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Quantum teleportation for continuous variables by means of a phase sensitive nondegenerate optical parametric amplifier

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Abstract

A simple scheme to realize quantum teleportation with $\pi/2$ unitarily rotated transformation for continuous variable is proposed, in which the bright entangled EPR beam is produced by phase sensitive nondegenerate optical parametric amplifier with a fixed relative phase of $\pi/4$ between the subharmonic signal and the harmonic pump field. The measurement of "Bell state" is accomplished by means of direct detection of photocurrents. © 2001 Elsevier Science B.V. All rights reserved.

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Recently, the theoretical and experimental studies on quantum communication based on the use of the nonlocal quantum correlations of entangled states show great promise for establishing new kinds of information processing without classical counterparts. One of the most striking features of quantum information is quantum teleportation [1] which is the disembodied transport of an unknown quantum state from one place (Alice) to another (Bob). Towards possible applications in quantum communication, both theoretical and experimental investigations increasingly focus on quantum states EPR for a continuous variable in an infinite-dimensional Hilbert space. EPR state can be efficiently generated using squeezed light and beam splitters, for instance, the entangled two-mode squeezed vacuum state that has already proven its usefulness for quantum teleportation [2]. The schemes of quantum teleportation and dense coding by means of bright amplitude squeezed light and direct measurement of Bell state were described in our previous paper [3]. The amplitude and phase quadrature are obtained from the direct detection implemented with two photo-detectors and two RF splitters. In this Letter we propose a experimental scheme in that EPR beam is generated from a phase sensitive nondegenerate optical parametric amplifier (NOPA) with a fixed relative phase of $\pi/4$ between the subharmonic signal and the harmonic pump field. In the proposed system the output state from the receiver (Bob) is an unitarily rotated input state with a rotated transformation of $\pi/2$. The measurement of "Bell state" is accomplished only by means of direct detection of photocurrents in which the local oscillators for the balanced homodyne detection are not needed. In the experiment of Ref. [2], an entanglement source states combined at a beamsplitter and two sets of balanced homo-

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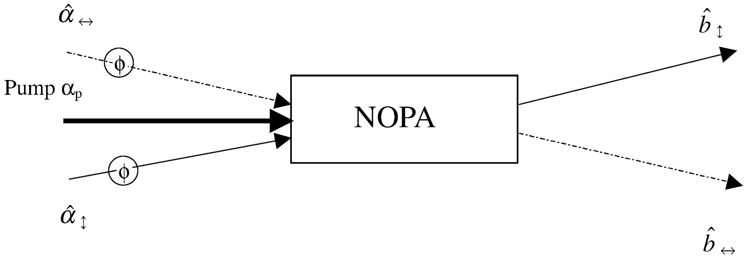

Fig. 1. Schematic diagram for phase sensitive NOPA.

dyne detector were employed. Since the two independent squeezed beams were generated in a traveling-wave degenerate resonator with two counterpropagation pump beam [2], the experimental complexity was increased. For a NOPA with a type-II nonlinear crystal, the output subharmonic light field itself consists of nondegenerate signal and idler modes with orthogonal polarizations, which are easily divided by a polarized-beam-splitter to form a pair of space-separate EPR beam, so the system is relatively simple [6]. The efficiency of mode matching between the squeezed vacuum and local beam usually should be high for ensuring high efficiency of homodyne detection, that must increase the difficulty on experiments with respect to the proposed direct detection.

The schematic diagram for phase sensitive NOPA is shown in Fig. 1. Two coherent input signals $a_{\updownarrow}$ and $a_{\leftrightarrow}$ with same frequency ω_0 and orthogonal polarization are injected into a NOPA. For simplification, the polarizations of the injected signal and idler field are orientated along the vertical and horizontal directions, and their intensities and original phases before NOPA are considered to be identical. The amplifier is pumped with the second harmonic wave of $\omega_p = 2\omega_0$ and amplitude of pump field $a_p \gg a_{\updownarrow}, a_{\leftrightarrow}$, in this case the pump field can be considered as a classical field without depletion during the amplification process. The output signal and idler fields polarized along the vertical and horizontal direction are denoted with $b_{\updownarrow}$ and $b_{\leftrightarrow}$. We define the operators of the light fields at the center frequency ω_0 in the rotating frame,

$$\hat{O}(t) = \hat{o}(t)e^{i\omega_0 t}, \tag{1}$$

where $O = [\hat{a}_{\updownarrow}, \hat{a}_{\leftrightarrow}, \hat{b}_{\updownarrow}, \hat{b}_{\leftrightarrow}]$ are the field envelope operators and $o = [\hat{A}_{\updownarrow}, \hat{A}_{\leftrightarrow}, \hat{B}_{\updownarrow}, \hat{B}_{\leftrightarrow}]$ are the field operators corresponding to input and output signal and idler fields. By the Fourier transformation we have

$$\hat{O}(\Omega) = \frac{1}{\sqrt{2\pi}} \int dt\, \hat{O}(t) e^{-i\Omega t}. \tag{2}$$

Here, the fields are described as functions of the modulation frequency Ω with commutation relation $[\hat{O}(\Omega), \hat{O}^{+}(\Omega')] = 2\pi\delta(\Omega - \Omega')$. A practical light field can be decomposed to a carrier $\hat{O}(0)$ oscillating at the center frequency ω_0 with an average amplitude (O_{ss}) which equals to the amplitude of its steady state field, and surrounded by "noise sidebands" $\hat{O}(\Omega)$ oscillating at frequency $\omega_0 + \Omega$ with zero average amplitude [4]:

$$\langle \hat{O}(\Omega = 0) \rangle = O_{\mathrm{ss}} \qquad \langle \hat{O}(\Omega \neq 0) \rangle = 0. \tag{3}$$

The noise spectral component at frequency Ω is the hereodyne mixing of the carrier and the noise sidebands. The amplitude and phase quadrature are expressed by

$$\begin{aligned} \hat{X}_O(\Omega) &= \hat{O}(\Omega) + \hat{O}^{+}(-\Omega), \\ \hat{Y}_O(\Omega) &= \frac{1}{i}\big[\hat{O}(\Omega) - \hat{O}^{+}(-\Omega)\big], \end{aligned} \tag{4}$$

with

$$\big[\hat{X}_O(\Omega), \hat{Y}_O(\Omega')\big] = i\delta(\Omega + \Omega'). \tag{5}$$

The input–output Heisenberg evolutions of the field modes of the NOPA are given by [5]

$$\begin{aligned} \hat{b}_{0\updownarrow} &= \mu\hat{a}_{0\updownarrow} + \nu\hat{a}^{+}_{0\leftrightarrow}, & \hat{b}_{0\leftrightarrow} &= \mu\hat{a}_{0\leftrightarrow} + \nu\hat{a}^{+}_{0\updownarrow}, \\ \hat{b}_{+\updownarrow} &= \mu\hat{a}_{+\updownarrow} + \nu\hat{a}^{+}_{+\leftrightarrow}, & \hat{b}_{+\leftrightarrow} &= \mu\hat{a}_{+\leftrightarrow} + \nu\hat{a}^{+}_{+\updownarrow}, \\ \hat{b}_{-\updownarrow} &= \mu\hat{a}_{-\updownarrow} + \nu\hat{a}^{+}_{-\leftrightarrow}, & \hat{b}_{-\leftrightarrow} &= \mu\hat{a}_{-\leftrightarrow} + \nu\hat{a}^{+}_{-\updownarrow}, \end{aligned} \tag{6}$$

where $\hat{a}$, $\hat{a}^{+}$ and $\hat{b}$, $\hat{b}^{+}$ denote the annihilation and creation operators of the input and the output modes.

The subindex 0 and $\pm$ stand for the central mode at frequency ω_0 and the sidebands at frequency $\omega_0 \pm \Omega$, respectively. The parameters $\mu = \cosh r$ and $\nu = e^{i\theta_p}\sinh r$ are the function of the squeezing factor r ($r \propto L\chi^2|a_p|$, L is the nonlinear crystal length, χ^2 is the effective second-order susceptibility of the nonlinear crystal in NOPA, a_p is the amplitude of pump field) and the phase θ_p of pump. In the following calculation the phase θ_p is set to zero as the reference of relative phase of all other light fields. For bright optical field, the quadratures of the output orthogonal polarization modes at a certain rotated phase θ are expressed by

$$\begin{aligned}\hat{X}_{\hat{b}_\updownarrow}(\theta) &= \frac{b_{0\updownarrow}^*\hat{b}_{+\updownarrow}e^{-i\theta} + b_{0\updownarrow}\hat{b}_{-\updownarrow}^+e^{i\theta}}{|b_{0\updownarrow}|} \\ &= \hat{b}_{+\updownarrow}e^{-i(\theta+\varphi)} + \hat{b}_{-\updownarrow}^+e^{i(\theta+\varphi)}, \\ \hat{X}_{\hat{b}_\leftrightarrow}(\theta) &= \hat{b}_{+\leftrightarrow}e^{-i(\theta+\varphi)} + \hat{b}_{-\leftrightarrow}^+e^{i(\theta+\varphi)},\end{aligned} \tag{7}$$

where $\varphi = \arg(b_{0\updownarrow}) = \arg(b_{0\leftrightarrow}) = \arg(e^{i\Phi} + e^{-i\Phi} \times \tanh r)$ is the phase of the modes $\hat{b}_{0\updownarrow}, \hat{b}_{0\leftrightarrow}$ relative to θ_p and Φ is the phase of the modes $\hat{a}_{0\updownarrow}, \hat{a}_{0\leftrightarrow}$ relative to θ_p. Taking $\theta = 0$ and $\theta = \pi/2$ in Eq. (7), the amplitude and phase quadrature of the output field are obtained:

$$\begin{aligned}\hat{X}_{\hat{b}_\updownarrow}^0 &= \hat{X}_{\hat{b}_\updownarrow}(0) = \hat{b}_{+\updownarrow}e^{-i\varphi} + \hat{b}_{-\updownarrow}^+e^{i\varphi}, \\ \hat{X}_{\hat{b}_\leftrightarrow}^0 &= \hat{X}_{\hat{b}_\leftrightarrow}(0) = \hat{b}_{+\leftrightarrow}e^{-i\varphi} + \hat{b}_{-\leftrightarrow}^+e^{i\varphi}, \\ \hat{X}_{\hat{b}_\updownarrow}^{\pi/2} &= \hat{X}_{\hat{b}_\updownarrow}\left(\frac{\pi}{2}\right) = -i\left(\hat{b}_{+\updownarrow}e^{-i\varphi} - \hat{b}_{-\updownarrow}^+e^{i\varphi}\right), \\ \hat{X}_{\hat{b}_\leftrightarrow}^{\pi/2} &= \hat{X}_{\hat{b}_\leftrightarrow}\left(\frac{\pi}{2}\right) = -i\left(\hat{b}_{+\leftrightarrow}e^{-i\varphi} - \hat{b}_{-\leftrightarrow}^+e^{i\varphi}\right).\end{aligned} \tag{8}$$

When the injected subharmonic signal and harmonic pump field are in phase ($\Phi = \varphi = 0$), the maximum parametric amplification is achieved [6]. The difference of the amplitude quadratures and the sum of the phase quadratures between two orthogonal polarization modes are

$$\begin{aligned}\hat{X}_{\hat{b}_\updownarrow}^0 - \hat{X}_{\hat{b}_\leftrightarrow}^0 &= e^{-r}\hat{X}_{\hat{a}_\updownarrow}^0 - e^{-r}\hat{X}_{\hat{a}_\leftrightarrow}^0, \\ \hat{X}_{\hat{b}_\updownarrow}^{\pi/2} + \hat{X}_{\hat{b}_\leftrightarrow}^{\pi/2} &= e^{-r}\hat{X}_{\hat{a}_\updownarrow}^{\pi/2} - e^{-r}\hat{X}_{\hat{a}_\leftrightarrow}^{\pi/2}.\end{aligned} \tag{9}$$

Under the limit $r \to \infty$, the output orthogonal polarization modes are the perfect EPR beams with quadrature amplitude correlation and quadrature phase anticorrelation [6]. When the injected subharmonic signal and harmonic pump field are out of phase, i.e., $\Phi = \varphi = \pi/2$, NOPA operates at parametric deamplification [7]. Therefore the sum of the amplitude quadratures and the difference of the phase quadratures of the orthogonal polarization modes are as follows:

$$\begin{aligned}\hat{X}_{\hat{b}_\updownarrow}^0 + \hat{X}_{\hat{b}_\leftrightarrow}^0 &= e^{-r}\hat{X}_{\hat{a}_\updownarrow}^0 - e^{-r}\hat{X}_{\hat{a}_\leftrightarrow}^0, \\ \hat{X}_{\hat{b}_\updownarrow}^{\pi/2} - \hat{X}_{\hat{b}_\leftrightarrow}^{\pi/2} &= e^{-r}\hat{X}_{\hat{a}_\updownarrow}^{\pi/2} - e^{-r}\hat{X}_{\hat{a}_\leftrightarrow}^{\pi/2}.\end{aligned} \tag{10}$$

Obviously, the EPR beams with the quadrature amplitude anticorrelation and quadrature phase correlation are obtained for $r > 0$ [3]. When $\varphi = \pi/4$, the sum of $\theta = \pi/4$ quadratures and the difference of $\theta = -\pi/4$ quadratures between the orthogonal polarization modes are written as follows:

$$\begin{aligned}\hat{X}_{\hat{b}_\updownarrow}^{\pi/4} + \hat{X}_{\hat{b}_\leftrightarrow}^{\pi/4} &= e^{-r}\hat{X}_{\hat{a}_\updownarrow}^0 - e^{-r}\hat{X}_{\hat{a}_\leftrightarrow}^0, \\ \hat{X}_{\hat{b}_\updownarrow}^{-\pi/4} - \hat{X}_{\hat{b}_\leftrightarrow}^{-\pi/4} &= e^{-r}\hat{X}_{\hat{a}_\updownarrow}^{\pi/2} - e^{-r}\hat{X}_{\hat{a}_\leftrightarrow}^{\pi/2}.\end{aligned} \tag{11}$$

It can be seen that if $r > 0$, the $\pi/4$ quadratures of two output polarization modes are anticorrelated and $-\pi/4$ quadratures of that are correlated. In the following we propose a simple scheme for quantum teleportation by means of EPR beams with $\pi/4$ quadratures anticorrelation and $-\pi/4$ quadratures correlation.

The proposed scheme is shown in Fig. 2. One of the EPR beams is sent to Alice where it mixes with the input signal beam $\hat{a}_{\text{in}}$ with same intensity and $\pi/2$ phase shift on the 50% beamsplitter (BS1). The resulting output beams, $\hat{c}$ and $\hat{d}$, are expressed by

$$\begin{aligned}\hat{c} &= \frac{\sqrt{2}}{2}\left(\hat{a}_{\text{in}} + i\hat{b}_\leftrightarrow\right), \\ \hat{d} &= \frac{\sqrt{2}}{2}\left(\hat{a}_{\text{in}} - i\hat{b}_\leftrightarrow\right),\end{aligned} \tag{12}$$

which are directly detected by photoelectronic detectors D_1 and D_2. The normalized output photocurrents spectra are given [3] by

$$\begin{aligned}\hat{i}_c(\Omega) &= \frac{1}{\langle\hat{c}\rangle}\int dt\, e^{-i\Omega t}\hat{c}^+\hat{c} \\ &= \frac{1}{2}\big(\hat{X}_{\hat{a}_{\text{in}}}^0(\Omega) + \hat{X}_{\hat{a}_{\text{in}}}^{\pi/2}(\Omega) - \hat{X}_{\hat{b}_\leftrightarrow}^{\pi/2}(\Omega) \\ &\quad + \hat{X}_{\hat{b}_\leftrightarrow}^0(\Omega)\big)\end{aligned}$$

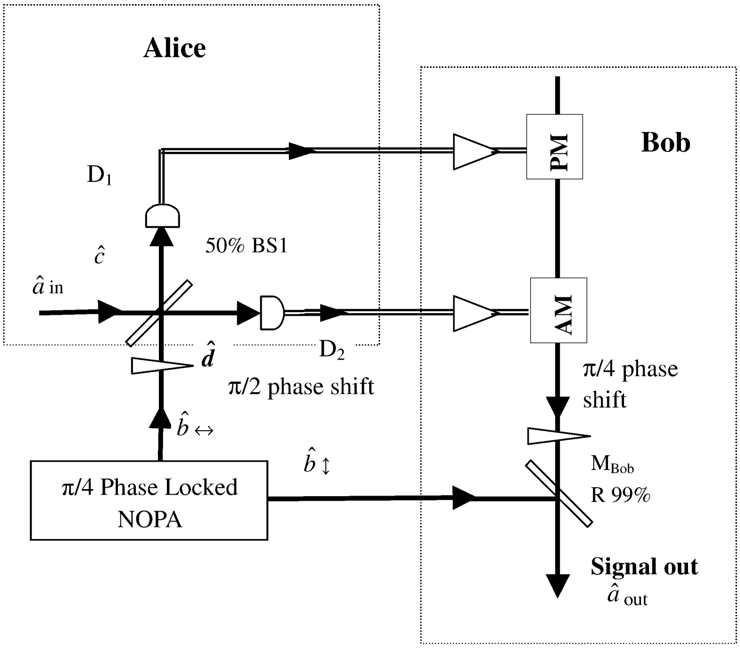

Fig. 2. Schematic of $\pi/4$ phase locked NOPA for teleportation.

$$= \frac{1}{\sqrt{2}}\big(\hat{X}^{-\pi/4}_{\hat{a}_{\text{in}}}(\Omega) + \hat{X}^{\pi/4}_{\hat{b}_{\leftrightarrow}}(\Omega)\big),$$

$$\hat{i}_d(\Omega) = \frac{1}{2}\big(\hat{X}^{0}_{\hat{a}_{\text{in}}}(\Omega) - \hat{X}^{\pi/2}_{\hat{a}_{\text{in}}}(\Omega) + \hat{X}^{\pi/2}_{\hat{b}_{\leftrightarrow}}(\Omega) + \hat{X}^{0}_{\hat{b}_{\leftrightarrow}}(\Omega)\big)$$
$$= \frac{1}{\sqrt{2}}\big(\hat{X}^{\pi/4}_{\hat{a}_{\text{in}}}(\Omega) + \hat{X}^{-\pi/4}_{\hat{b}_{\leftrightarrow}}(\Omega)\big). \tag{13}$$

Then the photocurrents are sent to amplitude and phase modulators in the receiver (Bob), respectively. The amplitude and phase modulators transform the photocurrent signals into a coherent state light field, which is then combined with the other half $\hat{b}_{\updownarrow}$ of the EPR beams after $\pi/4$ phase shift at the mirror M_{Bob} of high reflectivity, ∼99%. With Eq. (11) we can easily find the expression of output beam from the mirror

$$\hat{a}_{\text{out}} = \hat{b}_{\updownarrow} + g_c\hat{i}_c e^{i(\pi/4)} + g_d\hat{i}_d e^{-i(\pi/4)}$$
$$= \frac{1}{\sqrt{2}}\Big[\hat{X}^{-\pi/4}_{\hat{b}_{\updownarrow}}(\Omega) + \hat{X}^{\pi/4}_{\hat{b}_{\updownarrow}}(\Omega) + i\big(\hat{X}^{-\pi/4}_{\hat{b}_{\updownarrow}}(\Omega) - \hat{X}^{\pi/4}_{\hat{b}_{\updownarrow}}(\Omega)\big)\Big] + g_{\hat{c}}\hat{i}_{\hat{c}}e^{-i(\pi/4)} + g_{\hat{d}}\hat{i}_{\hat{d}}e^{i(\pi/4)}$$
$$= \frac{1}{\sqrt{2}}\big[\hat{X}^{-\pi/4}_{\hat{a}_{\text{in}}}(\Omega)(1-i) - X^{\pi/4}_{\hat{a}_{\text{in}}}(\Omega)(1+i)\big]$$
$$= \hat{Y}_{\hat{a}_{\text{in}}}(\Omega) - i\hat{X}_{\hat{a}_{\text{in}}}(\Omega) = -i\hat{a}_{\text{in}}, \tag{14}$$

where $g_{\hat{c}}$ and $g_{\hat{d}}$ describes Bob's (suitably normalized) amplitude and phase gain of the transformation from photocurrent to output beam, in Eq. (14) we have taken the normalized gain $g_{\hat{c}} = -g_{\hat{d}} = \sqrt{2}$ and $r \to \infty$. Eq. (14) shows a process in which a quantum state of input state is teleported and at the same time an unitarily $\pi/2$ rotated transformation of output state with respect to input state is complete. Although the unitarily rotated transformation can be achieved by some optical elements, such as empty cavity [8], it is interested that the teleportation and the unitarily $\pi/2$ rotated transformation are completed simultaneously.

In conclusion, we propose an experimental scheme of the quantum teleportation with $\pi/2$ rotated transformation of output state by using a NOPA with $\pi/4$ phase difference between the subharmonic signal and the harmonic pump field. Due to exploiting the bright EPR beams generated from NOPA and the directly measuring technique of "Bell state", the trouble to meet high efficiency of mode matching in experiment is eliminated. The unitary $\pi/2$ rotated transformation teleportation system might be useful in future quantum communication network.

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