Bright entanglement characteristics of subharmonic modes reflected from cavity for type II second-harmonic generation

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Received May 11, 2004; accepted October 21, 2004

Quantum fluctuation and quantum entanglement in competing nonlinear processes of type II second-harmonic generation in a triple-resonant cavity is analyzed. Operating above threshold with intracavity loss, the mean amplitude and the entanglement of reflected subharmonic field as a function of pump and loss parameters are calculated. It is found that the two eigenmodes become unbalanced when internal loss intervenes. The dependence of the entanglement characteristics of two reflected subharmonic modes on output coupling efficiency μ_b/μ, pump parameter σ, and normalized frequency Ω are discussed. Under appropriate conditions, this system can be exploited as a bright entanglement resource. © 2005 Optical Society of America

OCIS codes: 190.4410, 270.2500.

1. INTRODUCTION

Quantum entanglement plays a pivotal role in the basic problem of quantum mechanics and quantum information science. It has been proved that optical parametric oscillators (OPOs) process is one of the important and effective ways of generating continuous-variable entanglement states. Several important experiments in continuous-variable quantum information were completed with OPO. In 1998 and 2003, Furusawa *et al.*[1] and Zhang *et al.*[2] created an entanglement state by combining two independent squeezed fields from two degenerate OPOs on a 50/50 beam splitter, with which continuous-variable quantum teleportation was successfully demonstrated. Through combination of two bright amplitude-squeezed states, a bright entanglement state and the corresponding continuous-variable quantum teleportation were realized in Bowen *et al.*[3] With type II phase-matching (polarization-nondegenerate) OPO, an entanglement state was established through direct separation of two downconversion modes. Recently, the continuous-variable quantum teleportation, quantum dense coding, and controlled quantum dense coding[4–6] were realized with type II phase-matching OPO, and very recently a tripartite entanglement state was also generated by means of three degenerate OPOs.[7] It was proved that a stable entanglement resource is the key for the completion of the quantum information experiment. So far, the continuous-variable entanglement states are generated from degenerate or nondegenerate optical parametric processes in each of the aforementioned experiments.[1–7] Apart from parametric downconversion, the second-harmonic generation (SHG) process also yields nonclassical light fields. It has been demonstrated experimentally that the pump fields reflected from an optical cavity for SHG and OPO are squeezed because of the cascaded nonlinear interaction between subharmonic and harmonic fields inside the cavity.[8,9] In particular, the reflected pump modes of type II SHG possess two-mode squeezing characteristics[10]; therefore the generation of an entanglement with SHG is interesting.[11–13] Ou[10] theoretically analyzed quantum fluctuation and squeezing characteristics of the reflected subharmonic modes of type II SHG and calculated the spectra of squeezing. It was pointed out that for the case of type II harmonic generation there exists a threshold that is identified as the onset of an OPO formed by a subharmonic mode with its polarization orthogonal to the input polarization (not the original modes) and the output of the orthogonal polarization mode from the OPO exhibits phase squeezing. References 12 and 13 analyzed the quadrature squeezing and entanglement subharmonic modes from double-resonant and triple-resonant type II SHG operating below threshold. It is shown that perfect entanglement can be accessed by triple-resonant SHG. Jack *et al.*[11] generalized the work in Ref. 10 to an asymmetric pumping case. Under ideal conditions (without intracavity loss), Jack *et al.* discussed the effect that asymmetric pumping has on the squeezing of the transformed modes, and gave the spectra of EPR correlation operating below and above threshold in a symmetric pumping case with different intrcavity amplitudes. In this paper, we present classical and quantum characteristics of the two reflected eigenmodes of the triple-resonant type II SHG system operating above threshold, and the quadrature entanglement between the two eigenmodes are interpreted in terms of experiment-related parameters such as pump parameter $(P/P_{\mathrm{th}})^{1/2}$, output coupling efficiency, and normalized frequency. It is a continuous work of Ref. 13 (below threshold). Under appropriate conditions, a triple resonant type II SHG system can be exploited as a good bright entanglement resource.

2. DYNAMIC EQUATIONS AND THEIR STATIONARY SOLUTIONS

As shown in Fig. 1, consider a one-sided optical cavity in which a type II phase-matching $\chi^{(2)}$ crystal is placed. Subharmonic modes and harmonic modes resonate in the cavity simultaneously. Assuming perfect phase matching and cavity resonating, the semiclassical dynamic equations can be expressed as

$$\tau\dot{\alpha}_0(t) = -\gamma_0\alpha_0(t) - \chi\alpha_1(t)\alpha_2(t) + \sqrt{2\gamma_0}c_0(t), \tag{1a}$$

$$\tau\dot{\alpha}_1(t) = -\gamma_1\alpha_1(t) + \chi\alpha_2^*(t)\alpha_0(t) + \sqrt{2\gamma_{b1}}\alpha_1^{\text{in}}\exp(i\phi_1) + \sqrt{2\gamma_{c1}}c_1(t), \tag{1b}$$

$$\tau\dot{\alpha}_2(t) = -\gamma_2\alpha_2(t) + \chi\alpha_1^*(t)\alpha_0(t) + \sqrt{2\gamma_{b2}}\alpha_2^{\text{in}}\exp(i\phi_2) + \sqrt{2\gamma_{c2}}c_2(t), \tag{1c}$$

where α_0, α_1, and α_2 are the amplitude of harmonic and two subharmonic modes, respectively. α_1^{in} and α_2^{in} are the amplitudes of two pumping subharmonic modes outside the cavity. The cavity round-trip time of three modes is τ. The total loss parameter for each mode is $\gamma_i = \gamma_{bi} + \gamma_{ci}$, $(i = 0, 1, 2)$, where γ_{bi} is related to the amplitude reflection coefficients r_i and amplitude transmission coefficients t_i of the coupler by the formula

$$r_i = 1 - \gamma_{bi},$$

$$t_i = \sqrt{2\gamma_{bi}}.$$

γ_{ci} represents intracavity loss parameter. $c_i(t)$ is the noise term corresponding to intracavity loss.

In general, assuming the two pumping modes have the same positive real amplitude β, zero initial phase and the balanced loss in the cavity, we have

$$\gamma_1 = \gamma_2 = \gamma, \tag{2a}$$

$$\gamma_{b1} = \gamma_{b2} = \gamma_b, \tag{2b}$$

$$\gamma_{c1} = \gamma_{c2} = \gamma_c. \tag{2c}$$

Stationary mean field solutions $\bar{\alpha}_0$, $\bar{\alpha}_1$, and $\bar{\alpha}_2$ can be obtained by setting $\dot{\alpha}_0$, $\dot{\alpha}_1$, and $\dot{\alpha}_2$ to be zero. The steady-state equations are obtained:

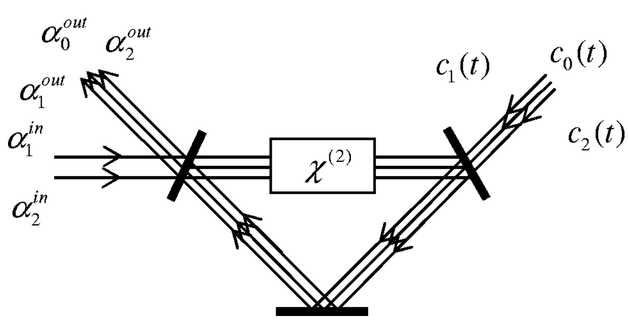

Fig. 1. Sketch of experimental setup.

$$\bar{\alpha}_0 = \frac{-\chi\bar{\alpha}_1\bar{\alpha}_2}{\gamma_0}, \tag{3a}$$

$$\left(-\gamma - \frac{\chi^2}{\gamma_0}|\bar{\alpha}_2|^2\right)\bar{\alpha}_1 + \sqrt{2\gamma_b}\beta = 0, \tag{3b}$$

$$\left(-\gamma - \frac{\chi^2}{\gamma_0}|\bar{\alpha}_1|^2\right)\bar{\alpha}_2 + \sqrt{2\gamma_b}\beta = 0. \tag{3c}$$

Equations (3b) and (3c) show that both $\bar{\alpha}_1$ and $\bar{\alpha}_2$ are real numbers. Pumping threshold β^{th} and pump parameter σ can be expressed as

$$\beta^{\text{th}} = (2\gamma^3\gamma_0/\chi^2\gamma_b)^{1/2}, \tag{4a}$$

$$\sigma = \beta/\beta^{\text{th}}. \tag{4b}$$

After Eqs. (3b) and (3c) are solved, stationary solutions of three modes can be obtained both below and above threshold. Below threshold $(\sigma \leqslant 1)$, the solutions are given as

$$\bar{\alpha}_1 = \bar{\alpha}_2 = \alpha, \tag{5a}$$

$$\alpha = \sqrt{\gamma\gamma_0}\sigma'/\chi \tag{5b}$$

$$\bar{\alpha}_0 = -\gamma\sigma'^2/\chi, \tag{5c}$$

$$\sigma' = [\sigma + (\sigma^2 + 1/27)^{1/2}]^{1/3} - \frac{1}{3}(\sigma + (\sigma^2 + 1/27)^{1/2})^{-1/3},$$

and above threshold $(\sigma \geqslant 1)$ as

$$\bar{\alpha}_1 = \sqrt{\gamma\gamma_0}\sigma/\chi - [\gamma\gamma_0(\sigma^2 - 1)]^{1/2}/\chi, \tag{6a}$$

$$\bar{\alpha}_2 = \sqrt{\gamma\gamma_0}\sigma/\chi + [\gamma\gamma_0(\sigma^2 - 1)]^{1/2}/\chi, \tag{6b}$$

$$\bar{\alpha}_0 = -\gamma/\chi. \tag{6c}$$

Using the input-output relations $\alpha_i^{\text{in}} + \alpha_i^{\text{out}} = \sqrt{2\gamma_b}\alpha_i$, the amplitude of output field below threshold $(\sigma \leqslant 1)$ can be written as

$$\alpha_1^{\text{out}} = \alpha_2^{\text{out}} = \alpha^{\text{out}} = (2\gamma\gamma_0/\chi^2\gamma_b)^{1/2}(\sigma'\gamma_b - \sigma\gamma), \tag{7a}$$

$$\alpha_0^{\text{out}} = -\sigma'^2\gamma\sqrt{2\gamma_0}/\chi, \tag{7b}$$

and above threshold $(\sigma \geqslant 1)$ can be written as

$$\alpha_1^{\text{out}} = (2\gamma\gamma_0/\chi^2\gamma_b)^{1/2}[\sigma\gamma_b - \sigma\gamma - \gamma_b(\sigma^2 - 1)^{1/2}], \tag{8a}$$

$$\alpha_2^{\text{out}} = (2\gamma\gamma_0/\chi^2\gamma_b)^{1/2}[\sigma\gamma_b - \sigma\gamma + \gamma_b(\sigma^2 - 1)^{1/2}], \tag{8b}$$

$$\alpha_0^{\text{out}} = -\gamma\sqrt{2\gamma_0}/\chi. \tag{8c}$$

Dependence of the output amplitude of two eigenmodes on pump parameter σ in condition of with $(\gamma > \gamma_b$, dashed curve) and without $(\gamma = \gamma_b$, solid curve) intracavity loss are shown in Fig. 2, which indicates that coherent output appears when $\sigma > 1$. It is shown in Fig. 2 and Eqs. (8a) and (8b) that the amplitudes of the two modes have balanced absolute value and opposite phase without internal loss. But when internal loss intervenes, the coherent output of the two modes becomes unbalanced. It

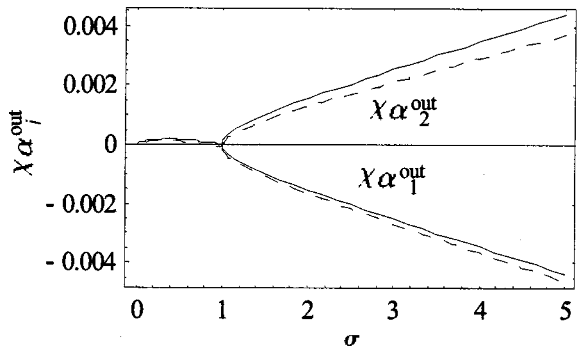

Fig. 2. Dependence of amplitude of the two output eigenmodes on pump parameter (σ) with $\gamma = \gamma_b = 0.02$, $\gamma_0 = 0.001$ for solid curve and $\gamma = 0.02$, $\gamma_b = 0.018$, $\gamma_0 = 0.001$ for dashed curve.

is easy to verify that, when $\gamma = \gamma_b$, the output field of all three modes both below and above threshold satisfies the law of energy conservation $2\alpha_0^{\text{out}^2} + \alpha_1^{\text{out}^2} + \alpha_2^{\text{out}^2} = 2\beta^2$.

3. FLUCTUATION AND CORRELATION SPECTRA OF SUBHARMONIC MODES ABOVE THRESHOLD

Quantum fluctuation and the squeezing of subharmonic modes of type II SHG below threshold in double- and triple-resonant cases were discussed in Refs. 12 and 13, respectively. It was shown that almost-perfect entanglement can be accessed in a triple-resonant case. In this section, entanglement between two pumping eigen subharmonic modes of triple-resonant SHG operating above threshold is considered.

Quantum fluctuations can be obtained through linearization of the evolution Eqs. (1) around the mean values given by Eqs. (6). Setting $\alpha_i = \bar{\alpha}_i + \delta\alpha_i$, we obtain

$$\tau\delta\dot{\alpha}_0(t) = -\gamma_0\delta\alpha_0(t) - \chi[\bar{\alpha}_2\delta\alpha_1(t) + \bar{\alpha}_1\delta\alpha_2(t)] + \sqrt{2\gamma_0}c_0(t), \tag{9a}$$

$$\tau\delta\dot{\alpha}_1(t) = -\gamma\delta\alpha_1(t) + \chi[\bar{\alpha}_0\delta\alpha_2^*(t) + \bar{\alpha}_2\delta\alpha_0(t)] + \sqrt{2\gamma_b}b_1(t) + \sqrt{2\gamma_c}c_1(t), \tag{9b}$$

$$\tau\delta\dot{\alpha}_2(t) = -\gamma\delta\alpha_2(t) + \chi[\bar{\alpha}_0\delta\alpha_1^*(t) + \bar{\alpha}_1\delta\alpha_0(t)] + \sqrt{2\gamma_b}b_2(t) + \sqrt{2\gamma_c}c_2(t), \tag{9c}$$

introducing amplitude quadrature (X) and phase quadrature (Y) of optical modes (O), which are defined by

$$O = \frac{1}{2}(X + iY), \tag{10}$$

where $O = [\alpha_0, \alpha_1, \alpha_2, b_0, c_0, b_1, c_1, b_2, c_2]$, $X = [X_0, X_1, X_2, X_{b0}, X_{c0}, X_{b1}, X_{c1}, X_{b2}, X_{c2}]$, and $Y = [Y_0, Y_1, Y_2, Y_{b0}, Y_{c0}, Y_{b1}, Y_{c1}, Y_{b2}, Y_{c2}]$.

Substitution of Eq. (10) into Eqs. (9a)–(9c), yields the following fluctuations of amplitude and phase quadratures:

$$\tau\delta\dot{X}_0(t) = -\gamma_0\delta X_0(t) - \chi[\bar{\alpha}_2\delta X_1(t) + \bar{\alpha}_1\delta X_2(t)] + \sqrt{2\gamma_0}\delta X_{c0}(t), \tag{11a}$$

$$\tau\delta\dot{X}_1(t) = -\gamma\delta X_1(t) + \chi[\bar{\alpha}_0\delta X_2(t) + \bar{\alpha}_2\delta X_0(t)] + \sqrt{2\gamma_b}\delta X_{b1}(t) + \sqrt{2\gamma_c}\delta X_{c1}(t), \tag{11b}$$

$$\tau\delta\dot{X}_2(t) = -\gamma\delta X_2(t) + \chi[\bar{\alpha}_0\delta X_1(t) + \bar{\alpha}_1\delta X_0(t)] + \sqrt{2\gamma_b}\delta X_{b2}(t) + \sqrt{2\gamma_c}\delta X_{c2}(t), \tag{11c}$$

and

$$\tau\delta\dot{Y}_0(t) = -\gamma_0\delta Y_0(t) - \chi[\bar{\alpha}_2\delta Y_1(t) + \bar{\alpha}_1\delta Y_2(t)] + \sqrt{2\gamma_0}\delta Y_{c0}(t), \tag{12a}$$

$$\tau\delta\dot{Y}_1(t) = -\gamma\delta Y_1(t) - \chi[\bar{\alpha}_0\delta Y_2(t) - \bar{\alpha}_2\delta Y_0(t)] + \sqrt{2\gamma_b}\delta Y_{b1}(t) + \sqrt{2\gamma_c}\delta Y_{c1}(t), \tag{12b}$$

$$\tau\delta\dot{Y}_2(t) = -\gamma\delta Y_2(t) - \chi[\bar{\alpha}_0\delta Y_1(t) - \bar{\alpha}_1\delta Y_0(t)] + \sqrt{2\gamma_b}\delta Y_{b2}(t) + \sqrt{2\gamma_c}\delta Y_{c2}(t). \tag{12c}$$

Under condition of above threshold ($\sigma \geqslant 1$), substituting stationary solutions (6a) and (6b) into Eqs. (11) and (12), performing Fourier transformation, and solving these equations with quadratures $X_1(\Omega)$, $X_2(\Omega)$, $Y_1(\Omega)$, and $Y_2(\Omega)$ in frequency domain, one can get

$$\begin{aligned}\delta X_1(\Omega) = {} & \delta X_{co}\frac{\sqrt{2\gamma_0}}{D}\{2\gamma[\gamma\gamma_0(\sigma^2 - 1)]^{1/2} \\ & + i\gamma\sqrt{\gamma\gamma_0}\Omega[\sigma + (\sigma^2 - 1)^{1/2}]\} \\ & + \frac{1}{D}(\sqrt{2\gamma_b}\delta X_{b1} + \sqrt{2\gamma_c}\delta X_{c1})[2\gamma\gamma_0\sigma \\ & \times[\sigma - \sqrt{\sigma^2 - 1}] - \gamma^2\Omega^2 + i\gamma\Omega(\gamma + \gamma_0)] \\ & + \frac{1}{D}(\sqrt{2\gamma_b}\delta X_{b2} + \sqrt{2\gamma_c}\delta X_{c2}) \\ & \times[-2\gamma\gamma_0 - i\gamma^2\Omega],\end{aligned} \tag{13a}$$

$$\begin{aligned}\delta X_2(\Omega) = {} & \delta X_{co}\frac{\sqrt{2\gamma_0}}{D}\{-2\gamma[\gamma\gamma_0(\sigma^2 - 1)]^{1/2} \\ & + i\gamma\Omega\sqrt{\gamma\gamma_0}[\sigma - (\sigma^2 - 1)^{1/2}]\} \\ & + \frac{1}{D}(\sqrt{2\gamma_b}\delta X_{b2} + \sqrt{2\gamma_c}\delta X_{c2})\{2\gamma\gamma_0\sigma \\ & \times[\sigma + \sqrt{(\sigma^2 - 1)}] \\ & - \gamma^2\Omega^2 + i\gamma\Omega(\gamma + \gamma_0)\} \\ & + \frac{1}{D}(\sqrt{2\gamma_b}\delta X_{b1} + \sqrt{2\gamma_c}\delta X_{c1}) \\ & \times[-2\gamma\gamma_0 - i\gamma^2\Omega],\end{aligned} \tag{13b}$$

$$\delta Y_1(\Omega) = \delta Y_{co}\frac{\sqrt{2\gamma_0}}{B}\{2\sigma\gamma\sqrt{\gamma\gamma_0} + i\Omega\gamma\sqrt{\gamma\gamma_0} \times [\sigma + (\sigma^2 - 1)^{1/2}]\} + \frac{1}{B}(\sqrt{2\gamma_b}\delta X_{b1} + \sqrt{2\gamma_c}\delta X_{c1})\{2\gamma\gamma_0\sigma \times [\sigma - (\sigma^2 - 1)^{1/2}] - \Omega^2\gamma^2 + i\Omega\gamma(\gamma + \gamma_0)\} + \frac{i\gamma^2\Omega}{B}(\sqrt{2\gamma_b}\delta X_{b2} + \sqrt{2\gamma_c}\delta X_{c2}), \quad (13c)$$

$$\delta Y_2(\Omega) = \delta Y_{co}\frac{\sqrt{2\gamma_0}}{B}\{2\sigma\gamma\sqrt{\gamma\gamma_0} + i\Omega\gamma\sqrt{\gamma\gamma_0} \times [\sigma - (\sigma^2 - 1)^{1/2}]\} + \frac{1}{B}(\sqrt{2\gamma_b}\delta X_{b2} + \sqrt{2\gamma_c}\delta X_{c2})\{2\gamma\gamma_0\sigma \times [\sigma + (\sigma^2 - 1)^{1/2}] - \Omega^2\gamma^2 + i\Omega\gamma(\gamma + \gamma_0)\} + \frac{i\gamma^2\Omega}{B}(\sqrt{2\gamma_b}\delta X_{b1} + \sqrt{2\gamma_c}\delta X_{c1}), \quad (13d)$$

where $\Omega = \omega\tau/\gamma$ is normalized frequency. By use of the input-output relations of the optical cavity for the fluctuations of quadrature-phase amplitude $\delta X(\delta Y)^{\text{out}} + \delta X(\delta Y)^{\text{in}} = \sqrt{2\gamma_b}\delta X(\delta Y)$, one can easily derive the fluctuations of sum (difference) of amplitude (phase) quadratures inherent in the output field of a triple-resonant SHG system in terms of pumping parameter σ, normalized frequency Ω, and the ratio of cavity loss between subharmonic and harmonic fields:

$$\delta X(\Omega)_1^{\text{out}} + \delta X(\Omega)_2^{\text{out}} = \frac{4i\Omega\sigma\mu\sqrt{\mu\mu_b}}{D}\delta X_{c0} - (\delta X_{b1} + \delta X_{b2}) + \frac{1}{D}(2\mu_b\delta X_{b1} + 2\sqrt{\mu_c\mu_b}\delta X_{c1}) \times \{2\mu[\sigma^2 - \sigma(\sigma^2 - 1)^{1/2} - 1] - \Omega^2\mu^2 + i\Omega\mu\} + \frac{1}{D}(2\mu_b\delta X_{b2} + 2\sqrt{\mu_c\mu_b}\delta X_{c2})\{2\mu[\sigma^2 + \sigma(\sigma^2 - 1)^{1/2} - 1] - \Omega^2\mu^2 + i\Omega\mu\}, \quad (14a)$$

$$\delta Y(\Omega)_1^{\text{out}} - \delta Y(\Omega)_2^{\text{out}} = \frac{4i\Omega\mu\sqrt{\mu\mu_b}}{B}(\sigma^2 - 1)^{1/2}\delta Y_{co} - (\delta Y_{b1} - \delta Y_{b2}) + \frac{1}{B}(2\mu_b\delta Y_{b1} + 2\sqrt{\mu_b\mu_c}\delta Y_{c1}) \times \{2\mu[\sigma^2 - \sigma(\sigma^2 - 1)^{1/2}] - \Omega^2\mu^2 + i\Omega\mu\} - \frac{1}{B}(2\mu_b\delta Y_{b2} + 2\sqrt{\mu_b\mu_c}\delta Y_{c2}) \times \{2\mu[\sigma^2 + \sigma(\sigma^2 - 1)^{1/2}] - \Omega^2\mu^2 + i\Omega\mu\}, \quad (14b)$$

where

$$B = 4\mu^2\sigma^2 - \Omega^2\mu^2(1 + 2\mu) + 4i\Omega\mu^2\sigma^2 - i\Omega^3\mu^3,$$

$$D = 4\mu^2(\sigma^2 - 1) - \Omega^2\mu^2(1 + 2\mu) + 4i\Omega\mu^2\sigma^2 - i\Omega^3\mu^3,$$

$$\mu_b = \gamma_b/\gamma_0, \qquad \mu_c = \gamma_c/\gamma_0, \qquad \mu = \mu_b + \mu_c.$$

The correlation spectra are defined as

$$V_{X_1+X_2}^{\text{out}} = \langle[\delta X_1^{\text{out}}(\Omega) + \delta X_2^{\text{out}}(\Omega)][\delta X_1^{\text{out}}(\Omega) + \delta X_2^{\text{out}}(\Omega)]^+\rangle, \quad (15a)$$

$$V_{Y_1-Y_2}^{\text{out}} = \langle[\delta Y_1^{\text{out}}(\Omega) - \delta Y_2^{\text{out}}(\Omega)][\delta Y_1^{\text{out}}(\Omega) - \delta Y_2^{\text{out}}(\Omega)]^+\rangle. \quad (15b)$$

For continuous-variable Gaussian states, under the constraint that the variance of all quadrature fluctuations of entangled beams is equal, the Peres–Horodecki criterion of entanglement proposed by Duan *et al.*[14] is

$$V_{X_1+X_2}^{\text{out}} + V_{Y_1-Y_2}^{\text{out}} < 2. \quad (16)$$

For unequal correlations between orthogonal quadratures, a symmetrization procedure must be carried out to use this criterion for entanglement verification. This can be done through performance of local unitary squeezing operations on the correlated beams,[15] and thus the criterion can be rewritten as

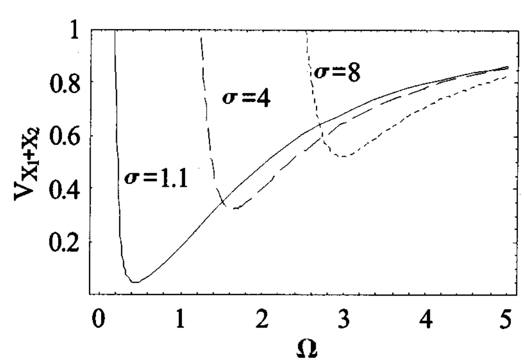

Fig. 3. Correlation spectra $V_{X_1+X_2}^{\text{out}}$ as a function of normalized frequency Ω with $\mu = \mu_b = 40$, $\sigma = 1.1$ (solid curve), $\sigma = 4$ (dashed curve), and $\sigma = 8$ (dotted curve).

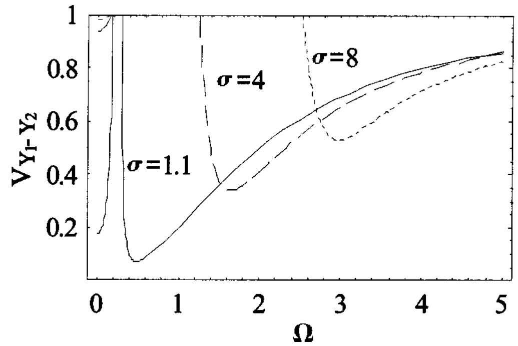

Fig. 4. Correlation spectra $V_{Y_1-Y_2}(\Omega)$ as a function of normalized frequency Ω with $\mu = \mu_b = 40$, $\sigma = 1.1$ (solid curve), $\sigma = 4$ (dashed curve), and $\sigma = 8$ (dotted curve).

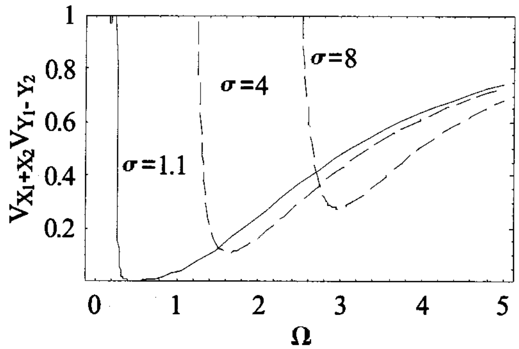

Fig. 5. $V_{X_1+X_2}(\Omega)V_{Y_1-Y_2}(\Omega)$ as a function of normalized frequency Ω with $\mu = \mu_b = 40$, $\sigma = 1.1$ (solid curve), $\sigma = 4$ (dashed curve), and $\sigma = 8$ (dotted curve).

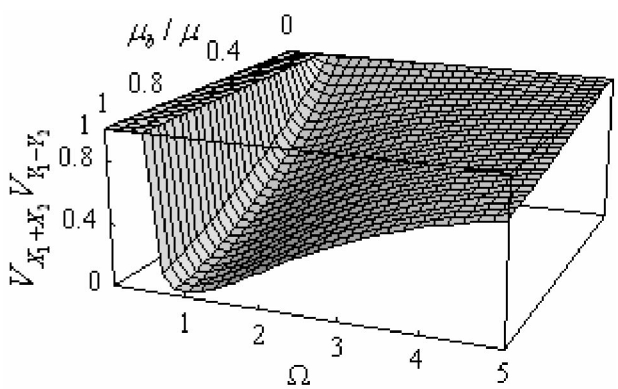

Fig. 6. $V_{X_1+X_2}(\Omega)V_{Y_1-Y_2}(\Omega)$ as a function of normalized frequency Ω and output coupling efficiency μ_b/μ with $\mu = 40$, $\sigma = 2$.

$$V_{X_1+X_2}^{\text{out}} V_{Y_1-Y_2}^{\text{out}} < 1, \tag{17}$$

which is suitable to our case. Figures 3, 4, and 5 show the dependence of $V_{X_1+X_2}^{\text{out}}$, $V_{Y_1-Y_2}^{\text{out}}$ and the product $V_{X_1+X_2}^{\text{out}} V_{Y_1-Y_2}^{\text{out}}$ on normalized frequency Ω at different values of pump parameter σ, respectively. It is clear that the best correlation and entanglement can be accessed when σ approaches 1, and obvious entanglement can also be obtained when pump power is far from threshold (In Fig. 5, $V_{X_1+X_2}^{\text{out}} V_{Y_1-Y_2}^{\text{out}} < 0.6$ when $\sigma = 8$). Furthermore, different pump parameters correspond to different optimum normalized frequencies. Figure 6 shows the product $V_{X_1+X_2}^{\text{out}} V_{Y_1-Y_2}^{\text{out}}$ as a function of output coupling efficiency μ_b/μ and normalized frequency Ω. Meanwhile, the broad-bandwidth entanglement can be obtained at the value of an output-coupling efficiency larger than 0.6, and an almost-perfect entanglement can be obtained when the output-coupling efficiency equals 1.

By means of semiclassical method, we have analyzed quantum fluctuation and the entanglement of subharmonic fields reflected from the cavity of type II SHG operating above threshold. Correlation spectra as a function of normalized frequency, pump parameter, and output-coupling efficiency are calculated. Compared with general OPO, a type II SHG system can be exploited as a very useful and relatively simple bright entanglement resource with broadband.[16]

ACKNOWLEDGMENTS

The National Fundamental Research Program (2001CB309304), the National Natural Science Foundation of China (Approved 66238010, 10274045, 60278010), the Teaching and Research Award Program for Outstanding Young Teachers in High Education Institute of MOE of China, and the Shanxi Provincial Science Foundation supported this work.

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