Generating multiplexed entanglement frequency comb in a nondegenerate optical parametric amplifier

Rongguo Yang,[1,2,*] Jing Zhang,[1] Shuqin Zhai,[1] Kui Liu,[2] Junxiang Zhang,[2] and Jiangrui Gao[1,2]

[1]*College of Physics and Electronics Engineering, Shanxi University, Taiyuan 030006, China*
[2]*State Key Laboratory of Quantum Optics and Quantum Optics Devices, Shanxi University, Taiyuan 030006, China*
**Corresponding author: yrg@sxu.edu.cn*

Received June 26, 2012; revised October 30, 2012; accepted November 13, 2012;
posted December 13, 2012 (Doc. ID 171426); published January 9, 2013

In this paper, we propose a scheme to produce a multiplexed entanglement frequency comb in a type II phase-matching nondegenerate optical parametric amplifier (NOPA) operating below threshold. The entanglement of the signal and idler frequency combs in the wide frequency range, which is limited by the phase-matching bandwidth of the NOPA, is investigated by the inseparability criterion. Furthermore, N Einstein–Podolsky–Rosen pairs can be created from the two combs without any reduction in the correlations by using frequency-dependent beam splitters.

OCIS code: 270.2500, 270.0270.

1. INTRODUCTION

With the development of science and technology, the unique properties based on isolated quantum systems are going to be the key ingredients for advanced quantum communication of quantum-key distribution [1,2], ultrahigh-precision sensing [3,4], quantum computers [5], and quantum encoding. In real applications, these valuable quantum resources present low noise, small wastage, high intelligence bandwidth, and the ability to encode information into multiple degrees of freedom [6]. We know that key resources required in quantum optical systems are nonclassical states of light, which are commonly generated via nonlinear optical processes. One of such process is parametric downconversion, in which a high-energy pump photon is converted into two lower-energy photons by a second-order nonlinear interaction [7]. Devices of optical parametric, as sources of entangled photon pairs in discrete-variable systems [8] and squeezed states in continuous-variable systems, are widely used in quantum optics experiments [9]. Therefore, researchers have demonstrated many schemes to produce the functional quantum resources based on the optical parametric processes. One of such field is the optical frequency comb, which has been investigated in recent years. A broadband optical frequency comb has been generated by a parametric oscillator with an intracavity electro-optic phase modulator [10]. Valcárcel *et al.* demonstrated theoretically that multimode squeezing of frequency combs can be produced in a type I optical parametric oscillator pumped by a pulse laser [11]. The scheme to produce a continuous-variable pair-entanglement frequency comb by enhanced Raman scattering in an optical oscillator cavity has been proposed [12]. A frequency comb of squeezing was theoretically predicted in an optical parametric oscillator [13]. These schemes can be applied in quantum information and fundamental science because of the importance of frequency standards [14]. Recently, Pinel *et al.* have reported the first experimental evidence of a multimode nonclassical frequency comb in a femtosecond synchronously pumped optical parametric oscillator [15]. The scheme for mode-locked biphoton frequency-comb generation by engineering quasi-phase-matching materials has been proposed [16].

In this paper, we present that the signal comb and the idler comb are simultaneously generated in a nondegenerate optical parametric amplifier (NOPA). Nonclassical correlation between the corresponding teeth of the signal and idler combs is found because a strong squeezing exists in the amplitude sum and phase difference of two combs. The multiplexed entanglements of two combs are investigated by the inseparability criterion proposed by Duan. N Einstein–Podolsky–Rosen (EPR) pairs can be created from the combs by using frequency-dependent beam splitters (FDBS). Furthermore, the low-frequency technical noise of the seed has no effect on the correlation of high-order teeth.

2. OPTICAL FREQUENCY-COMB GENERATION

NOPA is one of the best sources of entangled light in the so-called continuous-variable regime. It is considered the generator for a frequency comb, which consists of a two-sided cavity. Input modes a_0, $b_1^{\rm in}$, and $b_2^{\rm in}$, intracavity modes a_0, a_1, and a_2, signal mode $A_1^{\rm out}$, and idler mode $A_2^{\rm out}$ are displayed in Fig. 1. Here, $i = 0, 1, 2$ correspond to the pump field, signal, and idler fields for the expression of a_i, b_i, and $A_i^{\rm out}$, respectively.

We consider the optical parametric process in the triply resonating optical cavity with a nonlinear type II phase-matching $\chi^{(2)}$ crystal.

The interaction Hamiltonian of the optical parametric process in this NOPA is

$$H = i\hbar\chi^{(2)}(\hat{a}_0^\dagger \hat{a}_1 \hat{a}_2 - \hat{a}_0 \hat{a}_1^\dagger \hat{a}_2^\dagger). \tag{1}$$

In the ideal case—perfect phase matching and without any detuning—the equations of motion for the signal and idler modes can be expressed as

0740-3224/13/020314-05$15.00/0

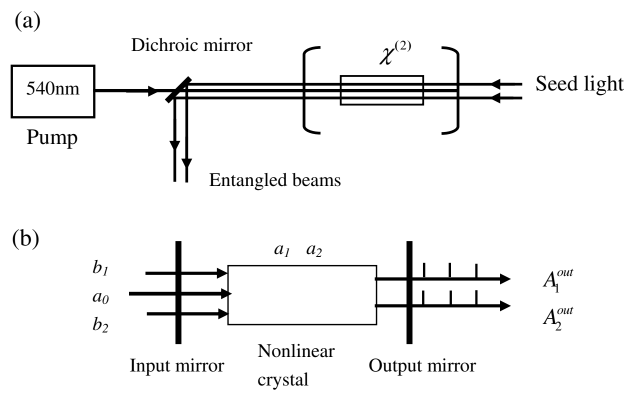

Fig. 1. Multiplexed entanglement frequency-comb generation from a NOPA. (a) Layout of quantum quadrature entanglement system. (b) Detailed project of NOPA as an optical frequency-comb generator.

$$\dot{\hat{a}}_1 = -k_1\hat{a}_1 - \chi^{(2)}\hat{a}_0\hat{a}_2^\dagger + \sqrt{2k_1}\hat{b}_1^{\text{in}}(t), \quad (2)$$

$$\dot{\hat{a}}_2 = -k_2\hat{a}_2 - \chi^{(2)}\hat{a}_0\hat{a}_1^\dagger + \sqrt{2k_2}\hat{b}_2^{\text{in}}(t). \quad (3)$$

Here $\hat{a}_0$, $\hat{a}_1$, and $\hat{a}_2$ are the amplitude operators of intracavity modes a_0, a_1, and a_2, and $\hat{b}_1^{\text{in}}$ and $\hat{b}_2^{\text{in}}$ denote the amplitude operators of input fields. $\chi^{(2)}$ is the effective nonlinear coupling parameter. It can be merged into pump-field amplitude α_0 and considered as χ. The cavity decay rate for two modes is assumed to be equal. $k_1 = k_2 = k$. Operator equations of cavity mode $\hat{a}_i$ after a single cavity round trip can be written as

$$\hat{a}_1(t+\tau) = -\chi\tau\hat{a}_2^\dagger + (1-k\tau)\hat{a}_1 + \sqrt{2k\tau}\hat{b}_1^{\text{in}}(t), \quad (4)$$

$$\hat{a}_2(t+\tau) = -\chi\tau\hat{a}_1^\dagger + (1-k\tau)\hat{a}_2 + \sqrt{2k\tau}\hat{b}_2^{\text{in}}(t). \quad (5)$$

Here, τ is a cavity round-trip time. It is convenient to separate the field operators into the mean value and quantum fluctuations in the form $\hat{a}_i = \alpha_i + \delta\hat{a}_i$ in the steady-state condition; thus we get the equation for the operator fluctuations as

$$\delta\hat{a}_1(t+\tau) = -\chi\tau\delta\hat{a}_2^\dagger + (1-k\tau)\delta\hat{a}_1 + \sqrt{2k\tau}\delta\hat{b}_1^{\text{in}}(t), \quad (6)$$

$$\delta\hat{a}_2(t+\tau) = -\chi\tau\delta\hat{a}_1^\dagger + (1-k\tau)\delta\hat{a}_2 + \sqrt{2k\tau}\delta\hat{b}_2^{\text{in}}(t). \quad (7)$$

The Fourier transforms of Eqs. (6) and (7) are $\delta\hat{a}(t+\tau) \to \delta\hat{a}(\omega)e^{i\omega\tau}$, $\delta\hat{a}(t) \to \delta\hat{a}(\omega)$. So we obtain the equations in the frequency domain as

$$\delta\hat{a}_1(\omega)[e^{i\omega\tau} - (1-k\tau)] = -\chi\tau\delta\hat{a}_2^\dagger(\omega) + \sqrt{2k\tau}\delta\hat{b}_1^{\text{in}}(\omega), \quad (8)$$

$$\delta\hat{a}_2(\omega)[e^{i\omega\tau} - (1-k\tau)] = -\chi\tau\delta\hat{a}_1^\dagger(\omega) + \sqrt{2k\tau}\delta\hat{b}_2^{\text{in}}(\omega). \quad (9)$$

By the complex calculation for Eqs. (8) and (9), we can get the operator fluctuations of intracavity fields as

$$\delta\hat{a}_1(\omega) = \frac{\sqrt{2k}\tau\delta\hat{b}_1^{\text{in}}(\omega)[e^{i\omega\tau} - (1-k\tau)] - \sqrt{2k}\chi\tau^2\delta\hat{b}_2^{\text{in}\dagger}(\omega)}{[e^{i\omega\tau} - (1-k\tau)]^2 - \chi^2\tau^2}, \quad (10)$$

$$\delta\hat{a}_2(\omega) = \frac{\sqrt{2k}\tau\delta\hat{b}_2^{\text{in}}(\omega)[e^{i\omega\tau} - (1-k\tau)] - \sqrt{2k}\chi\tau^2\delta\hat{b}_1^{\text{in}\dagger}(\omega)}{[e^{i\omega\tau} - (1-k\tau)][e^{i\omega\tau} - (1-k\tau)] - \chi^2\tau^2}. \quad (11)$$

Then the operator fluctuations of the output fields ($\delta A_1^{\text{out}}(\omega)$, $\delta A_2^{\text{out}}(\omega)$) can be obtained by using the boundary condition [17]

$$\hat{A}_{1(2)}^{\text{out}} = \sqrt{2k}\hat{a}_{1(2)} - \hat{b}_{1(2)}^{\text{in}}. \quad (12)$$

We get

$$\delta\hat{A}_1^{\text{out}}(\omega) = \frac{\left[\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)\left(k + \frac{1-e^{i\omega\tau}}{\tau}\right) + \chi^2\right]\delta\hat{b}_1^{\text{in}}(\omega) - 2k\chi\delta\hat{b}_2^{\text{in}\dagger}(\omega)}{\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)^2 - \chi^2}, \quad (13)$$

$$\delta\hat{A}_2^{\text{out}}(\omega) = \frac{\left[\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)\left(k + \frac{1-e^{i\omega\tau}}{\tau}\right) + \chi^2\right]\delta\hat{b}_2^{\text{in}}(\omega) - 2k\chi\delta\hat{b}_1^{\text{in}\dagger}(\omega)}{\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)^2 - \chi^2}. \quad (14)$$

Using the definitions of the amplitude and phase quadrature fluctuations ($\delta\hat{X}^+$, $\delta\hat{X}^-$) of the output and input beams,

$$\delta\hat{X}_{1(2)}^+(\omega) = \delta\hat{A}_{1(2)}^{\text{out}}(\omega) + \delta\hat{A}_{1(2)}^{\text{out}}(-\omega)^\dagger, \quad (15)$$

$$\delta\hat{X}_{1(2)}^-(\omega) = i[\delta\hat{A}_{1(2)}^{\text{out}}(\omega) - \delta\hat{A}_{1(2)}^{\text{out}}(-\omega)^\dagger], \quad (16)$$

$$\delta\hat{X}_{1(2)}^{\text{in}+}(\omega) = \delta\hat{b}_{1(2)}^{\text{in}}(\omega) + \delta\hat{b}_{1(2)}^{\text{in}}(-\omega)^\dagger, \quad (17)$$

$$\delta\hat{X}_{1(2)}^{\text{in}-}(\omega) = i[\delta\hat{b}_{1(2)}^{\text{in}}(\omega) - \delta\hat{b}_{1(2)}^{\text{in}}(-\omega)^\dagger], \quad (18)$$

we can obtain the amplitude and phase quadrature variances of signal and idler beams by the formulas $V_{1(2)}^{\text{out}\pm}(\omega) = \langle|\delta\hat{X}_{1(2)}^\pm|^2\rangle$ and $V_{1(2)}^{\text{in}\pm}(\omega) = \langle|\delta\hat{X}_{1(2)}^{\text{in}\pm}|^2\rangle$ as

$$V_1^{\text{out}\pm}(\omega) = \left|\frac{k^2 + \chi^2 - \left(\frac{1-e^{i\omega\tau}}{\tau}\right)^2}{\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)^2 - \chi^2}\right|^2 V_1^{\text{in}\pm} + \left|\frac{2k\chi}{\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)^2 - \chi^2}\right|^2 V_2^{\text{in}\pm}, \quad (19)$$

$$V_2^{\text{out}\pm}(\omega) = \left|\frac{k^2 + \chi^2 - \left(\frac{1-e^{i\omega\tau}}{\tau}\right)^2}{\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)^2 - \chi^2}\right|^2 V_2^{\text{in}\pm} + \left|\frac{2k\chi}{\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)^2 - \chi^2}\right|^2 V_1^{\text{in}\pm}. \quad (20)$$

The parameters are given as follows: $V_{1(2)}^{\text{in}\pm}(\omega) = \langle|\delta\hat{X}_{1(2)}^{\text{in}\pm}|^2\rangle = 1$, the free spectral range (FSR) Ω is 10 GHz, and the transmission rate of the output coupler is $t = 5\%$ for signal and idler beams. Using these parameters, we get all the figures in this paper. The combs about amplitude and phase quadrature noise spectra of signal and idler modes versus the analysis frequency are given in Fig. 2. As a consequence, the "up" combs of signal and idler modes are

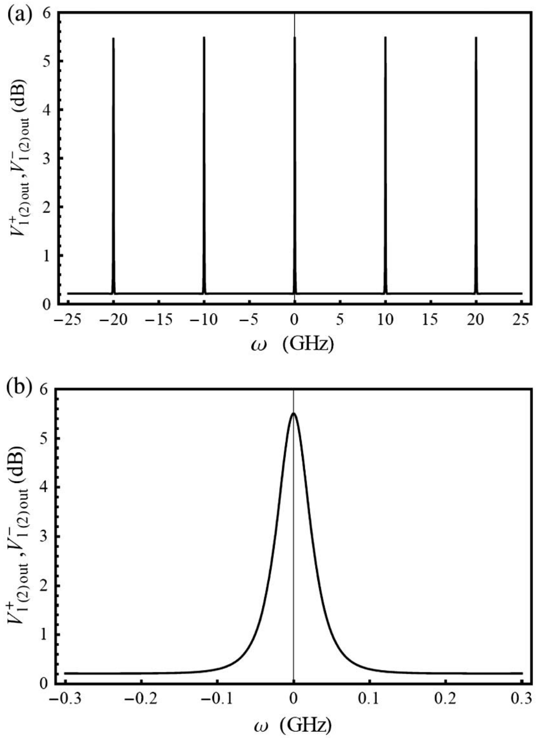

Fig. 2. Same amplitude and phase quadrature variances for the signal and idler modes, corresponding to the same comb. (a) Two sets of "up" frequency combs above the short noise level. (b) Zoom for the zeroth-order comb.

produced at low Fourier frequencies, as expected, and also at every subsequent resonance band of the NOPA.

Then we consider the correlation of amplitude and phase quadrature variances (the sum of the amplitude quadrature and the difference of the phase quadrature) for the two output modes. By using Eqs. (15)–(18), we get

$$V^+_{1\text{out}+2\text{out}}(\omega) = \left| \frac{(k-\chi)^2 - \left(\frac{1-e^{i\omega\tau}}{\tau}\right)^2}{\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)^2 - \chi^2} \right|^2 V_1^{\text{in}+} + \left| \frac{(k-\chi)^2 - \left(\frac{1-e^{i\omega\tau}}{\tau}\right)^2}{\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)^2 - \chi^2} \right|^2 V_2^{\text{in}+}, \tag{21}$$

$$V^-_{1\text{out}-2\text{out}}(\omega) = \left| \frac{(k-\chi)^2 - \left(\frac{1-e^{i\omega\tau}}{\tau}\right)^2}{\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)^2 - \chi^2} \right|^2 V_1^{\text{in}-} + \left| \frac{(k-\chi)^2 - \left(\frac{1-e^{i\omega\tau}}{\tau}\right)^2}{\left(k - \frac{1-e^{i\omega\tau}}{\tau}\right)^2 - \chi^2} \right|^2 V_2^{\text{in}-}. \tag{22}$$

The correlation variances of the sum of amplitude quadratures and the difference of phase quadratures for the output modes versus the analysis frequency are shown in Fig. 3. The correlated frequency combs can be produced at every resonance band of the NOPA cavity.

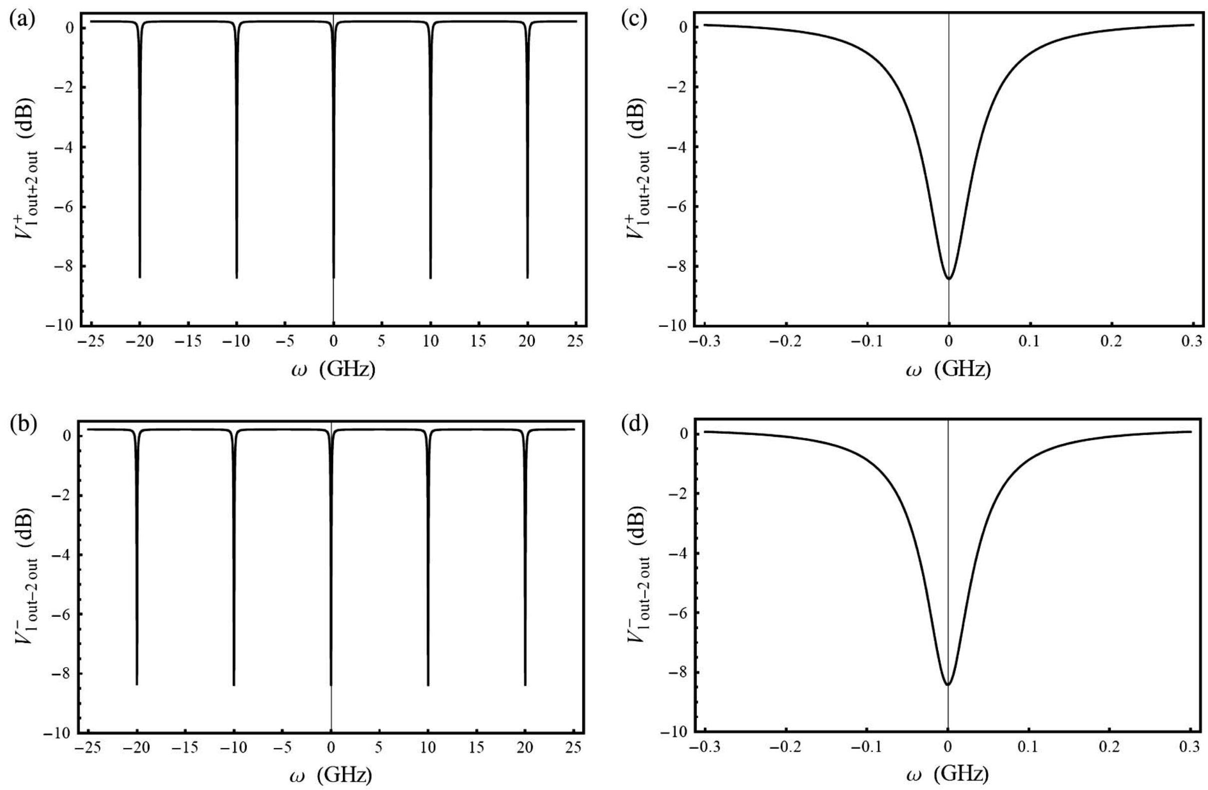

Fig. 3. Correlation spectra of amplitude and phase quadrature variances for the two output modes (signal and idler beams), which are below shot noise level. (a) Amplitude quadrature sum $V^+_{1\text{out}+2\text{out}}(\omega)$. (b) Phase quadrature difference $V^-_{1\text{out}-2\text{out}}(\omega)$. (c), (d) Zooms of the zeroth-order correlation for amplitude and phase quadrature variances.

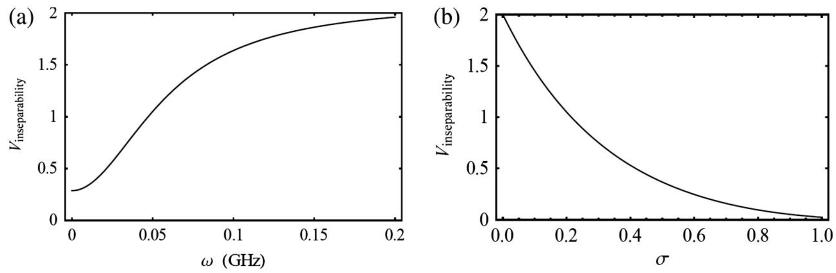

Fig. 4. (a) $V_{inseparability}$ versus analysis frequency ω. (b) $V_{inseparability}$ versus pump coefficient σ at the center of one random resonance band frequency.

In addition, the technical noise of the seed beam is larger than the quantum-noise limit inevitably when ω is low. But the extra noise only reduces the correlation of the zeroth-order comb [13].

3. MULTIPLEXED ENTANGLEMENT

The whole bandwidth of the comb is limited by the phase-matching bandwidth of the NOPA. The optical cavity, which is used to increase the strength of the nonlinear process, leads to spectral filtering of the downconverted output. The spectrum is separated in frequency by the cavity FSR. The downconversion bandwidth in a bulk nonlinear material of length $L_{crystal}$ can be estimated to be $10c/L_{crystal}$ (c is the speed of light). The FSR of a cavity with length L_{cavity} is $c/2L_{cavity}$ [18]. If we assume $L_{cavity} = 15$ mm, $L_{crystal} = 5$ mm, the result of $N = 60$ can be obtained. Sixty pairs of frequency can be excited. We can assume the phase-matching bandwidth of the crystal is much broader than the FSR.

The entanglement can be quantified according to the inseparability criterion [19] $V_{inseparability} = \Delta^2 X^+_{1+2}(N\Omega + \omega) + \Delta^2 X^-_{1-2}(N\Omega + \omega) < 2$, where $\Delta^2 X^\pm_{1\pm2}(N\Omega + \omega)$ is the minimum variance of the sum or difference for the quadratures between the Nth-order resonance band of two combs. The inseparability coefficients $V_{inseparability}$ versus ω and $V_{inseparability}$ versus pump coefficient σ are given in Fig. 4. Here, χ_{th} is the pump threshold and $\sigma = \chi/\chi_{th}$. It is obvious that are smaller than 2 in the range of 0.2 GHz and on a subthreshold condition. Additionally, the largest correlation degree can be obtained at the resonance frequency and near threshold of the NOPA. In a word, multiplexed entanglement indeed exists between the signal and idler fields.

Figure 5 shows N EPR pairs can be produced from the multiplexed entanglement comb by using a polarization beam splitter and $2N$ FDBSs. For example, a high-finesse mode cleaner [20] or an unbalanced Mach–Zehnder interferometer [21] can be used to spatially separate the quantum sidebands of an optical field.

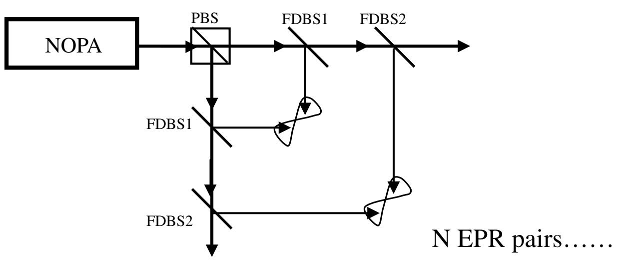

Fig. 5. Schematic illustration of N EPR entanglement-pair generation by way of linear optics from the two combs using a polarization beam splitter (PBS) and $2N$ FDBS.

4. CONCLUSION

In conclusion, we have theoretically investigated the new and special properties of signal and idler beams generated by an optic parametric process. A multiplexed entanglement frequency comb can be generated from a NOPA. And we believe that such an optical device and scheme might be used to provide a method for multiple degrees of freedom encoding, multichannel quantum-key distribution, multimode parallel quantum information processing, and multipartite optimal clone states, and can also be helpful for quantum storage with a comb field of an atom transition line. It will be very valuable in quantum information.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant Nos. 61108003, 61008006, and 11274210) and the State Key Basic Research and Development Plan of China (Grant No. 2010CB923102).

REFERENCES

1. N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, “Quantum cryptography,” Rev. Mod. Phys. **74**, 145–195 (2002).
2. F. Grosshans, G. Van Assche, J. Wenger, R. Brouri, N. J. Cerf, and P. Grangier, “Quantum key distribution using Gaussian-modulated coherent states,” Nature **421**, 238–241 (2003).
3. C. M. Caves, “Quantum-mechanical noise in an interferometer,” Phys. Rev. D **23**, 1693–1708 (1981).
4. B. J. Meers and K. A. Strain, “Modulation, signal, and quantum noise in interferometers,” Phys. Rev. A **44**, 4693–4703 (1991).
5. M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University, 2000).
6. R. J. Senior, G. N. Milford, J. Janousek, A. E. Dunlop, K. Wagner, H.-A. Bachor, T. C. Ralph, E. H. Huntington, and C. C. Harb, “Observation of a comb of optical squeezing over many gigahertz of bandwidth,” Opt. Express **15**, 5310–5317 (2007).
7. R. W. Boyd, *Nonlinear Optics* (Academic, 1992).
8. P. G. Kwiat, K. Mattle, H. Weinfurter, and A. Zeilinger, “New high-intensity source of polarization-entangled photon pairs,” Phys. Rev. Lett. **75**, 4337–4341 (1995).
9. L. A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, “Generation of squeezed states by parametric down conversion,” Phys. Rev. Lett. **57**, 2520–2523 (1986).
10. S. A. Diddams, L.-S. Ma, J. Ye, and J. L. Hall, “Broadband optical frequency comb generation with a phase-modulated parametric oscillator,” Opt. Lett **24**, 1747–1749 (1999).
11. G. J. de Valcárcel, G. Patera, N. Treps, and C. Fabre, “Multimode squeezing of frequency combs,” Phys. Rev. A **74**, 061801 (2006).
12. Y. B. Yu, S. N. Zhu, X. Q. Yu, P. Xu, J. F. Wang, Z. D. Xie, and H. Y. Leng, “Continuous-variable pair-entanglement frequency comb generated from an optical superlattice by enhanced Raman scattering,” Phys. Rev. A **77**, 032317 (2008).

13. A. E. Dunlop and E. H. Huntington, "Generation of a frequency comb of squeezing in an optical parametric oscillator," Phys. Rev. A **73**, 013817 (2006).
14. D. Meiser, J. Ye, D. R. Carlson, and M. J. Holland, "Prospects for a millihertz-linewidth laser," Phys. Rev. Lett **102**, 163601 (2009).
15. O. Pinel, P. Jian, R. Medeiros de Araujo, J. Feng, B. Chalopin, C. Fabre, and N. Treps, "Generation and characterization of multimode quantum frequency combs," Phys. Rev. Lett **108**, 083601 (2012).
16. Y. F. Bai, P. Xu, Z. D. Xie, Y. X. Gong, and S. N. Zhu, "Mode-locked biphoton generation by concurrent quasi-phase-matching," Phys. Rev. A **85**, 053807 (2012).
17. M. J. Collett and C. W. Gardiner, "Squeezing of intracavity and traveling-wave light fields produced in parametric amplification," Phys. Rev. A **30**, 1386–1391 (1984).
18. M. G. Raymer, J. Noh, K. Banaszek, and I. A. Walmsley, "Pure-state single-photon wave-packet generation by parametric down-conversion in a distributed microcavity," Phys. Rev. A **72**, 023825 (2005).
19. L.-M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, "Inseparability criterion for continuous variable systems," Phys. Rev. Lett. **84**, 2722–2725 (2000).
20. J. Zhang, "Einstein-Podolsky-Rosen sideband entanglement in broadband squeezed light," Phys. Rev. A **67**, 054302 (2003).
21. E. H. Huntington, G. N. Milford, C. Robilliard, and T. C. Ralph, "Demonstration of the spatial separation of the entangled quantum sidebands of an optical field," Phys. Rev. A **71**, 041802 (2005).