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HOW DOES AN IMPERFECT SYSTEM AFFECT THE MEASUREMENT OF SQUEEZING? *

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The effect of an imperfect system on the measurement of squeezing is studied. It is shown that the imperfectly efficient detectors and the unbalance of the beam-splitter will reduce the measured squeezing. The effects on quadrature phase squeezing and photon number squeezing are different from the usual balanced homodyne detection.

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I. INTRODUCTION

The phase-sensitive detection of quantum noise, such as homodyne detection, has been extensively discussed since it was originated principally by Yuen and Shapiro.[1-3] With this detection, the quantum noise of nonclassical light was measured, such as quadrature phase squeezing.[4,5] Measurement of photon number squeezing can also be realized by using an alternative scheme of homodyne detection[6,7], i.e., the balanced detection. However, most of the discussions about the squeezing measurement considered that the beam-splitter and the photodetectors are perfect or balanced. Practically, the beam-splitter and the photodetectors are imperfect. Loudon and Knight considered the imperfectly efficient detector in the direct-detection experiment.[5] However, the effects of the imperfect detectors and beam-splitter in the usual balanced homodyne detection is not clear. When the two parts of the 50-50 beam-splitter are not exactly equal and the quantum efficients of the two detectors are unbalanced (this is the usual case), how is the measured squeezing affected? Does the imperfect detection make the same effect on differently balanced homodyne systems?

The effect of the imperfect beam-splitter and detectors in the general homodyne detection on the squeezing of light is studied in this paper. Two typically schematic arrangement of homodyne detectors are discussed. The first one is the usual balanced homodyne detection, with which the quadrature squeezing is measured with strong local oscillator. The second one is for the measurement of photon number squeezing, in

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which the input beam is split into two equal parts by a beam-splitter. It is shown that the measured quantum noise is affected in different ways by the non-ideal system.

II. GENERAL HOMODYNE DETECTION

Figure 1 is the general scheme of balanced homodyne detection in which a and b are the input modes, BS is a beam-splitter with the amplitude reflection r and transmission coefficients t. The output mode destruction operators are given by [5]

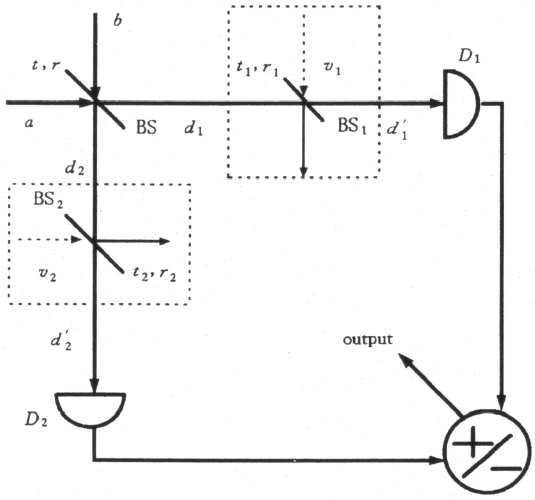

Fig. 1. General homodyne detection. $a(b)$: input modes; v's: vacuum inputs; d's: combined modes; r's and t's: amplitue reflection and transmission coefficients, respectively; BS': beam-splitters; D's: photodetectors; + / − :combiner.

$$d_1 = rb + ta, \quad d_1^+ = r^* b^+ + t^* a^+, \tag{1}$$
$$d_2 = tb + ra, \quad d_2^+ = t^* b^+ + r^* a^+, \tag{2}$$

where unitarity of the coupling coefficients requires that

$$|t|^2 + |r|^2 = 1, \tag{3}$$
$$t^* r + r^* t = 0. \tag{4}$$

The quantum efficients of the photodetectors D_1 and D_2 are η_1 and η_2, respectively. The imperfectly efficient detectors can be modelled by the arrangement shown in the dashed box in Fig. 1, where the inefficiency in detection is ascribed to the loss of light at the beam-splitters BS$_1$ and BS$_2$, which transmit only fractions $\eta_1{}^{1/2}$ and $\eta_2{}^{1/2}$ of the incident amplitudes, respectively. [5,8] The destruction operators after BS$_1$ and BS$_2$ are

$$d_1{}' = t_1 d_1 + r_1 v_1 = t_1 rb + t_1 ta + r_1 v_1, \tag{5}$$
$$d_2{}' = t_2 d_2 + r_2 v_2 = t_2 tb + t_2 ra + r_2 v_2, \tag{6}$$

and their Hermitian conjugates, where t_1, r_1 and t_2, r_2 are the amplitude transmission and reflection coefficients of the BS$_1$ and BS$_2$, respectively, and $|t_1| = \eta_1^{1/2}$, $|t_2| = \eta_2^{1/2}$, $|t_1|^2 + |r_1|^2 = |t_2|^2 + |r_2|^2 = 1$. And v_1 and v_2 represent an inevitable admixture of a vacuum field from the other input port to the beam-splitters BS_1 and BS_2, respectively. The field d_1' and d_2' are now assumed to be detected with 100% efficiencies. The photocurrents are then subtracted or summed by the combiner +/−. What we are concerned is about the quantum noise output from the +/− combiner. Though the photodetectors only detect the slowly varying of the optical field, they response the photon number operator effectively. The measured noise is the variance of the difference or sum of the photon numbers detected by D_1 and D_2, because the response time of the photodetectors is much shorter compared with the fluctuation period.[5] That is to say, the measured noise is

$$< \Delta^2 I_\pm > = < (d_1'^+ d_1' \pm d_2'^+ d_2')^2 > - < d_1'^+ d_1' \pm d_2'^+ d_2' >^2, \quad (7)$$

where + and − correspond to the +/− combiner and

$$\begin{aligned} d_1'^+ d_1' = & t_1^* r^* t_1 r b^+ b + t_1^* r^* t_1 t b^+ a + t_1^* r^* r_1 b^+ v_1 \\ & + t_1^* t^* t_1 r a^+ b + t_1^* t^* t_1 t a^+ a + t_1^* t^* r_1 a^+ v_1 + r_1^* t_1 r v_1^+ b \\ & + r_1^* t_1 t v_1^+ a + r_1^* r_1 v_1^+ v_1, \end{aligned} \quad (8)$$

$$\begin{aligned} d_2'^+ d_2' = & t_2^* t^* t_2 t b^+ b + t_2^* t^* t_2 r b^+ a + t_2^* t^* r_2 b^+ v_2 \\ & + t_2^* r^* t_2 t a^+ b + t_2^* r^* t_2 r a^+ a + t_2^* r^* r_2 a^+ v_2 + r_2^* t_2 t v_2^+ b \\ & + r_2^* t_2 r v_2^+ a + r_2^* r_2 v_2^+ v_2. \end{aligned} \quad (9)$$

According to Eqs. (7) − (9), we have

$$\begin{aligned} < \Delta^2 I_\pm > = & A_\pm^2 < b^+ bb^+ b > + A_\pm B_\pm < b^+ bb^+ a > + A_\pm C_\pm < b^+ ba^+ b > \\ & + A_\pm D_\pm < b^+ ba^+ a > + B_\pm A_\pm < b^+ ab^+ b > + B_\pm^2 < b^+ ab^+ a > \\ & + B_\pm C_\pm < b^+ aa^+ b > + B_\pm D_\pm < b^+ aa^+ a > + C_\pm A_\pm < a^+ bb^+ b > \\ & + C_\pm B_\pm < a^+ bb^+ a > + C_\pm^2 < a^+ ba^+ b > + C_\pm D_\pm < a^+ ba^+ a > \\ & + D_\pm A_\pm < a^+ ab^+ b > + D_\pm B_\pm < a^+ ab^+ a > + D_\pm C_\pm < a^+ aa^+ b > \\ & + D_\pm^2 < a^+ aa^+ a > + (|t_1|^2 |r_1|^2 t^* r + |t_2|^2 |r_2|^2 r^* t) < a^+ b > \\ & + (|t_1|^2 |r_1|^2 |t|^2 + |t_2|^2 |r_2|^2 |r|^2) < a^+ a > \\ & + (|t_1|^2 |r_1|^2 r^* t + |t_2|^2 |r_2|^2 t^* r) < b^+ a > \\ & + (|t_1|^2 |r_1|^2 |r|^2 + |t_2|^2 |r_2|^2 |t|^2) < b^+ b > \\ & - [A_\pm < b^+ b > + B_\pm < b^+ a > + C_\pm < a^+ b > + D_\pm < a^+ a >]^2, \end{aligned} \quad (10)$$

where

$$A_\pm = |t_1|^2 |r|^2 \pm |t_2|^2 |t|^2, \quad (11a)$$

$$B_\pm = |t_1|^2 r^* t \pm |t_2|^2 t^* r, \quad (11b)$$

$$C_\pm = |t_1|^2 t^* r \pm |t_2|^2 r^* t, \quad (11c)$$

$$D_\pm = |t_1|^2 |t|^2 \pm |t_2|^2 |r|^2. \quad (11d)$$

We will consider two typical cases in the following.

III. MEASUREMENT OF SINGLE-MODE SQUEEZED VACUUM STATES

A single-mode squeezed vacuum-state is defined as [9]

$$| 0_\xi\rangle = S(\xi) | 0\rangle, \tag{12}$$

where

$$\xi = z e^{i\theta} \qquad (0 \leqslant z \leqslant \infty, 0 \leqslant \theta \leqslant 2\pi) \tag{13}$$

is the complex squeeze parameter; $S(\xi)$ is the unitary squeeze operator and $|0\rangle$ is the vacuum state. In order to measure the squeezing of the squeezed vacuum state, let the input a in Fig. 1 be the squeezed vacuum state $|0_\xi\rangle$ and b the coherent state $|\beta\rangle$ which is called local oscillator, thus we have

$$b |\beta\rangle = \beta |\beta\rangle. \tag{14}$$

Using the following relations[5]

$$S^{-1}(\xi) a S(\xi) = a \cosh z - a^+ e^{i\theta} \sinh z, \tag{15a}$$

$$S^{-1}(\xi) a^+ S(\xi) = a^+ \cosh z - a e^{-i\theta} \sinh z, \tag{15b}$$

we can prove for the squeezed vacuum and coherent states that

$$< b^+ bb^+ b > = |\beta|^4 + |\beta|^2, \tag{16a}$$

$$< b^+ ba^+ a > = < a^+ ab^+ b > = |\beta|^2 \sinh^2 z, \tag{16b}$$

$$< b^+ ab^+ a > = - \beta^{*2} e^{i\theta} \sinh z \cosh z, \tag{16c}$$

$$< b^+ aa^+ b > = |\beta|^2 \cosh^2 z, \tag{16d}$$

$$< a^+ bb^+ a > = (1 + |\beta|^2) \sinh^2 z, \tag{16e}$$

$$< a^+ ba^+ b > = - \beta^2 e^{-i\theta} \sinh z \cosh z, \tag{16f}$$

$$< a^+ aa^+ a > = 2\sinh^2 z \cosh^2 z + \sinh^4 z, \tag{16g}$$

$$< b^+ b > = |\beta|^2, \tag{16h}$$

$$< a^+ a > = \sinh^2 z. \tag{16i}$$

Other terms in Eq. (10) are zero. In this configuration, we just consider the "$-$" of the combiner. From Eqs. (16) we get

$$\begin{aligned} < \Delta^2 I_- > = {} & A_-^2 |\beta|^2 - B_-^2 \beta^{*2} \sinh z \cosh z\, e^{i\theta} - C_-^2 \beta^2 \sinh z \cosh z\, e^{-i\theta} + B_- C_- |\beta|^2 (\sinh^2 z \\ & + \cosh^2 z) + |\beta|^2 (|t_1|^2 |r_1|^2 |r|^2 + |t_2|^2 |r_2|^2 |t|^2) + B_- C_- \sinh^2 z \\ & + 2D_-^2 \sinh^2 z \cosh^2 z + (|t_1|^2 |r_1|^2 |t|^2 + |t_2|^2 |r_2|^2 |r|^2) \sinh^2 z, \end{aligned} \tag{17}$$

where

$$A_-^2 = (|t_1|^2 |r|^2 - |t_2|^2 |t|^2)^2, \tag{18a}$$

$$B_-^2 = C_-^{*2} = |t_1|^4 r^{*2} t^2 + |t_2|^2 t^{*2} r^2 - 2|t_1|^2 |t_2|^2 |t|^2 |r|^2, \tag{18b}$$

$$B_- C_- = |r|^2 |t|^2 (|t_1|^2 + |t_2|^2)^2, \tag{18c}$$

$$D_-^2 = (|t_1|^2 |t|^2 - |t_2|^2 |r|^2)^2. \tag{18d}$$

Consider $|\beta|^2 \gg 1$, i.e., the local oscillator is much stronger than the squeezed vacuum input, the last three terms in Eq. (17) are small and can be neglected. By using the relations (3) and (4), Eq. (17) can be simplified

$$\begin{aligned} < \Delta^2 I_- > \approx {} & [|t_1|^2 |r|^2 + |t_2|^2 |t|^2 + 2|t|^2 |r|^2 (|t_1|^2 \\ & + |t_2|^2)^2 (\sinh^2 z + \sinh z \cosh z \cos(\theta - 2\varphi_b))] |\beta|^2, \end{aligned} \tag{19}$$

where φ_b is the phase of the local light. When the input a is the vacuum, i.e., $z \doteq 0$, the output corresponds to the standard quantum limit (SQL), which is

$$\mathrm{SQL} = (|t_1|^2|r|^2 + |t_2|^2|t|^2)|\beta|^2. \tag{20}$$

For the perfect process, $|t_1|^2 = |t_2|^2 = 1$ and $|r|^2 = |t|^2 = 1/2$, thus

$$\mathrm{SQL}_{\mathrm{ideal}} = |\beta|^2. \tag{21}$$

For the squeezed vacuum input, the detected noise varies with the phase of the local light φ_b. When $\varphi_b = (\theta - \pi)/2$, the minimum noise, which corresponds to the noise of the squeezed quadrature phase, is

$$<\Delta^2 I_->_{\mathrm{QP}} = [|t_1|^2|r|^2 + |t_2|^2|t|^2 + 2|t|^2|r|^2(|t_1|^2 + |t_2|^2)^2(\sinh^2 z - \sinh z \cosh z)]|\beta|^2. \tag{22}$$

And for the ideal measurement, we get, similarly,

$$<\Delta^2 I_->_{\mathrm{QPideal}} = e^{-2z}|\beta|^2. \tag{23}$$

This is consistent with the results obtained by London and Knight.[5] The squeezing for the perfect measurement is

$$S_{\mathrm{QPideal}} = \frac{\mathrm{SQL}_{\mathrm{ideal}} - <\Delta^2 I_->_{\mathrm{QPideal}}}{\mathrm{SQL}_{\mathrm{ideal}}} = 1 - e^{-2z}, \tag{24}$$

and for the imperfect system, the measured quadrature phase squeezing is

$$S_{\mathrm{QPm}} = \frac{\mathrm{SQL} - <\Delta^2 I_->_{\mathrm{QP}}}{\mathrm{SQL}} = \frac{(\eta_1 + \eta_2)^2 RT}{\eta_1 R + \eta_2 T} S_{\mathrm{QPideal}}, \tag{25}$$

where R and T are, respectively, the intensity reflectance and transmittance of the beam-splitter BS.

If the beam-splitter is ideal, i.e., $R = T = 1/2$, then

$$S_{\mathrm{QPm}} = \frac{1}{2}(\eta_1 + \eta_2) S_{\mathrm{QPideal}}, \tag{26}$$

and for the perfect detectors, i.e., $\eta_1 = \eta_2 = 1$,

$$S_{\mathrm{QPm}} = 4RTS_{\mathrm{QPideal}}. \tag{27}$$

IV. MEASUREMENT OF THE PHOTON NUMBER SQUEEZING

In the measurement of photon number squeezing, we used an alternative scheme of balanced detection. Let the input a be a single-mode field and b the vacuum $|0\rangle$ in Fig. 1. The $+/-$ combiner gives the full intensity noise and the shot noise limit (SNL).[6,7] In this case, most of the terms in Eq. (10) are zero and the nonzero terms are the following:

$$C_\pm B_\pm < a^+ bb^+ a > = C_\pm B_\pm < a^+ a > = C_\pm B_\pm < N >, \tag{28a}$$

$$D_\pm^2 < a^+ aa^+ a > = D_\pm^2 < N^2 >, \tag{28b}$$

and

$$< a^+ a > = < N >, \tag{28c}$$

where we have let $N = a^+ a$, which is the photon number operator. It is easy to show that

$$<\Delta^2 I_\pm>_{\mathrm{PN}} = (B_\pm C_\pm + |t_1|^2|r_1|^2|t|^2 + |t_2|^2|r_2|^2|r|^2) < N > + D_\pm^2 (< N^2 > - < N >^2). \tag{29}$$

In an ideal measurement,

$$B_+ C_+ = 1, \quad D_+^2 = 0, \tag{30a}$$

$$B_- C_- = 0, \quad D_-^2 = 1. \tag{30b}$$

So we have

$$< \Delta^2 I_- >_{\text{PNideal}} = < N >, \tag{31}$$

and

$$< \Delta^2 I_+ >_{\text{PNideal}} = < N^2 > - < N >^2 = < \Delta^2 N >, \tag{32}$$

which are exactly the SNL and the full photon number noise of the input a mode, respectively. The ideal photon number squeezing is

$$S_{\text{PNideal}} = \frac{\text{SNL}_{\text{ideal}} - < \Delta^2 I_+ >_{\text{PNideal}}}{\text{SNL}_{\text{ideal}}} = \frac{< N > - < \Delta^2 N >}{< N >}. \tag{33}$$

Considering the non-ideal measurement, we have

$$\begin{aligned} S_{\text{PNm}} &= \frac{<\Delta^2 I_- >_{\text{PN}} - < \Delta^2 I_+ >_{\text{PN}}}{<\Delta^2 I_- >_{\text{PN}}} \\ &= \frac{(B_+ C_+ - B_- C_-) < N > + (D_+^2 - D_-^2) < \Delta^2 N >}{(|t_1|^2 |t|^2 + |t_2|^2 |r|^2) < N > - (|t_1|^2 |t|^2 - |t_2|^2 |r|^2)^2 (< N > - < \Delta^2 N >)} \\ &= \frac{4RT\eta_1 \eta_2 S_{\text{PNideal}}}{(\eta_1 T + \eta_2 R) - (\eta_1 T - \eta_2 R)^2 S_{\text{PNideal}}}, \end{aligned} \tag{34}$$

where $B_\pm$, $C_\pm$ and $D_\pm$ are given by Eqs. (11). If the beam-splitter is balanced, then

$$S_{\text{PNm}} = \frac{2\eta_1 \eta_2 S_{\text{PNideal}}}{(\eta_1 + \eta_2) - (\eta_1 - \eta_2)^2 S_{\text{PNideal}}}, \tag{35}$$

and if the quantum efficiencies of the detectors are also balanced, i.e., $\eta_1 = \eta_2 = \eta$, we have

$$S_{\text{PNm}} = \eta S_{\text{PNideal}}. \tag{36}$$

If the photodetectors are perfect, and the beam-splitter are unbalanced, then

$$S_{\text{PNm}} = \frac{4RTS_{\text{PNideal}}}{1 - (T - R)^2 S_{\text{PNideal}}}. \tag{37}$$

In order to discuss the effect of the unbalance on the measured squeezing, let

$$\tau_{\text{BS}} = R/T, \tag{38}$$

$$\tau_{\text{PD}} = \eta_1/\eta_2, \tag{39}$$

$$\eta = (\eta_1 + \eta_2)/2. \tag{40}$$

Here τ_{BS} and τ_{PD} represent the unbalance of the beam-splitter and the photodetectors.

We get from Eqs. (25) and (34)

$$S_{\text{QPm}} = \frac{2\eta\tau_{\text{BS}}(1 + \tau_{\text{PD}})}{(1 + \tau_{\text{BS}})(1 + \tau_{\text{BS}}\tau_{\text{PD}})} S_{\text{QPideal}}, \tag{41}$$

$$S_{\text{PNm}} = \frac{8\eta\tau_{\text{PD}}\tau_{\text{BS}} S_{\text{PNideal}}}{(\tau_{\text{PD}} + \tau_{\text{BS}})(1 + \tau_{\text{BS}})(1 + \tau_{\text{PD}}) - 2\eta(\tau_{\text{PD}} - \tau_{\text{BS}})^2 S_{\text{PNideal}}}. \tag{42}$$

Figures 2 and 3 show the measured squeezing due to the unbalances of the beam-splitter and the quantum efficiencies of photodetectors, where we have supposed the input mode a are perfectly squeezed, i.e., $S_{\text{QPideal}} = S_{\text{PNideal}} = 1$ and the mean quantum efficiency $\eta = (\eta_1 + \eta_2)/2 = 0.85$. Comparing Fig. 2 with Fig. 3, we can see that the photon number squeezing is less affected by the unbalance than the quadrature phase squeezing. In fact, in the measurement of photon number squeezing, there exists some noise reduction in SNL and the total intensity noise due to the effects of the input squeezed light and the unbalance of the system, and the noise reduction will compensate the reduction of squeezing due to the unbalance.

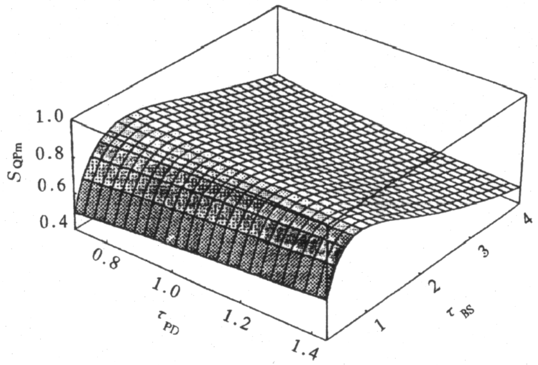

Fig. 2. Measured quadrature phase squeezing S_{QPm} affected by the imperfect system, where the input squeezing is 100% and the mean quantum efficiency of the two photodetectors is 0.85; τ_{BS} and τ_{PD} are defined by Eqs. (38) and (39), respectively.

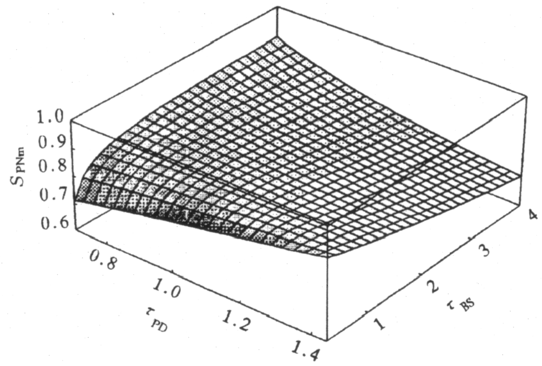

Fig. 3. Measured photon number squeezing S_{PNm} affected by the imperfect system, where the input squeezing is 100% and the mean quantum efficiency of the two photodetectors is 0.85; τ_{BS} and τ_{PD} are defined by Eqs. (38) and (39), respectively.

V. CONCLUSION

In conclusion, the effect of the non-ideally balanced homodyne detection on the squeezing have been discussed. It is shown that the imperfectly balanced beam-splitter and the photodetectors reduce the squeezing of the input field in different ways for the squeezings of squeezed vacuum state and the photon number squeezed state. This discussion will show practical benifit from the design of mesaurement of quantum state as well as the use of the nonclassical light.

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