} &= \frac{1}{\sqrt{2}}\left[\hat{C} - \hat{D}\exp\left(i\pi/2 + i\theta(t)\right)\right] \\ &= \frac{1}{2}\left\{\left[1-\exp\left(i\pi/2+i\theta(t)\right)\right]\hat{A} + \left[1+\exp\left(i\pi/2+i\theta(t)\right)\right]\hat{B}\exp\left(i\varphi\right)\right\},\end{aligned} \quad (4)$$

where $\theta(t) = \sum_s \theta_{w_s}\cos w_s t$ is a sum of the low-frequency cosine signal $\theta_{w_1}\cos w_1 t$ to be measured at frequency w_1 and some spurious signals which could be coupled to the photocurrent, where θ_{w_s} is the signal amplitude at frequency w_s. The frequency spacing is much larger than the measurement resolution bandwidth $\Delta\omega$. The quantum efficiency of photodiodes is supposed to be unity, so that the subtracted output photocurrent deduced from Eqs. (1)–(4) is

$$\begin{aligned}\hat{i}(t) &= \hat{F}^\dagger\hat{F} - \hat{E}^\dagger\hat{E} \\ &= N\sum_s \theta_{w_s}\cos w_s t\left(1+\cos 2\Omega t\right) + \sqrt{2N}\,\delta\hat{X}_b^{\varphi+\pi/2}(t)\cos\Omega t \\ &= s(t) + n(t),\end{aligned} \quad (5)$$

where the quadrature fluctuations are defined as $\delta\hat{X}_{\mathrm{b}}^{\varphi} = \delta\hat{b}e^{i\varphi} + \delta\hat{b}^\dagger e^{-i\varphi}$, and the product terms of quadrature fluctuations and the terms multiplying $\theta(t)$ and quadrature fluctuations are omitted for $\theta \ll 1$ and $\delta X_i^{\varphi} \ll \alpha \quad (i = +,-,b)$. We take the photocurrent duration as T ($1/T \ll w_s$), which is the reciprocal of measurement resolution bandwidth $\Delta\omega/2\pi$. The photocurrent $i(t)$ is separated into a signal part $s(t)$ and noise part $n(t)$, and the power spectral densities are calculated separately. By using the Wiener-Khinchine theorem, the power spectral density of the first term of Eq. (5), i.e. the signal term, is

$$\begin{aligned}
P_s(w) &= \int_{-\infty}^{\infty} d\tau e^{-iw\tau} R(\tau) = \frac{1}{T}\int_{-\infty}^{\infty} d\tau e^{-iw\tau}\int_{-T/2}^{T/2} dts(t)s(t-\tau) \\
&= \frac{\pi N^2}{2}\sum_s \left(2\theta_{w_s}^2 + \theta_{w_s}\theta_{w_s-2\Omega} + \theta_{w_s}\theta_{w_s+2\Omega}\right)\left[\delta(w+w_s)+\delta(w-w_s)\right] \\
&+ \frac{\pi N^2}{4}\sum_s \left(\theta_{w_s}^2 + \theta_{w_s}\theta_{w_s+4\Omega} + 2\theta_{w_s}\theta_{w_s+2\Omega}\right)\left[\delta(w+w_s+2\Omega)+\delta(w-w_s-2\Omega)\right] \\
&+ \frac{\pi N^2}{4}\sum_s \left(\theta_{w_s}^2 + \theta_{w_s}\theta_{w_s-4\Omega} + 2\theta_{w_s}\theta_{w_s-2\Omega}\right)\left[\delta(w-w_s+2\Omega)+\delta(w+w_s-2\Omega)\right],
\end{aligned} \tag{6}$$

where we use the approximation that $\sin\omega T/(\omega T)\approx 1$ when $\omega = 0$ and $\sin\omega T/(\omega T)\approx 0$ when $\omega \neq 0$. Function $R(\tau) = (1/T)\int_{-T/2}^{T/2} dts(t)s(t-\tau)$ is the autocorrelation function of signal $s(t)$. Function $\delta(w-w_1)$ is the Dirac delta function defined as $\int_{-\infty}^{\infty}\mathrm{dte}^{\text{-iwt}} = 2\pi\delta(w)$, and satisfies $\int_{w_1-\Delta\omega/2}^{w_1+\Delta\omega/2}\mathrm{d}w\delta(w-w_1)=1$. Also, $\theta_{w_s} = \theta_{-w_s}$ is used. The noise power spectral density of the second term on the other side of Eq. (5), i.e. the noise term, is

$$\begin{aligned}
P_n(w) &= \left\langle \frac{1}{T}\int_{-\infty}^{\infty} d\tau e^{-iw\tau}\int_{-T/2}^{T/2} dtn(t)n(t-\tau)\right\rangle_{en} \\
&= \left\langle \left|\mathbf{F}_T\{n(t)\}\right|^2\right\rangle_{en}\Big/T \\
&= \frac{N}{2}\left\langle \int_{-T/2}^{T/2} dt\delta X_b^{\varphi+\pi/2}(t)\left(e^{-i(w-\Omega)t}+e^{-i(w+\Omega)t}\right)\int_{-T/2}^{T/2} dt'\delta X_b^{\varphi+\pi/2}(t')\left(e^{i(w-\Omega)t'}+e^{i(w+\Omega)t'}\right)\right\rangle_{en}\Big/T \\
&= \frac{N}{2}\left[V_b^{\varphi+\pi/2}(w+\Omega)+V_b^{\varphi+\pi/2}(w-\Omega)\right],
\end{aligned} \tag{7}$$

where $\langle\ldots\rangle_{en}$ refers to the ensemble average, and $F_T\{n(t)\} = n_T(w) = \int_{-T/2}^{T/2} dtn(t)e^{-iwt}$ is the Fourier transform of function $n(t)$. $V_b^{\varphi+\pi/2}(w+\Omega)$, defined as

$$V_b^{\varphi+\pi/2}(w+\Omega) = \left\langle\left|\delta X_b^{\varphi+\pi/2}(w+\Omega)\right|^2\right\rangle_{en}\Big/T = \left\langle\left|\int_{-T/2}^{T/2} dt\delta X_b^{\varphi+\pi/2}(t)e^{-i(w+\Omega)t}\right|^2\right\rangle_{en}\Big/T \tag{8}$$

is the quadrature variance of the squeezed state at frequency $w+\Omega$. To evaluate the SNR at frequency w_1, we integrate the signal spectral density (Eq. (6)) and noise spectral density (Eq. (7)) in the frequency interval $\left[w_1-\Delta\omega/2,\ \ w_1+\Delta\omega/2\right]$, and take the ratio as the SNR:

$$\begin{aligned}
\mathrm{P}_s(w_1) &= \int_{w_1-\Delta\omega/2}^{w_1+\Delta\omega/2}\frac{dw}{2\pi}\mathrm{P}_s(w) \\
&= N^2\left(\theta_{w_1} + \theta_{w_1+2\Omega}/2 + \theta_{w_1-2\Omega}/2\right)^2,
\end{aligned} \tag{9}$$

and

$$\begin{aligned}
\mathrm{P}_n(w_1) &= \int_{w_1-\Delta\omega/2}^{w_1+\Delta\omega/2}\frac{dw}{2\pi}\mathrm{P}_n(w) \\
&= \frac{N\Delta\omega}{4\pi}\left[V_b^{\varphi+\pi/2}(w_1+\Omega)+V_b^{\varphi+\pi/2}(w_1-\Omega)\right].
\end{aligned} \tag{10}$$

Therefore

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$$SNR = \frac{P_s(w_1)}{P_n(w_1)} = \frac{2TN\left(\theta_{w_1} + \theta_{w_1+2\Omega}/2 + \theta_{w_1-2\Omega}/2\right)^2}{V_b^{\varphi+\pi/2}(w_1+\Omega) + V_b^{\varphi+\pi/2}(w_1-\Omega)}. \tag{11}$$

The integration of Eq. (10) takes the quadrature variances to be uniform in the integral frequency interval. Equations (9) and (11) show that the output photocurrent at frequency w_1 could be contaminated by the phase vibrations at frequencies $w_1 - 2\Omega$ and $w_1 + 2\Omega$, and it may not faithfully reflect the phase signal at w_1. However, frequency Ω is usually chosen to be at Megahertz so that the phase vibrations at $w_1 - 2\Omega$ and $w_1 + 2\Omega$ is rare in real experiment. The SNR of Eq. (11) can be simplified to

$$SNR = \frac{2NT\theta_{w_1}^2}{V_b^{\varphi+\pi/2}(w_1+\Omega) + V_b^{\varphi+\pi/2}(w_1-\Omega)}. \tag{12}$$

In this case, the photocurrent at frequency w_1 does reflect the phase vibration at w_1 and has sub-shot noise resolution if the quadrature variables of the squeezed state at the angle $\varphi + \pi/2$ are squeezed at frequencies $w_1 - \Omega$ and $w_1 + \Omega$.

The physics of this scheme can be explained as follows. Instead of using a single-frequency intense laser at frequency ω_0 in the interferometry, we use an intense laser with the carrier frequencies $\omega_0 - \Omega$ and $\omega_0 + \Omega$, with which the entangled upper and lower sidebands of vacuum squeezing interfere. Therefore, the low-frequency signal, which generates sidebands near the carrier fields, can be detected with sub-shot noise resolution, thanks to the entangled sidebands of squeezing. The terms with phase signals at frequencies $w_1 - 2\Omega$ and $w_1 + 2\Omega$ coupled to the SNR at frequency w_1 in Eq. (11) originate from the beating between one of the carriers and the sidebands at $w_1 - 2\Omega$ and $w_1 + 2\Omega$ of the other carrier. It's interesting to note that in Eq. (11) the signal appears at frequency w_1 and $w_1 \pm 2\Omega$ while the noise at frequency $w_1 \pm \Omega$. This difference can be attributed to the fact that, in Mach-Zehnder interferometer, signal sidebands are generated around carrier fields while noise sidebands come from the injected squeezing.

Conclusion

We have calculated the utility of high-frequency squeezed-state enhanced two-frequency interferometry for low-frequency phase measurement. By means of a two-frequency laser interferometer, the higher-frequency sidebands of the squeezed state can be used to enhance the lower-frequency phase measurement. A proof-of-principle experiment is in the reach of modern quantum optics technology and is in progress in our laboratory. Moreover, this scheme is also useful for many other squeezing-enhanced measurement schemes, and also provides a method to generate low-frequency squeezing with high-frequency squeezing.

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