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A FULL QUANTUM THEORY OF THE THREE-MODE INTERACTIONS INSIDE AN OPO CAVITY

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A full quantum treatment about the process of parametric down-conversion with frequency degenerate but polarization non-degenerate in an optical parametric oscillator (OPO) cavity is presented. Using the linearized Langevin equations and spectral matrix, we calculated the squeezing spectra of the coupled mode in the output field. The squeezing as a function of driving field and detection frequency is obtained. The results obtained, which are compared with those found semiclassically by Reynaud *et al.*, indicate that it is possible to generate a two-mode coherent squeezed state with large amplitude. The quantum correlation between the signal and the idler modes is also discussed. It is shown that there is an inseparable relationship between the two-mode squeezing and the intermode quantum correlation.

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I. INTRODUCTION

In recent years there has been great interest in nonclassical behaviours of the field in the optical parametric oscillator [1–4] (OPO). The single mode squeezed vacuum state which has 70% of maximal squeezing has been achieved through the degenerate parametric down-conversion [5]. But to our knowledge, the full quantum treatment on the three-mode interactions in an OPO cavity with a non-degenerate parametric process has not been discussed and this kind of treatment is very important to recognize the field nonclassical characters in the OPO cavity. In this paper, we calculate the squeezing spectrum of a coupled mode in output field using the linearized Langevin equations and spectral matrices, and discuss the squeezing of the coupled mode in detail. The quantum correlation between the signal and the idler modes is also discussed in terms of the definition given by Reid [6]. It is shown that there is an inseparable relationship between the two-mode squeezing and the intermode quantum correlation.

II. BASIC THEORY

A. Theoretical model and steady-state solutions

As shown in Fig. 1, a coherent pump field with a frequency ω_1 is incident on a Fabry-Perot cavity containing a crystal with a significant second-order susceptibility which converts the ω_1 photons into the correlated pairs of photons with the same frequency ($\omega_2 = \omega_1 / 2$) and perpendicular polarizations, i. e., this is a process with frequency degenerate but polarization non-degenerate. We assume that there is no mistuning of the cavity, then the

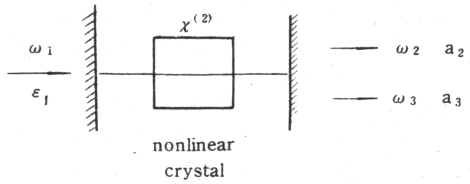

Fig. 1. The non-degenerate optical parametric oscillator. A pump field ε_1 drives a cavity containing a nonlinear crystal. Two photons with frequency degenerate but polarization non-degenerate are created from one ω_1 photon.

Hamiltonian of the system can be written as follows:

$$H = H_{\text{rev}} + H_{\text{irrev}} \quad , \tag{1}$$

$$H_{\text{rev}} = 2\hbar\omega a_1^{+} a_1 + \hbar\omega a_2^{+} a_2 + \hbar\omega a_3^{+} a_3 + i\hbar(\kappa/2)\,(a_1 a_2^{+} a_3^{+} - a_1^{+} a_2 a_3)$$
$$+ i\hbar\varepsilon_1 [a_1^{+} \exp(-2i\omega t) - a_1 \exp(2i\omega t)] \quad , \tag{2}$$

$$H_{\text{irrev}} = \sum_{i=1}^{3} (a_i \Gamma_i^{+} - a_i^{+} \Gamma_i) \quad , \tag{3}$$

where H_{rev} refers to the reversible part of the interaction and H_{irrev} the irreversible part due to cavity damping; a_i and a_i^{+} are the annihilation and creation operators for mode i ($i=1,2,3$, mode 1 is the pumping mode at frequency 2ω, modes 2 and 3 are the signal and idler modes, respectively); κ is the coupling constant, which is proportional to the second order susceptibility $\chi^{(2)}$ of the medium. Γ_i, Γ_i^{+} are heat bath operators which represent cavity losses for the three modes, and ε_1 is proportional to the coherent driving field amplitude. With the standard techniques to eliminate the heat baths, we obtain the master equation for the reduced density operator ρ of the system

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H_{\text{rev}}, \rho] + \sum_{i=1}^{3} \gamma_i (2a_i \rho a_i^{+} - a_i^{+} a_i \rho - \rho a_i^{+} a_i)$$
$$+ 2\sum_{i=1}^{3} \gamma_i n_i^{\text{th}} (a_i \rho a_i^{+} - \rho a_i^{+} a_i - a_i^{+} a_i \rho + a_i^{+} \rho a_i) \quad , \tag{4}$$

where γ_i are the mode damping constants and n_i^{th} are the mean numbers of thermal photons in the heat baths.

By means of generalization of the Glauber P representation developed by Drummond and Gardiner, the equation can be converted to a c-number Fokker-Planck equation:

$$\frac{\partial P(\vec{\alpha})}{\partial t} = \left\{ \frac{\partial}{\partial \alpha_1} (\gamma_1 \alpha_1 - \varepsilon_1 + \kappa \alpha_2 \alpha_3) + \frac{\partial}{\partial \alpha_1^{+}} (\gamma_1 \alpha_1^{+} - \varepsilon_1 + \kappa \alpha_2^{+} \alpha_3^{+}) \right.$$

$$
\begin{aligned}
&+\frac{\partial}{\partial\alpha_2}(\gamma_2\alpha_2-\kappa\alpha_1\alpha_3^+)+\frac{\partial}{\partial\alpha_2^+}(\gamma_2\alpha_2^+-\kappa\alpha_1^+\alpha_3)\\
&+\frac{\partial}{\partial\alpha_3}(\gamma_3\alpha_3-\kappa\alpha_1\alpha_2^+)+\frac{\partial}{\partial\alpha_3^+}(\gamma_3\alpha_3^+-\kappa\alpha_1^+\alpha_2)\\
&+\frac{1}{2}\Big[\frac{\partial^2}{\partial\alpha_2\partial\alpha_3}\kappa\alpha_1+\frac{\partial^2}{\partial\alpha_2^+\partial\alpha_3^+}\kappa\alpha_1^+\\
&+\sum_{i=1}^{3}2\gamma_i n_i^{\mathrm{th}}\frac{\partial^2}{\partial\alpha_i^+\partial\alpha_i}\Big]\Big\}P(\alpha)\quad, \qquad (5)
\end{aligned}
$$

where $\alpha=[\alpha_1,\alpha_1^+,\alpha_2,\alpha_2^+,\alpha_3,\alpha_3^+]$.

An alternative way of examining the statistical behaviour is to use the stochastic equations of motion corresponding to Eq. (5), that is, the Langevin equations

$$
\frac{\partial}{\partial t}\begin{vmatrix}\alpha_1\\ \alpha_1^+\\ \alpha_2\\ \alpha_2^+\\ \alpha_3\\ \alpha_3^+\end{vmatrix}=\begin{vmatrix}\varepsilon_1-\gamma_1\alpha_1-\kappa\alpha_2\alpha_3\\ \varepsilon_1-\gamma_1\alpha_1^+-\kappa\alpha_2^+\alpha_3^+\\ -\gamma_2\alpha_2-\kappa\alpha_1\alpha_3^+\\ -\gamma_2\alpha_2^+-\kappa\alpha_1^+\alpha_3\\ -\gamma_3\alpha_3-\kappa\alpha_1\alpha_2^+\\ -\gamma_3\alpha_3^+-\kappa\alpha_1^+\alpha_2\end{vmatrix}+\begin{vmatrix}0&\Gamma_1&0&0&0&0\\ \Gamma_1&0&0&0&0&0\\ 0&0&0&\Gamma_2&\kappa\alpha_1&0\\ 0&0&\Gamma_2&0&0&\kappa\alpha_1^+\\ 0&0&\kappa\alpha_1&0&0&\Gamma_3\\ 0&0&0&\kappa\alpha_1^+&\Gamma_3&0\end{vmatrix}\begin{vmatrix}\eta_1(t)\\ \eta_1^+(t)\\ \eta_2(t)\\ \eta_2^+(t)\\ \eta_3(t)\\ \eta_3^+(t)\end{vmatrix}\quad, \qquad (6)
$$

where $\Gamma_i=2n_i^{\mathrm{th}}\gamma_i$, $\eta_i(t)$ and $\eta_i^+(t)$ are delta-correlated stochastic forces with zero mean

$$
\begin{aligned}
&<\eta_i(t)> \;=\; <\eta_i^+(t)> \;=0\quad,\\
&<\eta_i(t)\,\eta_j^+(t')> \;=\; \delta_{ij}\,\delta(t-t')\quad.
\end{aligned}\qquad (7)
$$

Ignoring the stochastic forces $\eta_i(t)$, $\eta_i^+(t)$ and the stochastic nature of the α_i, we can obtain the steady-state solutions of Eq. (6)

$$
|\alpha_1^0| = \varepsilon_1/\gamma_1\;,\quad |\alpha_2^0| = |\alpha_3^0| = 0\;,\qquad (\varepsilon_1<\varepsilon_1^{\mathrm{thres.}}) \qquad (8)
$$

and

$$
|\alpha_1^0| = \gamma_2/\kappa\;,\quad |\alpha_2^0| = |\alpha_3^0| = \sqrt{(\varepsilon_1-\varepsilon_1^{\mathrm{thres.}})/\kappa}\;,\quad (\varepsilon_1\geqslant\varepsilon_1^{\mathrm{thres.}})\;, \qquad (9)
$$

where $\varepsilon_1^{\mathrm{thres.}}=\gamma_1\gamma_2/\kappa$ is the threshold driving field, and $\gamma_2=\gamma_3$ is assumed, i.e., both the signal and the idler modes have the same damping.

B. The fluctuation spectra of the coupled mode

In this section, we shall deal with the case in which there are small fluctuations around the steadystates. Taking the first order approximation of the expansion around the steady-state solutions, the Langevin equations are linearized

$$\frac{\partial}{\partial t}\left[\delta\alpha\right] = -A\left[\delta\alpha\right] + D^{1/2}\left[\eta(t)\right] \quad , \tag{10}$$

where

$$A = \begin{vmatrix} -\gamma_1 & 0 & -\kappa\alpha_3^0 & 0 & -\kappa\alpha_2^0 & 0 \\ 0 & -\gamma_1 & 0 & -\kappa\alpha_3^{0*} & 0 & -\kappa\alpha_2^{0*} \\ \kappa\alpha_3^{0*} & 0 & -\gamma_2 & 0 & 0 & \kappa\alpha_1^0 \\ 0 & \kappa\alpha_3^0 & 0 & -\gamma_2 & \kappa\alpha_1^{0*} & 0 \\ \kappa\alpha_2^{0*} & 0 & 0 & \kappa\alpha_1^0 & -\gamma_2 & 0 \\ 0 & \kappa\alpha_2^0 & \kappa\alpha_1^{0*} & 0 & 0 & -\gamma_2 \end{vmatrix} , \tag{11}$$

$$D = \begin{vmatrix} 0 & 2\gamma_1 n_1^{th} & 0 & 0 & 0 & 0 \\ 2\gamma_1 n_1^{th} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma_2 n_2^{th} & \kappa\alpha_1^0 & 0 \\ 0 & 0 & 2\gamma_2 n_2^{th} & 0 & 0 & \kappa\alpha_1^{0*} \\ 0 & 0 & \kappa\alpha_1^0 & 0 & 0 & 2\gamma_2 n_2^{th} \\ 0 & 0 & 0 & \kappa\alpha_1^{0*} & 2\gamma_2 n_2^{th} & 0 \end{vmatrix} , \tag{12}$$

$$[\delta\alpha] = \begin{vmatrix} \delta\alpha_1 \\ \delta\alpha_1^+ \\ \delta\alpha_2 \\ \delta\alpha_2^+ \\ \delta\alpha_3 \\ \delta\alpha_3^+ \end{vmatrix} , \quad [\eta(t)] = \begin{vmatrix} \eta_1(t) \\ \eta_1^+(t) \\ \eta_2(t) \\ \eta_2^+(t) \\ \eta_3(t) \\ \eta_3^+(t) \end{vmatrix} , \tag{13}$$

A and D are the drift and diffusion matrices respectively. The spectral matrix $S(\nu)$ is defined as the Fourier transform of the correlation matrix which consists of the normally ordered two time-correlation functions. The relation between the correlation matrices A and D is[7]

$$S(\nu) = (A + i\nu I)^{-1} D (A^T - i\nu I)^{-1} \quad , \tag{14}$$

where I is the identity matrix and T stands for transpose.

The coupled mode is defined as

$$d = (a_2 + a_3)/\sqrt{2}\ , \qquad d^+ = (a_2^+ + a_3^+)/\sqrt{2}\ , \tag{15}$$

and its quadrature components are

$$D_+ = (1/2\sqrt{2})\,(a_2 + a_3 + a_2^+ + a_3^+)\ ,$$
$$D_- = (1/2\sqrt{2}\mathrm{i})\,(a_2 + a_3 - a_2^+ - a_3^+)\ . \tag{16}$$

For a single-ended cavity, the two time-correlation functions for the output field can be calculated directly from correlation functions of the stochastic variables describing the internal field[8]. Then one can get the fluctuation spectra of the coupled mode in the output field

$$S_{D\pm}^{\text{out}}(v) = \pm\gamma_2\,[S_{33} + S_{35} \pm S_{34} \pm S_{36} + S_{53} + S_{55} \pm S_{54} \pm S_{56}$$
$$\pm S_{43} \pm S_{45} + S_{44} + S_{46} \pm S_{63} \pm S_{65} + S_{64} + S_{66}]/4\ . \tag{17}$$

Below the oscillating threshold, we have the squeezing spectrum of component $D-$:

$$S_{D-}^{\text{out}}(v) = -\frac{\gamma_2}{2}\,\frac{\kappa\alpha_1^0(\kappa^2\alpha_1^{02} + \gamma_2^2 + v^2) - 2\kappa^2\alpha_1^{02}\gamma_2}{(\gamma_2^2 + \kappa^2\alpha_1^{02} + v^2)^2 - 4\kappa^2\alpha_1^{02}\gamma_2^2}\ ; \tag{18}$$

and above the oscillating threshold, we have similarly

$$S_{D-}^{\text{out}}(v) = -\gamma_2\,\frac{\kappa\alpha_1^0[(A_1^2 + B_1^2) - (A_2^2 + B_2^2)]}{R^2 + I^2}\ , \tag{19}$$

where

$$A_1 = \kappa\alpha_1^0[(\gamma_1^2 - v^2)(\gamma_2^2 - v^2) - 4\gamma_1\gamma_2 v^2] + 2\kappa^3\alpha_1^0\alpha_2^{02}(\gamma_1\gamma_2 - v^2) - \kappa^3\alpha_1^{03}(\gamma_1^2 - v^2) + 2\kappa^5\alpha_1^0\alpha_2^{04}\ , \tag{20}$$

$$B_1 = 2\kappa\alpha_1^0 v[\gamma_2(\gamma_1^2 - v^2) + \gamma_1(\gamma_2^2 - v^2)] + 2\kappa^3\alpha_1^0\alpha_2^{02}(\gamma_1 + \gamma_2)v - 2\kappa^3\alpha_1^{03}\gamma_1 v\ , \tag{21}$$

$$A_2 = (\gamma_1^2 - v^2)(\gamma_2^3 - 3\gamma_2 v^2) - 2\gamma_1 v^2(3\gamma_2^2 - v^2) + 3\kappa^2\alpha_2^{02}[\gamma_1(\gamma_2^2 - v^2) - 2\gamma_2 v^2] + 2\kappa^4\gamma_2\alpha_3^{04}$$

$$-\kappa^4\gamma_1\alpha_1^{02}\alpha_2^{02} - \kappa^2\alpha_1^{02}[\gamma_2(\gamma_1^2 - v^2) - 2\gamma_1 v^2]\ , \tag{22}$$

$$B_2 = (\gamma_1^2 - v^2)(3\gamma_2^2 v - v^3) + 2\gamma_1 v(\gamma_2^3 - 3\gamma_2 v^2) + 3\kappa^2\alpha_2^{02}[2\gamma_1\gamma_2 v + v(\gamma_2^2 - v^2)] + 2\kappa^4\alpha_2^{04}v$$

$$-\kappa^4\alpha_1^{02}\alpha_2^{02}v - \kappa^2\alpha_1^{02}[(\gamma_1^2 - v^2)v + 2\gamma_1\gamma_2 v]\ , \tag{23}$$

$$R = \gamma_{11}^2\gamma_{22}^2 - 4\gamma_2^2\gamma_{11}^2 v^2 - 8\gamma_1\gamma_2 v^2\gamma_{22} - 2\kappa^2\alpha_1^{02}(\gamma_1\gamma_2 - v^2)^2 + 2\kappa^2\alpha_1^{02}(\gamma_1 + \gamma_2)^2 v^2 + 4\kappa^2\alpha_2^{02}[\gamma_1(\gamma_2^3 - 3\gamma_2 v^2)$$

$$-v^2(3\gamma_2^2 - v^2)] + 4\kappa^4\alpha_2^{04}\gamma_{22} - 4\kappa^4\alpha_1^{02}\alpha_2^{02}(\gamma_1\gamma_2 - v^2) + \kappa^4\alpha_1^{04}\gamma_{11} - 4\kappa^6\alpha_1^{02}\alpha_2^{04}\ , \tag{24}$$

$$I = 4\gamma_{11}\gamma_{22}\gamma_2 v + 2\gamma_1 v(\gamma_{22}^2 - 4\gamma_2^2 v^2) - 4\kappa^2\alpha_1^{02}v(\gamma_1 + \gamma_2)(\gamma_1\gamma_2 - v^2) + 4\kappa^2\alpha_2^{02}(3\gamma_1\gamma_2^2 - \gamma_1 v^2 + \gamma_2^3 - 3\gamma_2 v^2)v$$

$$+8\kappa^4\alpha_2^{04}\gamma_2 v - 4\kappa^4\alpha_1^{02}\alpha_2^{02}(\gamma_1 + \gamma_2)v + 2\kappa^4\alpha_1^{04}\gamma_1 v \tag{25}$$

and $\gamma_{11} = \gamma_1^2 - v^2$; $\gamma_{22} = \gamma_2^2 - v^2$. α_1^0 and α_2^0 are given by Eqs. (8) and (9).

Figures 2 and 3 are, respectively, the squeezing spectra of the coupled mode when the pump field is below or above the threshold. At $\nu=0$ and $\varepsilon_1=\varepsilon_1^{\text{thres.}}$, $S_{D_-\ \min}^{out}=-1/4$ corresponds to the perfect squeezing case.

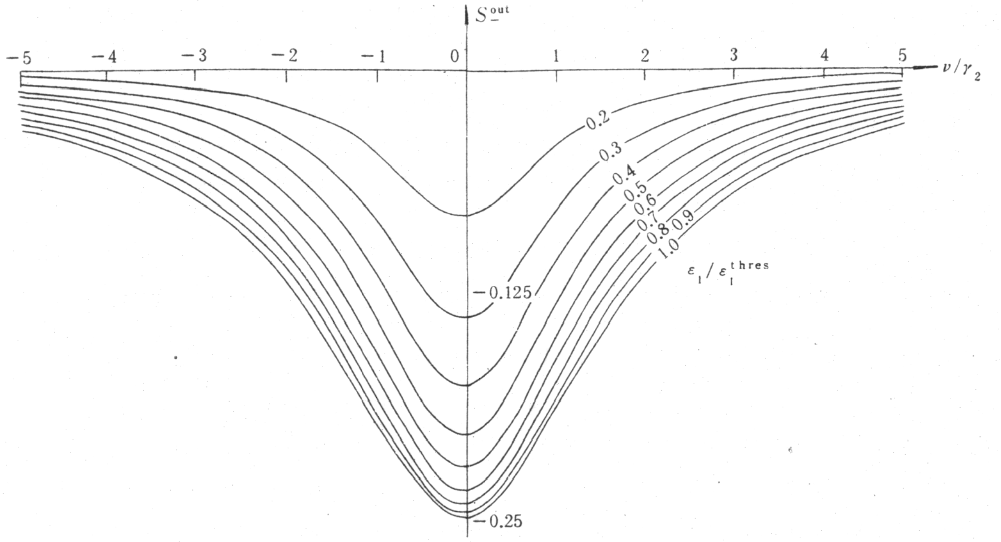

Fig. 2. The squeezing spectrum of the coupled mode with different driving field below threshold for $\gamma_1=10$, $\gamma_2=1$ and $\kappa=5$.

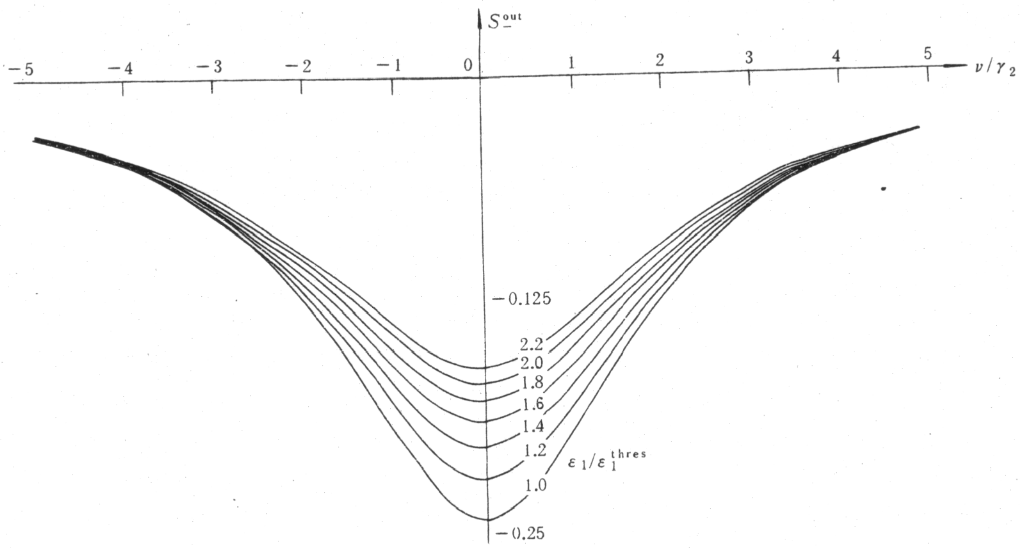

Fig. 3. The squeezing spectrum of the coupled mode with different driving field above threshold for $\gamma_1=10$, $\gamma_2=1$ and $\kappa=5$.

In Fig.4, we plot the fluctuation $<(\triangle D_-)^2>$ as a function of frequency ν and the pump field ε_1. As the pump field increases, at first the squeezing increases from zero and quickly reaches a maximum at the threshold, while the coupled mode is in the two-mode squeezed vacuum state, then if the pump field increases continuously, the squeezing will decrease gradually while the coupled mode is in the two-mode squeezed coherent state.

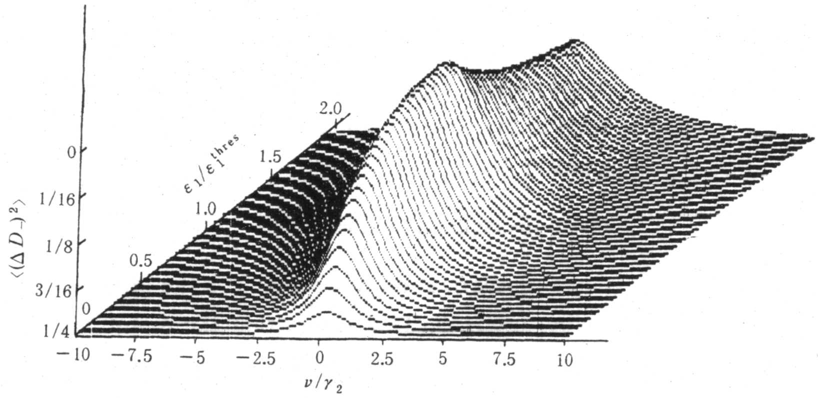

Fig. 4. Variation of the quadrature amplitudes of the coupled mode $<(\triangle D_-)^2>$ as a function of the dimensionless frequency ν/γ_2 and the driving field ε_1 for $\gamma_1=10$, $\gamma_2=1$ and $\kappa=5$.

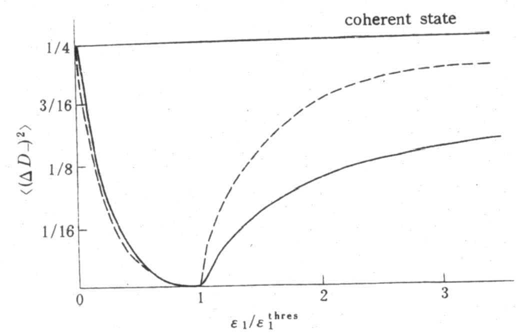

Fig. 5. Variation of the quadrature amplitudes of the coupled mode $<(\triangle D_-)^2>$ as a function of the pump field ε_1 (solid line). The dashed line is the result given by Reynaud *et al.* using the semiclassical treatment. ($\gamma_1=10$, $\gamma_2=1$ and $\kappa=5$)

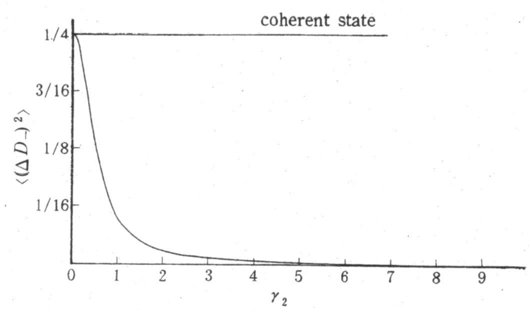

Fig. 6. Variation of the quadrature amplitudes of the coupled mode $<(\triangle D_-)^2>$ as a function of γ_2 of the signal mode at threshold. ($\gamma_1=10$, $\kappa=5$, $\nu=1$)

Figure 5 shows the variation of the squeezing versus the pump field, and the result of the semiclassical treatment obtained by Reynaud *et al.*[9] is also given here for comparison. It shows that both curves are almost the same below the threshold but are quite different above the threshold. The decrease of the squeezing above the threshold is not as fast as predicted by Reynaud, and there is still about 57 % squeezing at a field of double threshold.

In Fig. 6, we plot the squeezing as a function of the damping of signal at a fixed frequency and at the threshold. Here we consider only the damping caused by the output mirror, i.e., $\gamma_2=\gamma_{out}$. When the pump field is at the threshold, the larger the damping, the deeper the squeezing. But this effect is obvious only at very high frequency. Usually it will easily be saturated. Therefore, it is not realizable to increase the squeezing by increasing the damping of the output mirror and the pumping power.

C. The quantum correlation between the signal and idler modes

In the process of non-degenerate polarization, the signal and the idler modes themselves are not squeezed but their coupling mode is. This shows that the quantum correlation between the signal and idler modes plays a crucial role in the squeezing of the light fields. According to the definition given by Reid[6], we obtain the quantum correlation between the signal and idler modes

$$C=\frac{|\langle X_2\cdot X_3\rangle|}{\sqrt{(\langle X_2^2\rangle\ \langle X_3^2\rangle)}}\ , \tag{26}$$

where $X_2=a_2+a_2^+$; $X_3=a_3+a_3^+$.

For the case the light field is below the threshold, from Eq. (14) we have

$$C=\frac{4\kappa\alpha_1^0\gamma_2(A_{63}^2+B_{63}^2+A_{33}^2+B_{33}^2)}{8\kappa\alpha_1^0\gamma_2(A_{33}\ A_{33}+B_{63}\ B_{33})+D_1^2+D_2^2}\ , \tag{27}$$

where

$$A_{63}=\kappa\alpha_1^0(\gamma_1^2-v^2)(\gamma_2^2-v^2-\kappa^2\alpha_1^{02})-4\kappa\alpha_1^0\gamma_1\gamma_2 v^2\ , \tag{28}$$

$$B_{63}=2\kappa\alpha_1^0\gamma_2 v(\gamma_1^2-v^2)+2\kappa\alpha_1^0\gamma_1 v(\gamma_2^2-v^2-\kappa^2\alpha_1^{02})\ , \tag{29}$$

$$A_{33}=[\gamma_2(\gamma_1^2-v^2)-2\gamma_1 v^2](\gamma_2^2-v^2-\kappa^2\alpha_1^{02})-2\gamma_2 v^2[(\gamma_1^2-v^2)+2\gamma_1\gamma_2]\ , \tag{30}$$

$$B_{33}=2\gamma_2 v[\gamma_2(\gamma_1^2-v^2)-2\gamma_1 v^2]+[(\gamma_1^2-v^2)+2\gamma_1\gamma_2](\gamma_2^2-v^2-\kappa^2\alpha_1^{02})\ v\ , \tag{31}$$

$$D_1=(\gamma_1^2-v^2)(\gamma_2^2-v^2-\kappa^2\alpha_1^{02})-4\gamma_1\gamma_2 v^2\ , \tag{32}$$

$$D_2=2\gamma_2 v(\gamma_1^2-v^2)+2\gamma_1 v(\gamma_2^2-v^2-\kappa^2\alpha_1^{02})\ . \tag{33}$$

Similarly, for the field above the threshold, we have

$$C=\frac{4\kappa\alpha_1^0\gamma_2(A_1^2+B_1^2+A_2^2+B_2^2)}{8\kappa\alpha_1^0\gamma_2(A_1A_2+B_1B_2)+R^2+I^2}\ , \tag{34}$$

where A_1, A_2, B_1, B_2, R and I are given by Eqs. (20)–(25), and α_1^0, α_2^0 are given by Eqs. (8) and (9).

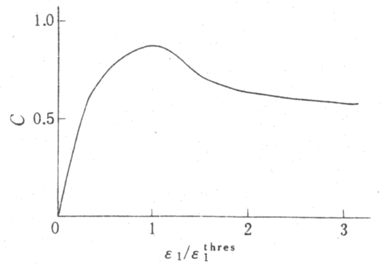

Fig. 7. The quantum correlation between the signal and the idler modes versus the pump field ε_1. ($\gamma_1=10$, $\gamma_2=1$, $\kappa=5$, $\nu=0.5$)

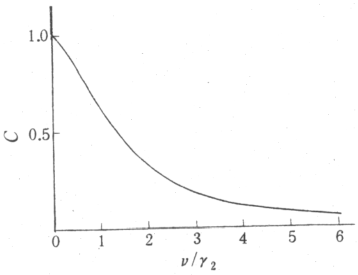

Fig. 8. The quantum correlation between the signal and the idler modes versus the dimensionless frequency ν/γ_2. ($\gamma_1=10$, $\gamma_2=1$, $\kappa=5$, $\varepsilon_1=2.5$)

The relationship between the quantum correlation and the pump field at a fixed frequency is depicted in Fig. 7, which shows that the quantum correlation is maximum at the threshold and gradually decreases as the field increases above the threshold, but the decrease of the correlation is much slower, as compared with that of the squeezing.

Figure 8 shows the quantum correlation as a function of the noise spectrum frequency ν. The perfect squeezing corresponds to $\nu=0$, $C=1$; and the quantum correlation reduces along with the increase of frequency. Generally, one can obtain a two-mode squeezed coherent state with considerable squeezing and quantum correlation in a quite wide bandwidth of frequency. The quantum correlation between the two modes expresses the characterisitics of the light field on a deeper level.

III. CONCLUSION

We have discussed the squeezing of the coupled mode in the output field for the process of the parametric down-conversion with its frequency degenerate but polarization non-degenerate. The coupled mode can be perfectly squeezed at the threshold. The decrease of squeezing above the threshold field is slower than that given according to the semiclassical treatment (Ref. [9]), and there is about 57% squeezing at the double threshold field. The result shows that it is possible to generate a two-mode squeezed coherent state with large amplitude.

We obtained the quantum correlation as a function of the noise frequency and the pump field. The maximum of the quantum correlation occurs at $\nu=0$ and $\varepsilon_1=\varepsilon_1^{thres.}$, and the quantum correlation gradually decreases as the pump field continuously increases above the threshold. But in a quite wide bandwidth the coupled mode still has very strong quantum correlation, if only the pump field is not too strong as compared with the threshold (for example $\varepsilon_1/\varepsilon_1^{thres.}=1.5$). The results indicate that it is very favourable to realize the quantum non-demolition measurement by the nonlinear process.

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