Sub-shot-noise measurement for slight absorption *

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Abstract Sub-shot-noise measurement for slight absorption is experimentally achieved with the twin beams of quantum correlation. The improvement in signal-to-noise ratio of 2.5 dB relative to the SNL is obtained. The experimental results demonstrate the predictions of the semi-classical theory.

Keywords: twin beams, shot-noise limit, photocurrent fluctuations, signal-to-noise ratio

In the past decade, a variety of non-classical lights, in which the quantum fluctuation of one physical component is squeezed below the correspondent shot-noise-limit (SNL), have been experimentally produced[1—3]. Injecting the quadrature squeezed vacuum state lights into the "dark" port of interferometers, Kimble's group and Bell Lab[4—6] carried out the optical measurements for phase shift, polarization and spectroscopy with precision beyond standard quantum limit (SQL). Besides the quadrature squeezed vacuum states, the intensity correlated twin beams, the quantum fluctuation of intensity difference between which is lower than that for a classical coherent states, were obtained from the nondegenerate parametric down-conversion processes. The theoretical analyses and experiments show that compared with quadrature-squeezing, the restriction on experimental conditions for the generation of the intensity difference squeezing is relatively less[3,7], which makes the study of its application more attractive.

Scientists are interested in the applications of twin beams in sub-shot noise measurements[8—11]. Based on the semi-classically theoretical analyses we demonstrated that quantum correlated twin beams can be used in sub-shot-noise measurements for absorption spectroscopy analysis[12] and small phase shift[13]. The signal-to-noise ratio beyond SNL would be proportional to the degree of the intensity difference squeezing. In principle, the minimum of the detectable values would tend to zero if perfect squeezed lights are employed[12,13].

Recently we experimentally generated quantum correlated twin beams[14] from a semimololithic optical parametric oscillator (OPO) pumped by a homemade CW intracavity frequency stabilized and doubled ring Nd:YAP laser. The twin beams with the character of intensity difference squeezing were used in the sub-shot-noise measurements for the slight absorption. Since the twin beams produced from our OPO are with perpendicular polarization and near-degenerate frequencies, the intensity unbalance between the two channels of the twin beams in different

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colors was minimized and the equipment for splitting beams was simplified to a normal polarization beamsplitter. Compared with that of ref. [9] the transmissivities of the measured sample designed in our lab need not be modulated, so it could be used to any practical material. To our knowledge this is the first sub-shot-noise measurement for slight absorption on an unmodulated sample by means of the intensity difference squeezed light. The signal-to-noise ratio relative to SNL is increased by 2.5dB. The other advantage of our system is that the intensity of "laser like" twin beams generated from an OPO is much higher than that from the spontaneous emission; therefore the absolute sensitivity must be increased and it would be more convenient for the applications. The experimental results are in good agreement with our theoretical predictions[12].

1 The principle of measurement

1.1 The absorption signal

The light field $E_i(t)$ of twin beams ($i = 1, 2$) generated from OPO can be expressed with its quadrature amplitude and phase components $P_i(t)$ and $Q_i(t)$[15].

$$E_i(t) = P_i(t)\cos\omega_i t + Q_i(t)\sin\omega_i t, \quad i = 1, 2, \tag{1}$$

where ω_i is the frequency of one of the twin beams. In the case of near-degenerate frequency, $\omega_1 \approx \omega_2$. The average values of amplitude and phase component are[12]

$$\langle P_1(t)\rangle = \langle P_2(t)\rangle = \overline{P}, \tag{2a}$$

$$\langle Q_1(t)\rangle = \langle Q_2(t)\rangle = 0. \tag{2b}$$

The average intensity (photons/s) is equal to[12]

$$\langle I_1(t)\rangle = \langle I_2(t)\rangle = \overline{I} = \frac{\overline{P}^2}{4}. \tag{3}$$

As shown in fig. 1, E_1 and E_2 are separated by polarizing-beam-splitter P_1. E_2 is directly detected by D_2. The amplitude of the signal field E_1 is modulated by the modulator consisting of the electro-optic crystal EO, $\lambda/4$-wave plate and polarizing-beam-splitter P_2. The modulated signal at frequency ω_m is separated into two equal parts of E_{s1} and E_{s2}. E_{s2} is detected by D_{s2}; E_{s1} traverses the absorption cell and then injects into the detector D_{s1}.

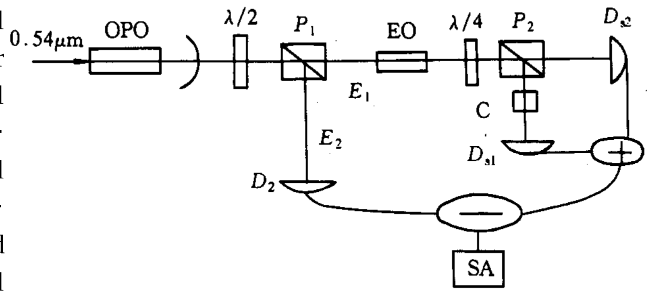

Fig. 1. The experimental diagram for slight absorption measurement. OPO, optical parametric oscillator; P_1 and P_2, polarizers; EO, electro-optic crystal; D_{s1}, D_{s2} and D_2, detectors; C, sample cell; SA, spectrum analyzer.

The polarization of field E_1 and the direction of electric field applied on the EO crystal are parallel with the x-axis of the crystal and the light propagates along Z-axis. The polarization orientations of polarizers P_1 and P_2 are identical. The average intensities of the modulated signals are[16]

$$\langle I_{s1}\rangle = \frac{\overline{I}}{2}\left(1 - 2M\sin\omega_m t\right), \tag{4a}$$

$$\langle I_{s2}\rangle = \frac{\overline{I}}{2}\left(1 + 2M\sin\omega_m t\right). \tag{4b}$$

The output intensity from the absorption cell can be expressed as

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$$\langle I_{SC}\rangle = \langle I_{s1}\rangle e^{-\delta l} = \langle I_{s1}\rangle(1 - \delta l)$$
$$= \frac{\bar{I}}{2}(1 - \delta l - 2M\sin\omega_m t + 2M\delta l\sin\omega_m t), \quad (5)$$

where δ and l are respectively the absorption coefficient and length of the sample. Suppose that all detectors have the same quantum efficiency η. The finally analyzed photocurrent signal $\langle i_{sig}\rangle$ on the spectrum analyzer (SA) is

$$\langle i_{sig}\rangle = e\eta\bar{I}[\langle I_{sc}\rangle + \langle I_{s2}\rangle - \langle I_2\rangle]$$
$$= e\eta\bar{I}\left(M\delta l\sin\omega_m t - \frac{\delta l}{2}\right). \quad (6)$$

The first term is a pulsating signal with the modulated frequency ω_m and the amplitude which is proportional to the absorption δl. The absorption of the sample can be measured by evaluating the height of this pulsating signal.

1.2 Background noise

The fluctuation of photocurrent resulting from the quantum noise of injected light field enters the spectrum analyzer (SA) along with the signal to form the background noise which confines the minimum detectable signal. According to the semiclassical theory, when a light field with amplitude $P(t)$ is injected into a detector of efficiency η, the detected amplitude component should be[17]

$$P_d(t) = \sqrt{\eta}P(t) + \sqrt{1-\eta}V(t), \quad (7)$$

where $V(t)$ is the amplitude component of the vacuum field. As is well known, $\langle V(t)\rangle = 0$. The detected intensity and intensity fluctuation of light are[12]

$$I_d(t) = \frac{\left[\sqrt{\eta}P(t) + \sqrt{1-\eta}V(t)\right]^2}{4}, \quad (8)$$

$$\delta I_d(t) = \eta\delta I(t) + \sqrt{\eta(1-\eta)}\frac{\langle P(t)\rangle}{2}\delta V(t). \quad (9)$$

The fluctuations of the output photocurrent from a detector is equal to

$$\delta i(t) = e\delta I_d(t)$$
$$= e\left\{\eta\delta I(t) + \sqrt{\eta(1-\eta)}\frac{\langle P(t)\rangle}{2}\delta V(t)\right\}. \quad (10)$$

The total photocurrent fluctuation entering the spectrum analyzer is expressed as

$$\delta i_\Delta(t) = \delta i_{s1}(t) + \delta i_{s2}(t) - \delta i_2(t), \quad (11)$$

where δi_{s1}, δi_{s2} and δi_2 respectively stand for the photocurrent noises from the detectors D_{s1}, D_{s2} and D_2. The noise power spectrum is defined as[2]

$$\Phi_\Delta(\Omega) = \int e^{i\Omega\tau}C_{\delta i}(\tau)\,d\tau, \quad (12)$$

where $C_{\delta i}(\tau)$ expresses the autocorrelation faction of $\delta_\Delta i(t)$, i.e.[2]

$$C_{\delta i}(\tau) = \langle\delta i_\Delta(t)\,\delta i\Delta(t+\tau)\rangle$$
$$= \left\langle\left\{\delta i_{s1}(t) + \delta i_{s2}(t) - \delta i_2(t)\right\}\left\{\delta i_{s1}(t+\tau) + \delta i_{s2}(t+\tau) - \delta i_2(t+\tau)\right\}\right\rangle. \quad (13)$$

Substituting eq. (10) into eq. (13), we obtain

$$C_{\delta i}(\tau) = e^2\left\langle\left[\eta\left\{\delta I_{s1}(t) + \delta I_{s2}(t) - \delta I_2(t)\right\} + \sqrt{\eta(1-\eta)}\left\{\frac{\langle P_{s1}(t)\rangle}{2}V_{s1}(t)\right.\right.\right.$$

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$$+\frac{\langle P_{s2}(t)\rangle}{2}\delta V_{s2}(t)-\frac{\overline{P}}{2}\delta V_2(t)\Big\}\Big]\Big[\eta\Big\{\delta I_{s1}(t+\tau)+\delta I_{s2}(t+\tau)-\delta I_2(t+\tau)\Big\}$$
$$+\sqrt{\eta(1-\eta)}\Big\{\frac{\langle P_{s1}(t)\rangle}{2}\delta V_{s1}(t+\tau)+\frac{\langle P_{s2}(t)\rangle}{2}\delta V_{s2}(t+\tau)-\frac{\overline{P}}{2}\delta V_2(t+\tau)\Big\}\Big]\Big\rangle, \tag{14}$$

where $\delta I_{s1}(t)$, $\delta I_{s2}(t)$, $\delta I_2(t)$ and $\delta V_{s1}(t)$, $\delta V_{s2}(t)$, $\delta V_2(t)$ are respectively the intensity fluctuations and the correspondent vacuum fluctuations introduced from the imperfect detectors D_{s1}, D_{s2} and D_2 with efficiency η in the light fields E_{s1}, E_{s2} and E_2. It has been theoretically demonstrated that the sum of $\delta I_{s1}(t)$ and $\delta I_{s2}(t)$ are equal to $\delta I_{s1}(t)$ [12]. If the inserting loss of the sample cell is neglected, eq. (14) can be expanded into

$$C_{\delta i}(\tau)=e^2\Big\{\eta^2 C_{\delta I}(\tau)+\eta(1-\eta)\big[C_{s1}(\tau)+C_{s2}(\tau)+C_2(\tau)\big]\Big\}. \tag{15}$$

The auto-correlation function $C_{\delta I}(\tau)$ of the intensity difference fluctuation between the twin beams is equal to

$$C_{\delta I}(\tau)=\Big\langle\big[\delta I_1(t)-\delta I_2(t)\big]\big[\delta I_1(t+\tau)-\delta I_2(t+\tau)\big]\Big\rangle. \tag{16}$$

The fourth-order field-correlation functions $C_{s1}(\tau)$, $C_{s2}(\tau)$ and $C_2(\tau)$ between the vacuum fluctuation and signal fields E_{s1}, E_{s2} and E_2 are given as follows:

$$C_k(\tau)=\frac{\langle P_k(t)\rangle^2}{4}\langle\delta V_k(t)\,\delta V_k(t+\tau)\rangle\quad(k=\mathrm{s1},\mathrm{s2},2). \tag{17}$$

In eq. (15) the cross terms of the independent operators without any quantum correlation are equal to zero. The fluctuation spectrum of the photocurrent noise is written as

$$\Phi_\Delta(\Omega)=e^2\Big\{\eta^2\int C_{\delta I}(\tau)e^{-i\Omega\tau}d\tau+\eta(1-\eta)\Big[\int C_{s1}(\tau)e^{-i\Omega\tau}d\tau$$
$$+\int C_{s2}(\tau)e^{-i\Omega\tau}d\tau+\int C_{s2}(\tau)e^{-i\Omega\tau}d\tau\Big]\Big\}. \tag{18}$$

The first term is the noise power spectrum $S_{\delta I}(\Omega)$ of the intensity difference between the twin beams[15]

$$S_{\delta I}(\Omega)=\int C_{\delta I}(\tau)e^{-i\Omega\tau}d\tau=S_0S_r(\Omega), \tag{19}$$

where $S_0=2\overline{I}$ is the shot-noise level, $S_r(\Omega)$ is the noise spectrum of the amplitude difference[15]. The left three terms stand for the coherent noise power spectra between the vacuum field V_k and the light field E_k of the twin beams,

$$S_k(\Omega)=\int C_k(\tau)e^{-i\Omega\tau}d\tau$$
$$=\frac{\langle P_k(t)\rangle^2}{4}\langle\delta V_k^2(\Omega)\rangle=\frac{\overline{P}_k^2}{4}=\langle I_k\rangle\quad k=\mathrm{s1},\mathrm{s1},2. \tag{20}$$

Since $\langle\delta V_k(\Omega)\rangle$ is the Fourier component of vacuum fluctuation $\delta V_k(t)$, we simplify it into $\langle\delta V_k^2(\Omega)\rangle=1$ [15] in eq. (20). From eq. (4), we have

$$S_{s1}(\Omega)+S_{s2}(\Omega)=S_2(\Omega)=\overline{I}. \tag{21}$$

Eq. (18) can be rewritten in the simple form

$$\Phi_\Delta=e^2\Big\{\eta^2S_0S_r(\Omega)+\eta(1-\eta)S_0\Big\}$$
$$=ei_0\Big\{\eta S_r(\Omega)+(1-\eta)\Big\}, \tag{22}$$

where $i_0=2e\eta\overline{I}$ is the mean photocurrent. The measured photocurrent fluctuations at frequency

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Ω within the detection bandwidth $\Delta\Omega = 2\pi B$ is given by

$$\langle(\delta i_\Delta(\Omega))^2\rangle = 2B\Phi_\Delta(\Omega) = 2ei_0BR(\Omega), \quad (23)$$

where $R(\Omega)$ is the noise spectrum factor of the intensity difference between the twin beams:

$$R(\Omega) = \eta S_r(\Omega) + 1 - \eta. \quad (24)$$

If parametric down conversion fields E_1 and E_2 are perfect correlation ($R(\Omega) \to 0$) and $\eta = 1$, the photocurrent fluctuation is also close to zero. If there is no quantum correlation between fields E_1 and E_2 ($R(\Omega) = 1$), the current fluctuation $\langle(\delta i_\Delta(\Omega))^2\rangle$ is just the shot-noise-level i_N of photocurrent:

$$i_N^2 = 2ei_0B. \quad (25)$$

After the sample cell is inserted, due to the extra optical loss σ the noise spectrum factor $R(\Omega)$ becomes[12]

$$R'(\Omega) = (1 - \sigma)R(\Omega) + \sigma. \quad (26)$$

1.3 The minimum detectable absorption

For the measurement with classical coherent lights the signal-to-noise ratio $\Psi_v = i_{\mathrm{sig}}^2 / i_N^2$, and the minimum detectable absorption $[\delta]_{\mathrm{SNL}} = (8B/\eta \bar{I})^{1/2}/Ml$. If the twin beams with quantum correlation are employed the signal-to-noise ratio will be increased to

$$\Psi_{\mathrm{sq}} = \frac{i_{\mathrm{sig}}^2}{i_N^2 R'(\Omega)}. \quad (27)$$

Correspondingly, the minimum detectable absorption will be decreased to $[\delta]_{\mathrm{sq}} = [\delta]_{\mathrm{SNL}}\sqrt{R'(\Omega)}$.

2 Experiments

2.1 The production and measurement of intensity difference squeezed lights

The measurement sensitivity for slight signal beyond SNL depends on the degree of quantum correlation between twin beams which is evaluated with the intensity difference squeezing. Hence before the measurement the noise power spectrum of the intensity difference fluctuation should be carefully detected. The experimental scheme for detective system of intensity difference fluctuating is shown in fig. 2. The green light at 0.54 μm emitted by intracavity frequency doubledm, and frequency stabilized laser Nd:YAP is coupled into OPO. The OPO is a semimonolithic F-P cavity consisting of an a-cut KTP crystal, the front plane face of which is also as the input coupler (transmission of 15 % for green light; high reflectivity for the infrared light) and an output coupler which is a concave mirror with the curvature of 20 mm (high reflectivity for the green light, transmission of 3 % for the infrared light). The output coupling efficiency is about 90 %.

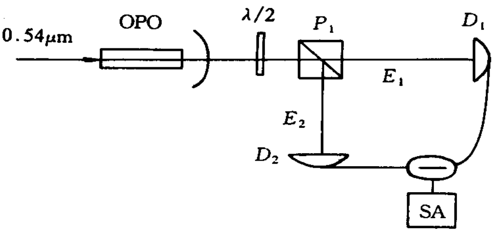

Fig. 2. The experimental diagram for detection of intensity difference squeezing.

The quantum correlated twin beams with near-degenerate frequency and cross polarizations were generated from OPO operating above the pump threshold (50mW). Under the pump power of 110 mW the output power of ~20 mW was obtained. The twin beams were separated by the

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polarizer P_1 and monitored by the photodiodes D_1 and D_2. The outputs of the photodiodes were amplified, and then subtracted in the power combiner (-). Finally the difference photocurrent was analyzed by a spectrum analyzer (SA). To ensure the balance of the detective system, the photodiodes were carefully chosen and the electronic compensations were included. The polarizations of the twin beams were rotated at an angle of 2β by a half-wave plate. When $\beta = 0°$, the signal recorded by a spectrum analyzer was the noise of the intensity difference between the twin beams; when $\beta = 22.5°$ the signal recorded by the spectrum analyzer was the correspondent shot-noise-level for a beam with the intensity of $(I_1 + I_2)$ [3]. Fig. 3 shows the measured noise power spectrum from 1 to 6 MHz. The noise power of the intensity difference is reduced by 55 % (3.5 dB) to around 3.6 MHz which corresponds to $R(\Omega) = 45\%$.

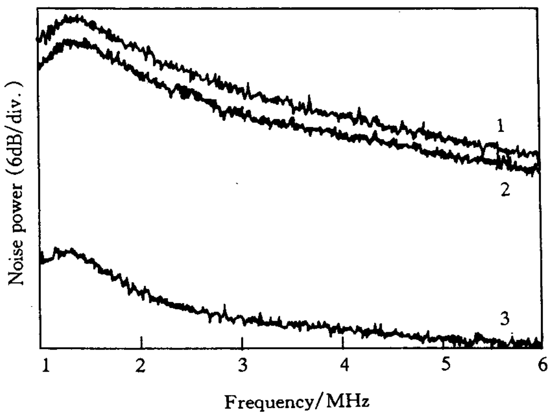

Fig. 3. Experimental results on intensity difference squeezing. 1, the shot noise lever; 2, the noise power spectrum of intensity difference fluctuation between twin beams; 3, the electrics noise floor.

2.2 The measurement of slight absorption

As shown in fig. 1, the modulated voltage at 3.6 MHz was applied on EO along x-axis with the modulation index of $M = 2 \times 10^{-4}$. Without the absorption cell the two arms of E_{s1} and E_{s2} were balanced and the modulation signal was canceled by the power combiner (+) (see eq. (4)). In this experiment we placed a sample cell with solvent (pure water) in E_{s1} at first, and then inserted an adjustable attenuator in E_{s2} to make the balance of AC signals between E_{s1} and E_{s2}. The extra losses from the modulator and absorption cell were $\sigma = 16\%$; therefore another attenuator of 16 % had to be placed in E_2 to balance the DC components between E_1 and E_2. As shown in fig. 4(a), the modulated signal at the frequency ω_m was canceled. Due to the extra losses σ, the intensity difference squeezing was decreased to 2.5 dB (44 %) below SNL which cor-

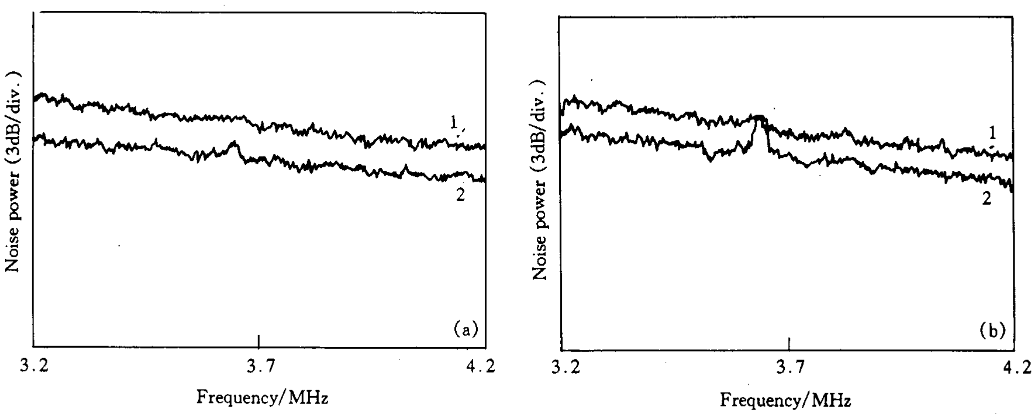

Fig. 4. Sub-shot-noise measurement for slight absorption (rf bandwidth, 30 kHz; video bandwidth, 30 Hz). (a) Measured results without absorption medium; (b) measured result with absorption medium. 1, with uncorrelatted two beams; 2, with quantum correlated twin beams.

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responds to $R' = 56\%$. Afterwards a little absorption medium (organic compound Sm$(CPFX)_2Cl_3(H_2O)_4$) was dropped into the solvent. The balance between E_{s1} and E_{s2} was destroyed and the pulsating signal with modulated frequency should appear. However, when the coherent lights without correlated quantum were used ($\beta = 22.5°$), the slight absorption signal was submerged in the shot noies. There is no pulasting signal to be observed (upper trace of fig. 4 (b)). When the twin beams with quantum correlation were employed ($\beta = 0°$), the absorption signal emerged from the squeezed noise background around the modulated frequency (lower trace of fig. 4(b)). The absorption δl can be evaluated from the height of this signal (equation (6)).

3 Conclusion

We designed an optical system for the slight absorption measurement by means of quantum correlated twin beams as the light source and analyzed the principle of the measurement with semi-classical theory. By calculating the noise spectrum of photocurrents we have demonstrated that the precision of measurement is beyond shot-noise-limit. In principle the minimum detectable absorption is able to close to zero if the quantum correlation between the twin beams is perfect. The signal-to-noise ratio relative to the SNL is increased by 2.5 dB, which is in good agreement with the prediction from the theoretical calculation.

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