Nonclassical Properties and Comparison of Two Squeezings*

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Abstract The criteria for the existence of nonclassical effects of the signal and coupled modes are obtained through the Fokker-Planck equation in the nondegenerate optical parametric oscillator (NOPO). For the first time, the relation between the nonclassical depth and the intensity correlation and the comparison between the two-mode squeezing (TMS) and the intensity difference squeezing (IDS) have been presented.

Keywords: nonclassical effect, two-mode squeezing, intensity difference squeezing.

1 Introduction

Since the HBT experiment[1] there has been great interest in the field of nonclassical states which can be understood only by quantum-mechanical description. The most well studied manifestations in recent years are the photon antibunching and squeezing of the light field. Generation of the nonclassical field, especially the squeezed states, by the nondegenerate optical parametric oscillator (NOPO) has been an important subject of theoretical and experimental studies. The NOPO can generate not only the two-mode squeezing (TMS)[2] but also the intensity difference squeezing (IDS)[3]. J. Perina[4] and Haul[5] and other authors have discussed the process of the three-mode transient interactions, but have not considered the damping of signal and idler modes in an optical parametric oscillator cavity and have not analysed the relationship bewteen the photon correlation and the nonclassical properties by means of the nonclassical criteria. To our knowledge, up to now the nonclassical properties of the subharmonic and the coupled modes in an intracavity three-mode interactions have not been comprehensively analysed and the generation conditions and parameter dependences of two-mode squeezing (TMS) and intensity difference squeezing (IDS) have not been discussed by comparison either.

We investigate the quantum statistics of the signal and coupled modes in NOPO around the steady-state solution of the Fokker-Planck equation and here only consider the adiabatical approximation of the pump mode. Based on the criteria for the existence of nonclassical effects, the nonclassical photon statistics of the signal and coupled modes has been studied and the inseparable relation between the nonclassical properties of the two-mode radiation and the two-mode intensity correlation (photon

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correlation) has been demonstrated. From Mandel's nonclassical criterion[6], we find that although there are not any squeezings existing in each signal mode, there are some antibunching effects when the pump field is strong enough. This is similar to the result obtained in Ref. [4] for the transient process. Besides, by comparing the parameter dependence of TMS with IDS we confirm that IDS is much larger than TMS on the same conditions. In the meanwhile, optimal parameters for the two kinds of squeezings have been obtained. These discussions not only involve the deep insight of the non-classical properties of the light field, but also give a valuable theoretical reference for the design of experimental systems.

2 Theoretical Model

We assume that the nondegenerate parametric oscillation will occur in a suitable nonlinear medium inside an optical cavity tuned to allow the resonance of three modes which are the injected driving field at frequency ω, and two subharmonic fields with the same frequencies ($\omega_1=\omega_2=\omega$) and orthogonal polarizations. The total Hamiltonian of the system is

$$H=2\hbar\omega a_0^+a_0+\hbar\omega a_1^+a_1+\hbar\omega a_2^+a_2+i\hbar\kappa/2\cdot(a_0a_1^+a_2^+-a_0^+a_1a_2)$$
$$+i\hbar\varepsilon[a_0^+\exp(-2i\omega t)-a_0\exp(2i\omega t)]+\sum_{i=0}^{2}(a_i\Gamma_i^+-a_i^+\Gamma_i), \qquad (1)$$

where a_i^+ and a_i are the creation and annihilation operators for mode i ($i=0,1,2$), κ is the coupling constant, ε is proportional to the amplitude of the driving coherent field and Γ_i, Γ_i^+ are the heat bath operators. Using the standard techniques and the generalized P representation, we obtain the following Fokker-Planck equation[7]:

$$\frac{\partial P(\overline{\alpha})}{\partial t}=\Big[-\frac{\partial}{\partial\alpha_0}(\varepsilon-\gamma_0\alpha_0+\kappa\,\alpha_1\alpha_2)-\frac{\partial}{\partial\alpha_0^+}(\varepsilon-\gamma_0\alpha_0^++\kappa\,\alpha_1^+\alpha_2^+)$$
$$-\frac{\partial}{\partial\alpha_1}(-\gamma_1\alpha_1+\kappa\,\alpha_0\alpha_2^+)-\frac{\partial}{\partial\alpha_1^+}(-\gamma_1\alpha_1^++\kappa\,\alpha_0^+\alpha_2)$$
$$-\frac{\partial}{\partial\alpha_2}(-\gamma_2\alpha_2+\kappa\,\alpha_0\alpha_1^+)-\frac{\partial}{\partial\alpha_2^+}(-\gamma_2\alpha_2^++\kappa\,\alpha_0^+\alpha_1)$$
$$+\frac{\partial^2}{\partial\alpha_1\partial\alpha_2}\kappa\,\alpha_0+\frac{\partial^2}{\partial\alpha_1^+\partial\alpha_2^+}\kappa\,\alpha_0^++\sum_{i=0}^{2}\gamma_i n_i^{\text{th}}\frac{\partial^2}{\partial\alpha_i^+\partial\alpha_i}\Big]P(\overline{\alpha}), \qquad (2)$$

where $\overline{\alpha}=[\alpha_0,\alpha_0^+,\alpha_1,\alpha_1^+,\alpha_2,\alpha_2^+]$. It is difficult to solve this equation straightforwardly. Here we assume that the damping of the pump mode is much larger than the dampings of the signal and idler modes, so that the pump mode can be adiabatically eliminated and the mean numbers of thermal photons $n_i^{\text{th}}=(e^{\hbar\omega/kT}-1)^{-1}\ll 1$ at normal to low temperatures. Thus Eq. (2) can be simplified as

$$\frac{\partial P(\bar{\alpha})}{\partial t} = \Big[-\frac{\partial}{\partial \alpha_1}(-\gamma_1\alpha_1+\kappa\,\alpha_0\alpha_2^+) - \frac{\partial}{\partial \alpha_1^+}(-\gamma_1\alpha_1^+ + \kappa\,\alpha_0^+\alpha_2)$$
$$-\frac{\partial}{\partial \alpha_2}(-\gamma_2\alpha_2+\kappa\,\alpha_0\alpha_1^+) - \frac{\partial}{\partial \alpha_2^+}(-\gamma_2\alpha_2^+ + \kappa\,\alpha_0^+\alpha_1)$$
$$+\frac{\partial^2}{\partial\alpha_1\partial\alpha_2}\kappa\,\alpha_0 + \frac{\partial^2}{\partial\alpha_1^+\partial\alpha_2^+}\kappa\,\alpha_0^+\Big]P(\bar{\alpha}), \tag{3}$$

where $\alpha_0=(\varepsilon-\kappa\,\alpha_1\alpha_2)/\gamma_0$, $\alpha_0^+=(\varepsilon-\kappa\,\alpha_1^+\alpha_2^+)/\gamma_0$ and $\bar{\alpha}=[\alpha_1, \alpha_1^+, \alpha_2, \alpha_2^+]$.

As a further simplification, we assume $\gamma_1=\gamma_2$. Let $\partial P/\partial t=0$ in Eq. (2). We obtain the steady-state solution of Eq. (3)[8,9].

$$P(\bar{\alpha}) = A\cdot \exp[\psi(\bar{\alpha})], \tag{4}$$

where

$$\psi(\bar{\alpha})=2\alpha_1\alpha_1^+ + 2\alpha_2^+\alpha_2 + \left(\frac{2\gamma_0\gamma_1}{\kappa^2}-1\right)\ln(\kappa\,\alpha_1\alpha_2-\varepsilon)+\left(\frac{2\gamma_0\gamma_1}{\kappa^2}-1\right)\ln(\kappa\,\alpha_1^+\alpha_2^+-\varepsilon), \tag{5}$$

A is the normalization factor which is determined by

$$\int P(\bar{\alpha})\, d\mu\,(\bar{\alpha}) = 1\,. \tag{6}$$

So

$$A=[\int \exp[\,\psi(\bar{\alpha})\; d\mu\,(\bar{\alpha})]^{-1}\,. \tag{7}$$

3 Statistics Properties of Light Field

As we know, any normally ordered averages $\langle a_1^{+m} a_1^m a_2^{+n} a_2^n\rangle$ is

$$\langle a_1^{+m} a_1^m a_2^{+n} a_2^n\rangle = \int (\alpha_1^+\alpha_1)^m (\alpha_2^+\alpha_2)^n P(\bar{\alpha})\, d\mu\,(\bar{\alpha}) = A I_{mn}, \tag{8}$$

where

$$I_{mn} = \int (\alpha_1^+\alpha_1)^m (\alpha_2^+\alpha_2)^n \exp[\,\psi(\bar{\alpha})]\, d\mu\,(\bar{\alpha})\;. \tag{9}$$

The integral (9) can be evaluated by using tedious substitutes of variables (see Appendix). The first several terms of results are as folows:

$$I_{00} = A^{-1} = \frac{N_0}{[\Gamma(q+2)]^2}\sum_{p=0}^{\infty}\frac{1}{[(q+2)_p]^2}\cdot u^{2p}, \tag{10}$$

$$I_{10} = I_{01} = \frac{2N_0(u/2)^2}{[\Gamma(q+3)]^2}\sum_{p=0}^{\infty}\frac{p+1}{[(q+3)_p]^2}\cdot u^{2p}, \tag{11}$$

$$I_{20} = I_{02} = \frac{N_0 u^4}{2[\Gamma(q+4)]^2}\sum_{p=0}^{\infty}\frac{p+1}{p!\,[(q+4)_p]^2}\cdot u^{2p}, \tag{12}$$

$$I_{11} = \frac{N_0 u^2}{4[\Gamma(q+3)]^2}\sum_{p=0}^{\infty}\frac{(p+1)^2}{[(q+3)_p]^2}\cdot u^{2p}, \tag{13}$$

where $u=2\,\varepsilon/\kappa$, $q=2\gamma_0\gamma_1/\kappa^2-1$, $N_0=(\varepsilon/\kappa)^2\,\varepsilon^{2q}\,(q!)^2$, and similarly

$$\langle a_1\rangle=\langle a_2\rangle=0,$$

$$\langle (a_1)^2\rangle=\langle (a_2)^2\rangle=0,$$

$$I_{12}=\langle a_1a_2\rangle=\frac{N_0u}{2\Gamma(q+2)\Gamma(q+3)}\sum_{p=0}^{\infty}\frac{p+1}{(q+2)_p(q+3)_p}\cdot u^{2p}. \tag{14}$$

3.1 *The Nonclassical Depth of the Signal Mode*

As pointed out in Ref. [10], for the signal mode there exists no squeezing on any conditions, and the fluctuation of one of the quadratures of the signal mode is in general the noise of the coherent state. But we shall point that the signal and idler modes themselves are not the classical fields, they possess the nonclassical effect on certain conditions.

For a single mode, the well-known criterion for the existence of nonclassical effect is[6]

$$D^{(2)}=\langle n^{(2)}\rangle-\langle n^2\rangle<0\ , \tag{15}$$

where $\langle n^{(2)}\rangle=\langle a^+a^+a\,a\rangle$. For the signal mode in our system we obtain

$$D_1^{(2)}=(I_{00}I_{20}-I_{10}^2)/I_{00}^2. \tag{16}$$

Figure 1 shows the relation between the nonclassical depth $D_1^{(2)}$ of the signal mode and the pump field for various thresholds. At first the values of $D_1^{(2)}$ increases from zero to the maximum and then decreases when the pump field increases and at about double threshold, $D_1^{(2)}$ is less than zero, and the stronger the pump field, the deeper the nonclassical depth.

In fact, for a single mode, criterion (15) is identical with $g_1^{(2)}(0)<1$ because the second-order correlation function of the signal mode is

$$g_1^{(2)}(0)=\frac{\langle a_1^{+2}a_1^2\rangle}{\langle a_1^+a_1\rangle^2}=\frac{I_{20}I_{00}}{I_{10}^2} \tag{17}$$

and $D_1^{(2)}<0$, i.e. $I_{00}I_{20}<I_{10}^2$ means $g_1^{(2)}(0)<1$. That is, the nonclassical effect of the signal mode is the photon antibunching.

3.2 *The Nonclassical Depth of the Two-Mode Radiation*

A Simple generalized criterion for the existence of nonclassical effect in the two-mode radiation was established by Ching Tsung Lee in 1990[11] as follows:

$$D_{12}^{(2)}=\langle n_1^{(2)}\rangle+\langle n_2^{(2)}\rangle-2\langle n_1n_2\rangle<0, \tag{18}$$

and the smaller the $D_{12}^{(2)}$, the deeper the nonclassical depth.

We calculate the $D_{12}^{(2)}$ for the coupled mode in our system and have

$$D_{12}^{(2)}=2(I_{20}-I_{11})/I_{00}. \tag{19}$$

Figure 2 is $D_{12}^{(2)}$ as a function of pump field at different thresholds. We see that $D_{12}^{(2)}$

is always less than zero. That is to say, the coupled mode formed from the signal and idler modes is always the nonclassical field and the stronger the pump field or the smaller the threshold, the deeper the nonclassical depth.

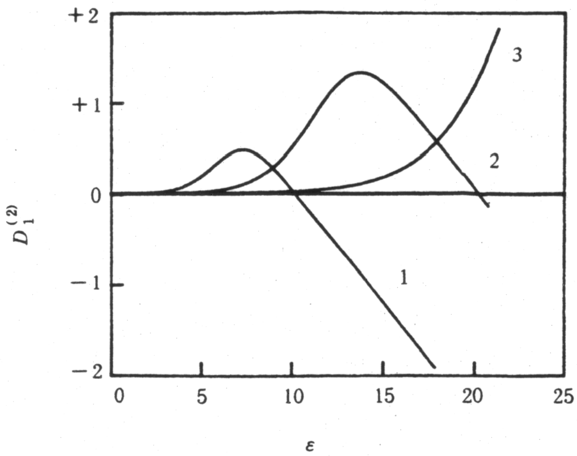

Fig. 1. The nonclassical depth of the signal mode as a function of pump field for $\kappa=1$. 1, $\varepsilon^{th}=5$; 2, $\varepsilon^{th}=10$; 3, $\varepsilon^{th}=20$.

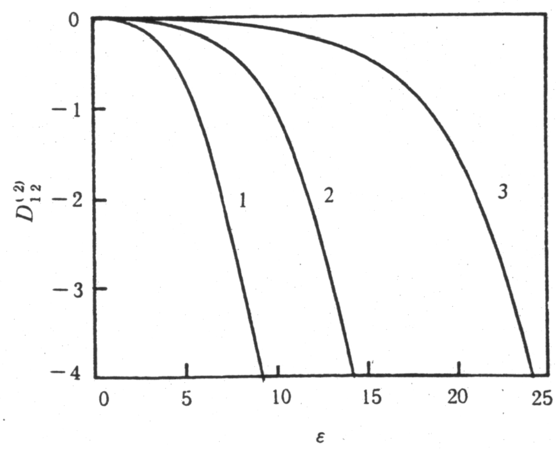

Fig. 2. The nonclassical depth of the two-mode radiation as a function of pump field for $\kappa=1$. 1, $\varepsilon^{th}=5$; 2, $\varepsilon^{th}=10$; 3, $\varepsilon^{th}=20$.

3.3 *The Intensity Correlation*

As discussed above, the nonclassical depth of the signal and idler modes are much less than that of the coupled mode. Even when there is no nonclassical properties in the signal or idler modes, there still exists deep nonclassical depth for the coupled mode. This results from the strong photon correlation or intensity correlation between the signal and idler photons which are generated and annihilated simultaneously. The intensity correlation can be characterized as follows[12]:

$$C=2[\langle a_1^+a_1a_2^+a_2\rangle-\langle a_1^+a_1\rangle\langle a_2^+a_2\rangle]$$
$$=2(I_{00}I_{11}-I_{10}^2)/I_{00}^2. \quad (20)$$

Figure 3 gives the relationship between the intensity correlation and the two-mode nonclassical depth. We can see that the stronger the intensity correlation is, the deeper the nonclassical depth is, and the linear dependence between them will appear when the intensity correlation is strong enough. It shows that the quantum correlation of the coupled mode results in the nonclassical properties of the two-mode radiation field.

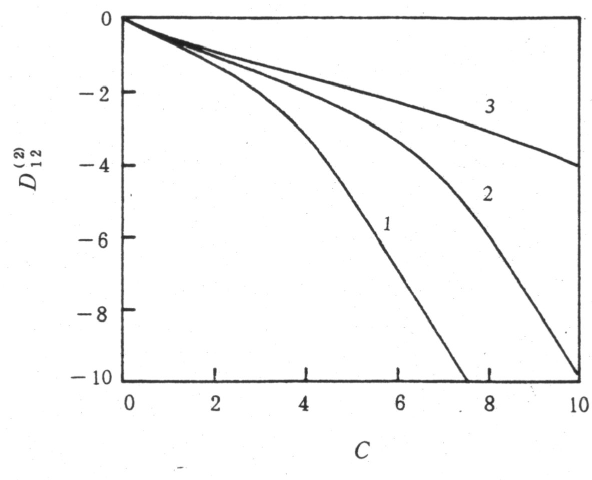

Fig. 3. The nonclassical depth of the two-mode radiation as a function of intensity correlation for $\kappa=1$. 1, $\varepsilon^{th}=5$; 2, $\varepsilon^{th}=10$; 3, $\varepsilon^{th}=20$.

4 The Comparison Between TMS and IDS

4.1 *The Two-Mode Squeezing*

The coupled mode is defined[10] as

$$d=(a_1+a_2)/\sqrt{2}\ ,\qquad d^+=(a_1^+ +a_2^+)/\sqrt{2}\ , \tag{21}$$

and its two quadrature components as

$$D_+=1/2\sqrt{2}\ .\ (a_1+a_2+a_1^+ +a_2^+)\ ,$$
$$D_-=1/2\sqrt{2}\,i\ .\ (a_1+a_2-a_1^+ -a_2^+)\ . \tag{22}$$

Then the fluctuations of the two components are

$$\langle(\Delta D_+)^2\rangle=1/4+(I_{10}+I_{12})/2I_{00}\ ,$$
$$\langle(\Delta D_-)^2\rangle=1/4+(I_{10}-I_{12})/2I_{00}\ . \tag{23}$$

If the fluctuations are less than 1/4, the coupled mode is squeezed and the squeezing degree is

$$S_p=[1/4-\langle\Delta D_-)^2\rangle]/(1/4)\ . \tag{24}$$

4.2 *The Intensity Difference Squeezing*

Consider the fluctuation of the intensity difference of the signal and idler modes (I_1-I_2):

$$\langle\Delta^2(I_1-I_2)\rangle=\langle(I_1-I_2)^2\rangle-\langle I_1-I_2\rangle^2=2(I_{20}+I_{10}-I_{11})/I_{00}, \tag{25}$$

where I_1 and I_2 are the intensities of the signal and idler modes, respectively. According to Ref. [13], the noise of the "mixed" field of the signal and idler modes $\langle\Delta^2(I_1+I_2)\rangle$ is the shot noise, i.e. the standard quantum limit:

$$\langle\Delta^2(I_1+I_2)\rangle=\langle(I_1+I_2)^2\rangle-\langle I_1+I_2\rangle^2=2(I_{20}+I_{11}+I_{10})/I_{00}-4(I_{10}/I_{00})^2. \tag{26}$$

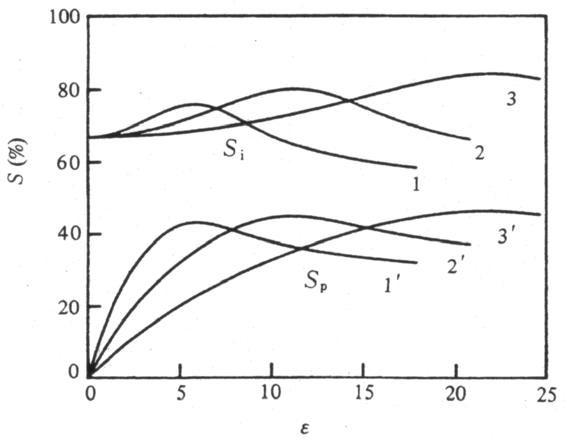

Fig. 4. The comparison of the two-mode squeezing S_p and the intensity difference squeezing S_i as the functions of pump field for $\kappa=1$ and 1, 1′, $\varepsilon^{th}=5$; 2, 2′, $\varepsilon^{th}=10$; 3, 3′, $\varepsilon^{th}=20$.

So the intensity difference squeezing degree is

$$S_i=[\langle\Delta^2(I_1+I_2)\rangle-\langle\Delta^2(I_1-I_2)\rangle]/\langle\Delta^2(I_1+I_2)\rangle. \tag{27}$$

Figure 4 gives the comparison of these two squeezings. Both squeezings increase and reach the maxima when the pump field increases from zero to a little above the threshold, and then decrease gradually. Comparing 1, 1′, 2, 2′, and 3, 3′, we find that both squeezings reach the maxima under nearly the same conditions. When the ε approaches zero, $S_i=66.7\%$, which means that the IDS always exists and has a large squeezing provided that the pump field exists. At $\varepsilon=0$, that is, $\langle\Delta_2(I_1+I_2)\rangle=\langle\Delta^2(I_1+I_2)\rangle=0$, the IDS is meaningless. But when ε approaches 0, the TMS approaches zero. The IDS is much larger than the TMS on the same conditions, and the former changes slower than the latter and this is more obvious below the threshold. For example, when $\varepsilon^{th}=5$, the maxima of S_{imax} and S_{pmax} are about 75% and 42%, respectively. When ε decreases to 2.5, S_{imax} and S_{pmax} decrease to 70% and 30%, respectively.

From Figs. 5—7, we give respectively the IDS and TMS as the functions of parameters γ_0, γ_1 and κ. Obviously, there is an optimal parameter value corresponding

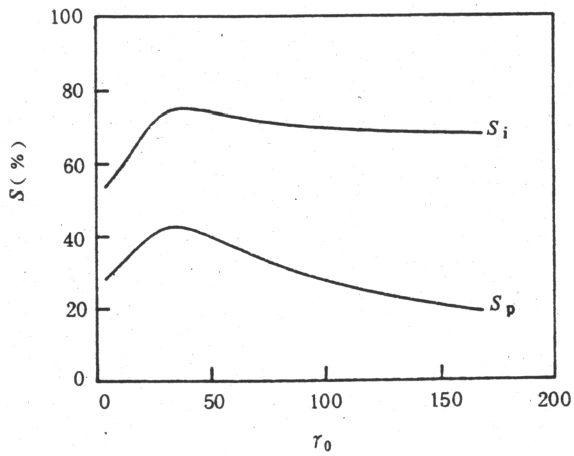

Fig. 5. The two-mode squeezing and the intensity difference squeezing as the functions of the damping of the pump mode for $\kappa=2$, $\varepsilon=5$ and $\gamma_1=1$.

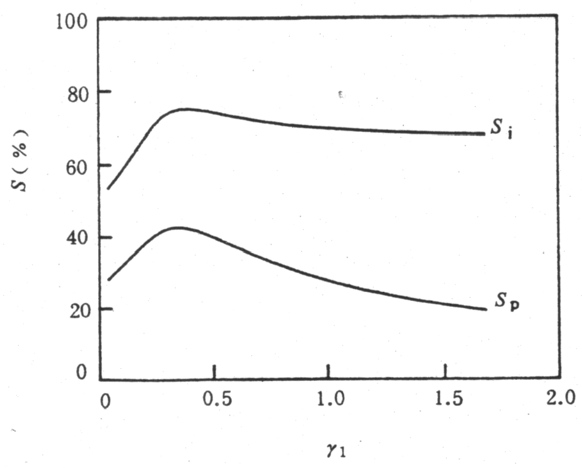

Fig. 6. The two-mode squeezing and the intensity difference squeezing as the functions of the damping of the signal mode for $\kappa=2$, $\varepsilon=5$ and $\gamma_0=1$.

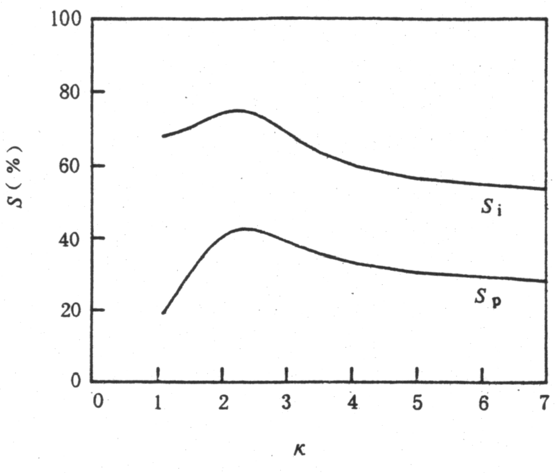

Fig. 7. The two-mode squeezing and the intensity difference squeezing as the functions of the coupling coefficient for $\varepsilon=5$, $\gamma_0=100$ and $\gamma_1=0.5$.

to the maximum squeezing in each case. For example, for $\varepsilon=5$, $\gamma_1=1$, $\kappa=2$, the optimal values of γ_0 are 35.5 and 39.0, corresponding to the maximum squeezings of TMS and IDS, respectively.

Having an insight into the physical implication, the IDS only involves the fluctuation of the intensity and it is independent of phase. In the parametric down conversion, pairs of signal and idler photons are created and annihilate simultaneously; therefore, counting the photons emitted in the signal and idler modes during a time much longer than $(1/2\ \gamma_1)$ should give exactly equal numbers. On the ideal conditions (no cavity damping and the quantum efficient is 1), the perfect quantum noise suppression for IDS can be obtained[3], and this IDS is not strongly dependent on the pump power, the coupling constant and the quality factor of the cavity. But the TMS is strongly dependent on the phase and the other cavity parameters[3, 12].

5 Conclusions

We have discussed the nonclassical effect of the signal and coupled modes in NOPO by the solution of the Fokker-Planck equation for generalized P representation of the signal and idler modes and obtained the criteria for the existence of non-classical effects of the signal and coupled modes. For the signal mode the nonclassical effect of photon antibunching appears when the pump field is strong enough (larger than double threshold). For the coupled mode formed by the signal and idler modes, the nonclassical effect resulting from the intensity correlation of the twin beams always exists and the nonclassical depth increases as the intensity of pump field is increased.

The comparison between the TMS and IDS shows that the IDS is much larger than the TMS under the same conditions and so it is easier to obtain IDS than TMS; in other words, it is more difficult to obtain the TMS with large squeezing. This is consistent with the experimental results up to now[2, 3]. On the other hand, there are some inherent correlations between the two kinds of squeezing. From Figs. 4—7, we see that the conditions for achieving the maximum squeezings are almost the same. This indicates that we may design an experiment to generate both of the squeezings.

Appendix

To calculate integrals of (9):

$$I_{mn}=\int(\alpha_1^+\alpha_1)^m(\alpha_2^+\alpha_2)^n\exp[\psi(\overline{\alpha})]d\mu(\overline{\alpha}), \tag{A1}$$

substituting

$$\begin{cases} t=\alpha_1^+, \quad v=\alpha_2, \\ x=(\kappa/\varepsilon)\,\alpha_1\alpha_2, \\ y=(\kappa/\varepsilon)\,\alpha_1^+\alpha_2^+ \end{cases} \tag{A2}$$

into (A1), we find

$$I_{mn}=\left(\frac{\varepsilon}{\kappa}\right)^{m+n+2}\varepsilon^{2q}\int\left(\frac{t}{v}\right)^{m-n}x^m y^n(x-1)^q(y-1)^q\exp\left[ux\frac{t}{v}+uy\frac{v}{t}\right]\frac{d\overline{x}}{tv}, \tag{A3}$$

where $u=2\,\varepsilon/\kappa$, $q=2\,\gamma_0\,\gamma_1/\kappa^2-1$ and $d\overline{x}=dxdydtdv$.

Take the further substitutions

$$z=(\kappa/\varepsilon)\,tv,\qquad w=t/v \tag{A4}$$

and consider the integral in z; then (A3) becomes

$$I_{mn}=(\varepsilon/\kappa)^{m+n+2}\varepsilon^{2q}\int w^{m-n-1}x^m y^n(1-x)^q(1-y)^q\exp\left[uwx+(u/w)y\right]dxdydw. \tag{A5}$$

Using formula[14]

$$[\Gamma(b-a)\,\Gamma(a)]\,/\,\Gamma(b)\,M\,[a,b;c]=\int t^{a-1}(1-t)^{b-a-1}e^{ct}dt, \tag{A6}$$

where $\Gamma(a)$ is the Γ Function and $M(a,b;c)$ is the Kummer function

$$M(a,b;c)=1+\frac{a}{b}\,c+\frac{(a)_2}{(b)_2}\,\frac{c^2}{2!}+\cdots+\frac{(a)_n}{(b)_n}\,\frac{c^n}{n!}+\cdots, \tag{A7}$$

$$(a)_0=1,\ (a)_n=a(a+1)(a+2)\cdots(a+n-1); \tag{A8}$$

and considering the integrals in x and y for (A5), we obtain

$$I_{mn}=N_0\left(\frac{\varepsilon}{\kappa}\right)^{m+n}\frac{\Gamma(m+1)\Gamma(n+1)}{\Gamma(m+q+2)\Gamma(n+q+2)}\int w^{m-n-1}M\,[\,m+1,m+q+2;uw\,]$$

$$\cdot\,M\,[\,n+1,n+q+2;\ u/w\,]\,dw, \tag{A9}$$

where $N_0=(\varepsilon/\kappa)^2\,\varepsilon^{2q}\,(\,q!\,)^2$.

As an example, we calculate I_{00}. From Eq. (A9) we have

$$I_{00}=N_0/\,[\Gamma(q+2)]^2\int w^{-1}M\,[1,q+2;uw\,]\cdot M\,[1,q+2;u/w\,]\,dw. \tag{A10}$$

Recalling the definition of Kummer function of (A7), we obtain

$$M\,[1,\ q+2;\ uw]=1+uw/(q+2)+u^2w^2/\,[(q+2)\,(q+3)]+u^3w^3/[(q+2)\,(q+3)\,(q+4)]+\cdots,$$

$$M\,[1,\ q+2;\ u/w]=1+u/w(q+2)]+u^2/\,[w^2(q+2)\,(q+3)]+u^3/[w^3(q+2)\,(q+3)\,(q+4)]+\cdots.$$

Thus we can readily obtain the coefficient of w^{-1} for the series integrand $w^{-1}M[1,\ q+2;\ uw]\cdot M[1,q+2;\ u/w]$ in (A10):

$$1+[u\,/(q+2)]^2+[u^2/(q+2)(q+3)]^2+[u^3/(q+2)\,(q+3)\,(q+4)]^2+\cdots$$

$$=\sum_{p=0}^{\infty}\frac{1}{[(q+2)_p]^2}\cdot u^{2p}. \tag{A11}$$

Using the residue theorem, taking a closed integration contour around $w=0$, we obtain

$$I_{00}=\frac{N_0}{[\Gamma(q+2)]^2}\sum_{p=0}^{\infty}\frac{1}{[(q+2)_p]^2}\cdot u^{2p}. \tag{A12}$$

That is the result of (10). Other moments of I_{mn} may be calculated similarly.

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