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Small-displacement measurements using high-order Hermite-Gauss modes

Hengxin Sun, Kui Liu, Zunlong Liu, Pengliang Guo, Junxiang Zhang, and Jiangrui Gao

State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan 030006, China

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We present a scheme for small-displacement measurements using high-order Hermite-Gauss modes and balanced homodyne detection. We demonstrate its use with experimental results of displacement measurements using fundamental transverse mode TEM$_{00}$ and first order transverse mode TEM$_{10}$ as signal modes. The results show a factor of 1.41 improvement in measurement precision with the TEM$_{10}$ mode compared with that with the TEM$_{00}$ mode. This scheme has potential applications in precision metrology, atomic force microscopy, and optical imaging. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4869819]

Optical displacement measurements are one of several basic techniques in the precision measurement toolbox. Even for very high precision measurements, they have been widely used in many areas such as atomic force microscopy, optical tweezers, optical imaging, and even gravitational wave detection. In general, measurement precision is limited ultimately by quantum noise and for that reason has become a very challenging field. Two methods have been used to improve the signal-to-noise ratio (SNR) or precision. One involves reducing the quantum noise using spatial squeezing and entanglement techniques, and the other involves enhancing the detection efficiency by optimizing the detection setup. For example, homodyne detection has been proved to be better than split detection in displacement measurements. Parallel research on weak-value measurements has also greatly improved the SNR of beam deflection measurements.

In this Letter, we present and demonstrate experimentally a scheme for optical displacement measurements based on high-order Hermite-Gauss modes and balanced homodyne detection (BHD). Displacement measurements with TEM$_{00}$ and TEM$_{10}$ modes as signal beams are realized. A factor of 1.41 improvement is obtained in measurement precision for the TEM$_{10}$ mode compared with that for the TEM$_{00}$ mode. The results obtained are equivalent to that with a 3-dB spatially squeezed light system although our scheme is the more practical and more robust to losses. Indeed, some applications based on high-order Hermite-Gauss modes have been reported in some areas such as optical imaging and tracking of single atoms.

There are two methods often used in beam displacement measurements: the split detection and balanced homodyne detection. The split detection is only 80% efficient compared with TEM$_{10}$ homodyne detection with a TEM$_{00}$ signal beam. Hence we shall only concentrate on the latter in this Letter.

Any small transverse displacement $d$ of a TEM$_{n,0}$ mode $u_n(x)$ leads to an excitation of the other order mode. Here, $n$ denotes the order of the $x$-axis Hermite-Gauss mode. Using a Taylor series expansion, we have

$$u_n(x + d) = u_n(x) + u'_n(x)d + \sum_{m=2}^{\infty} \frac{u^{(m)}_n(x)}{m!} d^m,$$

where $u'_n(x) = (\sqrt{2}/w_0) \left[ \sqrt{n/2}u_{n-1}(x) - \sqrt{n+1/2}u_{n+1}(x) \right]$ is the first derivative of $u_n(x)$ with respect to $x$ and $w_0$ denotes the beam waist of the TEM$_{00}$ mode. The excited modes are mostly the neighboring order modes for $(n+1)$ and $(n-1)$, which are included in the main displacement signal. It can be measured perfectly by BHD with the matched local-oscillator mode, which has normalized expression

$$u'_n^{LD}(x) = \sqrt{2/(2n + 1)} \left[ \sqrt{n/2}u_{n-1}(x) - \sqrt{n+1/2}u_{n+1}(x) \right].$$

After the BHD system, we obtain

$$\vec{F}^{BHD} = \sqrt{N_{Lo}} \left( 2\sqrt{2n + 1}\sqrt{Nd}/w_0 + \delta \vec{X}^+_x \right),$$

where $N$ and $N_{Lo}$ are, respectively, the signal and local-field mean photon number, $\delta \vec{X}^+_x$ is the quantum noise of the signal field, and $d$ is the displacement. In Eq. (3), the first term represents the signal, and the second represents noise. For coherent light ($\delta \vec{X}^+_x = 1$), the SNR is defined as $SNR^{BHD} = (2\sqrt{2n + 1}\sqrt{Nd})/w_0^2$. The minimum measurable displacement is, in general, defined as the displacement with SNR = 1; for the BHD measurement, it is given by

$$d_{min}^{BHD} = \frac{w_0}{2\sqrt{2n + 1}\sqrt{N}}.$$

Clearly, the higher the mode order is, the smaller the minimum measurable displacement is, and therefore the higher the measurement precision is. This will be important in displacement measurements, especially when the optical power density should not be too intense. Damage, for example, can occur to biological specimens if beams are too intense.
Indeed, the minimum measurable information of any parameter $\theta$ of an optical beam is ultimately constrained by the quantum Cramér-Rao (QCR) bound\textsuperscript{22}

$$\delta \theta \geq \delta \theta_{\min} = \frac{\sigma_{\min}}{\sqrt{QN_\theta}} \left[ 4\|u'_{0=0}\|^2 + \left( \frac{N'_{\theta}}{N_\theta} \right)^{1/2} \right], \quad (5)$$

where $\delta \theta_{\min}$ is the error limit in the estimation $\hat{\theta}$, $u'_{0=0}$ is the derivative of the normalized transverse field distribution $u_0$ with respect to $\theta$ at $\theta = 0$, $N_{\theta}$ is the mean photon number for single measurement, $N'_{\theta}$ is its derivative with respect to $\theta$, $Q$ is the number of measurement repetitions, and $\sigma_{\min}$ is quantum noise.

For displacement measurements using a high-order TEM$_{n,0}$ signal beam, $\theta$ is replaced by the displacement $d$, $u'_{0} = \partial[u_0(x + d)]/\partial[d]_{0=0}$, $N'_d = 0$, $\sigma_{\min} = 1$ as the coherent light is used, and $\|\partial[u_0(x + d)]/\partial[d]_{0=0}\|^2 = (2n + 1)/w_0^2$. We then obtain the QCR bound for the displacement measurement

$$d_{\text{QCR}} = \frac{w_0}{2\sqrt{2n + 1}/N}. \quad (6)$$

We can see from Eqs. (4) and (6) that the BHD scheme has reached the QCR bound limit. However, generating the local mode described in Eq. (2) in an experiment is a little complicated; hence, we preliminarily choose an intermediate mode TEM$_{n+1,0}$ for the local oscillator, i.e., $u^{\text{int,LO}}_n(x) = u_{n+1}(x)$. In the corresponding experiment, the BHD output and its SNR are easy to obtain

$$\hat{n}^\text{int,BHD}_- = \sqrt{N_{\text{LO}}}(2\sqrt{n + 1}/\sqrt{N}d/w_0 + \delta X_{s}^+), \quad (7)$$

$$\text{SNR}^\text{int,BHD} = (2\sqrt{n + 1}/\sqrt{N}d)^2/w_0^2. \quad (8)$$

The minimum measurable displacement with just the TEM$_{n+1,0}$ local-oscillator mode is then given by

$$d_{\text{min,BHD}}^{\text{int}} = \frac{w_0}{2\sqrt{2n + 1}/N}. \quad (9)$$

Equations (4) and (9) are plotted in Fig. 1, with green star and red point, respectively. Both are normalized using $w_0/2\sqrt{N}$, which represents the minimum measurable displacement with the coherent TEM$_{00}$-mode signal. The minimum measurable displacement decreases and the rate of decrease is slow as mode order increases. Furthermore, the optimal local oscillator BHD is better than the intermediate local oscillator. The difference between the two curves is relatively small, and hence the intermediate mode for the local oscillator is a good alternative in general.

A schematic of the experiment is presented in Fig. 2. A continuous wave solid-state YAG laser operating at 1064 nm is used to drive the experiment. Part of the beam is passed through mode-conversion cavity MC1, then modulated by the displacement modulation system (DMS) as the signal beam with displacement. In BHD, it is in phase with the local-oscillator mode from mode-conversion cavity MC2. The BHD output is analyzed by an electronic spectrum analyzer (ESA).

The DMS consists of a mirror mounted on a piezoelectric transducer (PZT1) and connected to a signal generator (SG). The PZT1 is driven by a sine wave signal at its mechanical resonance frequency of 3 MHz.

We generated and locked the cavity at different transverse modes by misaligning the input TEM$_{00}$ beams into the mode-conversion cavity.\textsuperscript{23,24} MC1 is locked to the TEM$_{00}$ mode (or TEM$_{10}$ mode) to produce the signal beam, and MC2 at the TEM$_{10}$ mode (or TEM$_{20}$ mode) as the local-oscillator mode. With the two cavities locked to the same modes, the interference visibilities measured for the TEM$_{00}$, TEM$_{10}$, and TEM$_{20}$ modes were 0.99 ± 0.01, 0.98 ± 0.01, and 0.96 ± 0.01, respectively. The experimental parameters are: signal beam power, $P_s = 100 \mu W$; beam waist of TEM$_{00}$, $w_0 = 53 \mu m$; resolution bandwidth, $RBW = 30$ kHz; video bandwidth, $VBW = 100$ Hz; and analyzing frequency, $f = 3$ MHz.

The measured displacement signal power vs displacement is shown in Fig. 3(a). Trace (a1) is the shot noise level (SNL) without displacement. The SNLs are the same for TEM$_{10}$ and TEM$_{20}$ local oscillators because the power is the same. Trace (b1) corresponds to the TEM$_{00}$ signal mode and TEM$_{10}$ local mode while trace (c1) to the TEM$_{10}$ signal mode and TEM$_{20}$ local mode. The TEM$_{00}$ and the TEM$_{10}$ signal-beam powers are also the same. The detector dark noise is approximately 14 dB below the shot noise level. The corresponding SNR vs displacement is shown in Fig. 3(b). The two smooth real lines are theoretical results obtained using Eq. (8). Trace (c2) clearly increases faster than trace...
With SNR of 1 in Fig. 3(b), we obtained a minimum measurable displacement using the TEM$_{00}$ mode out-performs that for the TEM$_{10}$ mode, in good agreement with theory. This scheme can be widely used in precision measurements, and, perhaps, even in gravitational wave detection and quantum measurements.

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FIG. 3. (a) Normalized signal power measured in the ESA vs beam displacement: (a1) corresponds to the quantum noise, (b1) TEM$_{00}$ signal mode, and (c1) TEM$_{10}$ signal mode. (b) Signal-to-noise ratio plotted against displacement. Traces (a2), (b2), and (c2) correspond to traces (a1), (b1), and (c1), respectively.

(b2) as displacement increases, signifying that the measured SNR increases with increasing mode order under the same conditions (displacement, optical power, and fundamental mode beam waist). Consequently, measurement precision improved using high-order modes.

From Eq. (9), the minimum measurable displacements for the TEM$_{00}$ and for TEM$_{10}$ mode are, respectively,

$$d_{\text{min}}^{\text{TEM}_{00}} = \frac{\hbar}{2P_s \lambda / (hc \times RBW)} = 2 \, \AA,$$

(10)

$$d_{\text{min}}^{\text{TEM}_{10}} = d_{\text{min}}^{\text{TEM}_{00}} / \sqrt{2} = 1.4 \, \AA,$$

(11)

where $\lambda = 1064$ nm is the wavelength, $h$ is the Planck’s constant, and $c$ is the speed of light in vacuum.

With SNR of 1 in Fig. 3(b), we obtained a minimum measurable displacement of almost 1.4 $\AA$ and 2 $\AA$ for the TEM$_{00}$ and TEM$_{10}$ signal beam, respectively, and a 1.41-factor improvement in measurement precision, in good agreement with theory.

In conclusion, we have theoretically presented a scheme for optical transverse-displacement measurements based on high-order Hermite-Gauss modes. Minimum measurable displacements attained the limit deduced from the QCR bound. With an intermediate local-oscillator mode, we also give preliminary experimental results of displacement measurements using coherent TEM$_{00}$ and TEM$_{10}$ as signal modes. The minimum measurable displacement using the TEM$_{10}$ mode out-performs that for the TEM$_{00}$ mode, in good agreement with theory. This scheme can be widely used in precision measurements, and, perhaps, even in gravitational wave detection and quantum measurements.