5 November 2001

PHYSICS LETTERS A

Physics Letters A 290 (2001) 1–5

www.elsevier.com/locate/pla

Application of amplitude-squeezed state light from injection-locked laser diode in quantum teleportation and dense coding

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Received 17 July 2001; accepted 18 September 2001
Communicated by P.R. Holland

Abstract

The novel quantum teleportation and dense coding systems based on applying the amplitude-squeezed state light produced from the injection-locked laser diodes and a simple direct measurement scheme for the Bell state are proposed.

In developing theoretical and experimental studies of quantum information the quantum teleportation which is the disembodied transport of an unknown quantum state from a sender to a remote receiver [1] and the dense coding, in that the single bit sent from a sender to a receiver can successfully carry two bit of classical information [2], have attracted extensive interests. The nonlocal quantum entanglement plays a determinant role in the quantum information processing. Towards possible applications in quantum communication, both theoretical and experimental investigations increasingly focus on quantum states of continuous variable in an infinite-dimensional Hilbert space, since the EPR entangled state can be efficiently generated using squeezed light and beam splitters, for instance, the entangled EPR pairs resulting from two-mode squeezed vacuum state have been successfully employed in demonstrating unconditional quantum teleportation [3]. Another scheme proposed in Ref. [4] is a similar arrangement but in which two bright squeezed sources were used to produce the EPR beams and one LO was needed at Alice for the Bell-state measurement. Later, the schemes realizing highly efficient dense coding for continuous variables are theoretically proposed, in that the two-mode squeezed-state entanglement are utilized to achieve unconditional signal transmission [5–7]. The bright EPR beams have been experimentally produced with a nondegenerate optical parametric amplifier [8] and the dense coding for continuous variables based on bright EPR beam has been demonstrated [9]. In this Letter we propose a teleportation and a dense coding system in which the EPR beams are generated by mixing two amplitude-squeezed lights from the injection-locked laser diode (LD). In the practical injection-locked LD system, the generated squeezed state is always not the minimum-uncertainty state ($\langle \delta X^2 \rangle \times$

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0375-9601/01/$ – see front matter
PII: S0375-9601(01)00620-X

$\langle \delta Y^2 \rangle = 1$, δX and δY are the quantum fluctuation of the quadrature components), and there is large excess noise in the unsqueezed quadrature phase component ($\langle \delta Y^2_{\rm unsquee} \rangle > 1/\langle \delta X^2_{\rm squee} \rangle$) [10]. Our analyses prove that the influence of the excess noise of the unsqueezed quadrature on the teleportation and dense coding is not significant. The measurement of the Bell state is accomplished by means of direct detection for photocurrent and two rf power splitter. As a local oscillator and balanced homodyne detector are not needed, the proposed system is easy to realize experimentally.

In recent years, the injection-locked LD systems have attracted widely attention in scientific and technic fields due to its ability of providing a convenient tunable nonclassical light source [10,11]. We propose a experimental setup to generate bright EPR beams using injection-locked LD systems shown in Fig. 1. A Ti:Sapphire laser is used as the master laser to inject into the slave laser diodes LD1 and LD2 placed in cryostat to produce two coherent amplitude-squeezed beams and then mixed on a 50% beamsplitter (BS1) to form a pair of EPR beams. The injection-locked technique of LD to generate amplitude-squeezed light is quite mature [10,11] and the system without optical parametric amplifier and a lot of phase-locking loops is favorable to practical application. The generated bright coherent amplitude-squeezed beams a, b have the nonzero average intensities $\langle a \rangle$ and $\langle b \rangle$, and the amplitude and phase quadratures of the output beams c and d are given [7]:

$$\begin{aligned}
X_c(\Omega) &= \frac{1}{2\bar{i}}\big[\langle a\rangle X_a(\Omega) - \langle a\rangle Y_b(\Omega) + \langle b\rangle Y_a(\Omega) \\
&\quad + \langle b\rangle X_b(\Omega)\big], \\
X_d(\Omega) &= \frac{1}{2\bar{i}}\big[\langle a\rangle X_a(\Omega) + \langle a\rangle Y_b(\Omega) - \langle b\rangle Y_a(\Omega) \\
&\quad + \langle b\rangle X_b(\Omega)\big], \\
Y_c(\Omega) &= \frac{1}{2\bar{i}}\big[\langle a\rangle Y_a(\Omega) - \langle a\rangle X_b(\Omega) + \langle b\rangle X_a(\Omega) \\
&\quad + \langle b\rangle Y_b(\Omega)\big], \\
Y_d(\Omega) &= \frac{1}{2\bar{i}}\big[\langle a\rangle Y_a(\Omega) + \langle a\rangle X_b(\Omega) - \langle b\rangle X_a(\Omega) \\
&\quad + \langle b\rangle Y_b(\Omega)\big];
\end{aligned} \tag{1}$$

here X_j and Y_j ($j = a, b, c, d$) express the quadrature amplitude and phase components of the optical field a, b, c and d, $\bar{i} = \langle c\rangle = \langle d\rangle = \sqrt{(\langle a\rangle^2 + \langle b\rangle^2)/2}$. The strange nature of entanglement was first pointed out by Einstein, Podolsky and Rosen (EPR) [12] for the continuous variable of position and momentum. The EPR effect for continuous variable was realized by Ou et al. [13] experimentally with a nondegenerate optical parametric oscillator. The observed quantum correlations were shown to demonstrate the EPR paradox. The paradox applying two (noncommuting) measurements on a system allows one to deduce the values of two other (noncommuting) observables at another remote system, in such a way that the product of the two inferred variances apparently violate Heisenberg inequality. This violation is apparent only, because the inferred variances are actually conditional variances,

$$\langle \delta X^2_{\rm inf}\rangle\langle \delta Y^2_{\rm inf}\rangle = V_{X_c|X_d} V_{Y_c|Y_d} < 1, \tag{2}$$

where $V_{X_c|X_d} = V_{X_c}(1 - C^2_{X_cX_d})$ and $C^2_{X_cX_d} = |\langle X_c \times X_d\rangle - \langle X_c\rangle\langle X_d\rangle|^2 / V_{X_c} V_{X_d}$, and $V_{Y_c|Y_d}$ and $C^2_{Y_cY_d}$ have similar expressions with $V_{X_c|X_d}$ and $C^2_{X_cX_d}$. $V_{X_c|X_d}$ is just $V_+^{g+} = \langle \delta(X_c + g_+ X_d)^2\rangle$ and $V_{Y_c|Y_d}$ is $V_-^{g-} = \langle \delta(Y_c + g_- Y_d)^2\rangle$ with g_+ and g_- optimized. We consider the two amplitude-squeezed lights from LD have same noise at both quadratures $V_{X_a} = V_{X_b} = V_{\rm squee}$ and $V_{Y_a} = V_{Y_b} = V_{\rm unsquee}$. From Eqs. (1) we can readily write out the conditional variances and gain coefficients:

$$\begin{aligned}
V_{X_c|X_d} &= V_{Y_c|Y_d} = \frac{2V_{\rm squee}V_{\rm unsquee}}{V_{\rm squee} + V_{\rm unsquee}}, \\
g_+^{\rm opt} &= -g_-^{\rm opt} = \frac{V_{\rm unsquee} - V_{\rm squee}}{V_{\rm squee} + V_{\rm unsquee}}.
\end{aligned} \tag{3}$$

From Eqs. (3) it is obvious that Heisenberg inequality (2) is violated when $4(V_{\rm squee}V_{\rm unsquee})^2 < (V_{\rm squee} + V_{\rm unsquee})^2$ and optimized gain coefficients depend on the noises of two quadrature components. If the squeezed lights are the minimum-uncertainty state, the Heisenberg inequality is violated just with a few squeezing, but if not, supposing that the unsqueezed quadrature has infinite large noise, the Heisenberg inequality is violated only when the squeezing is more than 3 dB and in this case $g^{\rm opt}$ is unity gain.

The diagram for teleportation of continuous variable using direct detection of Bell state and the bright EPR beams is shown in Fig. 1. A part of Ti:Sapphire output is used as the input signal field which is in a coherent state and the other parts of it are injected

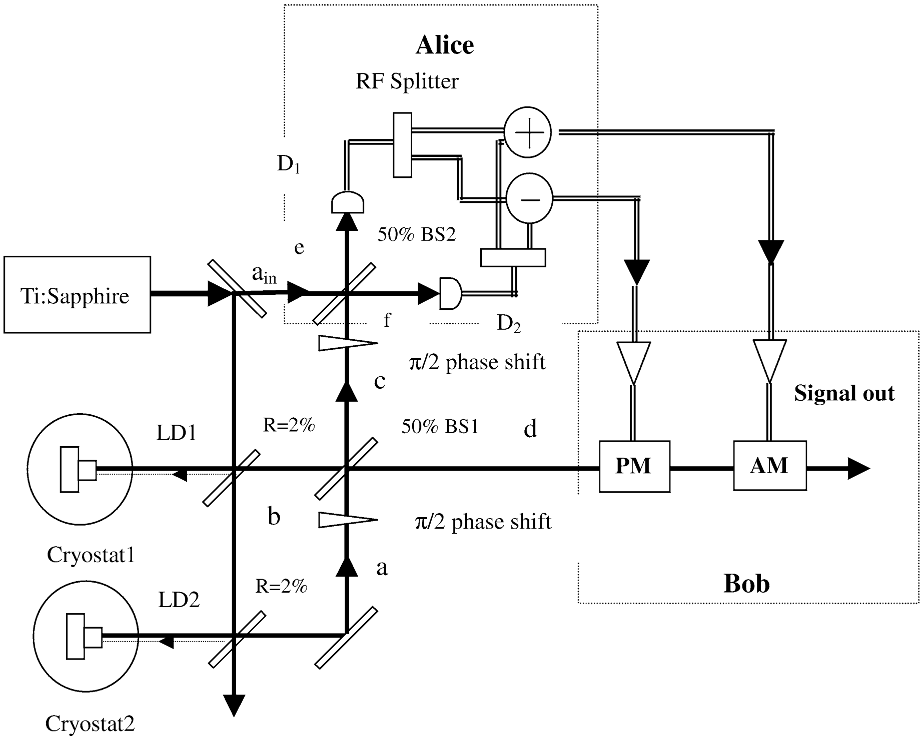

Fig. 1. Schematic of injection-locked LD systems for teleportation.

into LD1 and LD2 to produce two coherent amplitude-squeezed state lights a, b. a is phase-shifted $\pi/2$ then is mixed with b on a 50% beamsplitter (BS1) to generate the EPR beams. A half of the EPR beams is sent to the sender (Alice) where it mixed with the input signal beam of same intensity and $\pi/2$ phase shift on the second 50% beamsplitter (BS2). The bright output beams, e and f, are directly detected by D_1 and D_2. The e and f are given by

$$e = \frac{\sqrt{2}}{2}(a_{\text{in}} + ic), \qquad f = \frac{\sqrt{2}}{2}(a_{\text{in}} - ic). \tag{4}$$

Each of the detected photocurrents is divided into two parts by the RF power splitters. The sum and difference of the divided photocurrents are expressed by

$$i_+ = \frac{1}{\sqrt{2}}(X_{\text{in}} + X_c), \qquad i_- = \frac{1}{\sqrt{2}}(Y_{\text{in}} - Y_c). \tag{5}$$

Thus a Bell-state measurement of two beams is achieved with this simple direct detection. Then the photocurrents are sent to amplitude and phase modulators in the receiver (Bob), respectively. The amplitude and phase modulators transform the photocurrents into the other half d of the EPR beams. The output beam from modulators is found to be

$$a_{\text{out}} = d + g_+ i_+ + i g_- i_-, \tag{6}$$

where g_+ and g_- describe Bob's (suitably normalized) amplitude and phase gain for the transformation from photocurrent to output beam. The amplitude and phase variances of output beam are given by

$$V_{X_{\text{out}}} = g^2 V_{X_{\text{in}}} + (g+1)^2 \frac{V_{X_{\text{squee}}}}{2} + (g-1)^2 \frac{V_{Y_{\text{unsquee}}}}{2},$$
$$V_{Y_{\text{out}}} = g^2 V_{Y_{\text{in}}} + (g+1)^2 \frac{V_{X_{\text{squee}}}}{2} + (g-1)^2 \frac{V_{Y_{\text{squee}}}}{2}, \tag{7}$$

where $g_+ = g_- = \sqrt{2}g$. We can calculate the fidelity $F = |\langle a_{\text{in}}^+ a_{\text{out}} \rangle|^2$ characterizing the "quality" of teleportation with Eqs. (7) according to the definition in Ref. [3]. In the given teleportation scheme the gain coefficients g should be better taken as unity to ensure a

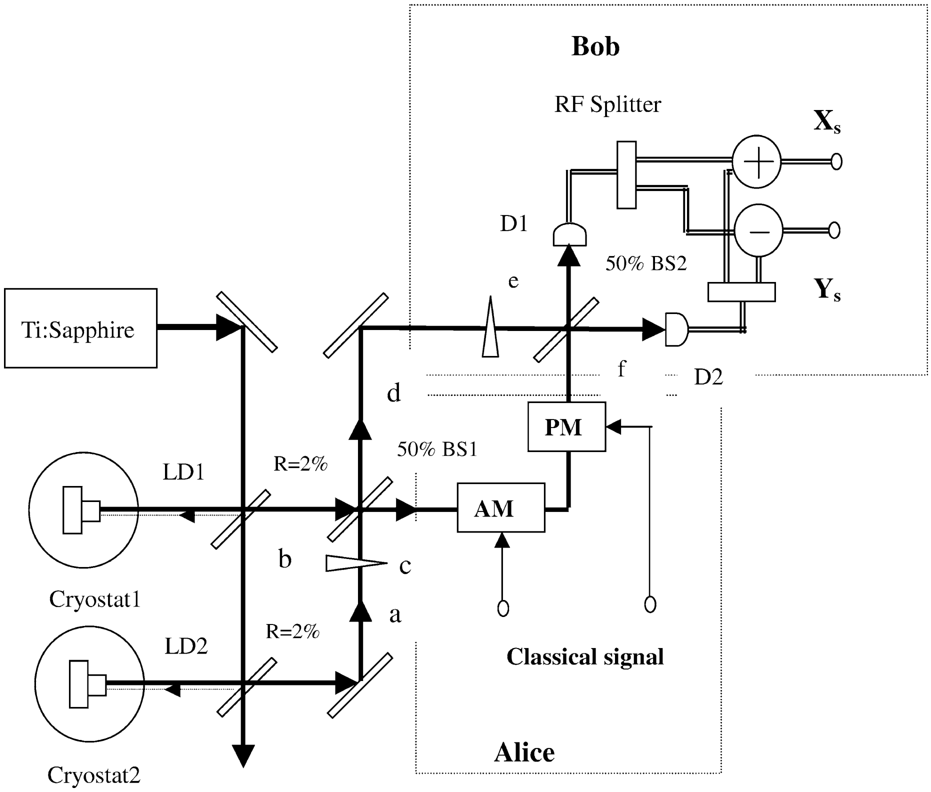

Fig. 2. Schematic of injection-locked LD systems for dense coding.

good fidelity, thus we get [3]

$$F_{g=1} = \frac{2}{\sqrt{(1 + V_{X_{\text{in}}} + V_{X_{\text{squee}}})(1 + V_{Y_{\text{in}}} + V_{X_{\text{squee}}})}}. \tag{8}$$

From Eq. (8) it is obvious that fidelity only depend on the squeezing of the amplitude quadrature and the noise of the phase quadrature is not significant. If the EPR beams are the coherent state, the variances $V_{X_{\text{out}}} = V_{Y_{\text{out}}} = 3$, the fidelity $F^{\text{Theo}}_{\text{Class}} = 0.5$ is the boundary between quantum and classical teleportation for coherent-state inputs. To meet the requirement of the quantum teleportation $F > 0.5$ Alice and Bob must share a nonlocal quantum source EPR pair. For satisfying the more stringent criteria of $F > 2/3$ proposed by Grangier et al. [14], where the inferred variables are required to violate the Heisenberg inequality (2) and at the same time with $g = 1$, the squeezing must be required to be more than 3 dB.

We propose also a scheme of dense coding, in which the simultaneous measurement of phase and amplitude signals with the sensitivity beyond the SQL, by means of bright amplitude-squeezed light from injection-locked LD and direct measurement of Bell state. As shown in Fig. 2, the classical amplitude and phase signals are modulated on a half of the entangled pair of EPR beams, that leads to a displacement of a_s,

$$c' = c + a_s, \tag{9}$$

where a_s is the sent signal via the quantum channel. From Eqs. (1) we know that there are very large noise $\langle\delta(X_c)^2\rangle \to \infty$, $\langle\delta(Y_c)^2\rangle \to \infty$ in both amplitude and phase quadratures of each of EPR beams, therefore the signal-noise ratios trend to zero:

$$\text{SNR}_X = \frac{\langle\delta(X_s)^2\rangle}{\langle\delta(X_c)^2\rangle} \to 0, \qquad \text{SNR}_Y = \frac{\langle\delta(Y_s)^2\rangle}{\langle\delta(Y_c)^2\rangle} \to 0. \tag{10}$$

No one other than Bob can attain any information of the signal from the modulated half of EPR beams in the ideal condition because the signal is submerged in large noises. The signal only can be demodulated with the aid of the other half of EPR beams which is quantum-correlated with the modulated first half. A phase-shifted $\pi/2$ is added between the two halves of the EPR beams at first and then they are combined on the 50% beamsplitter (BS2). The two bright output beams are directly detected by D_1 and D_2. Each of the photocurrents of D_1 and D_2 is divided into two parts through the RF power splitters. The sum and difference of the divided photocurrents are expressed by

$$\hat{i}_+(\Omega) = X_{\text{squee}} + \frac{1}{\sqrt{2}} X_s(\Omega),$$
$$\hat{i}_-(\Omega) = X_{\text{squee}} + \frac{1}{\sqrt{2}} Y_s(\Omega). \tag{11}$$

Thus we obtain the amplitude and phase quadrature of the signal with the sensitivity below the SQL by the simple direct detection. From Eqs. (11) we know that the precision of the dense coding only depends on the squeezing.

We proposed easily realized schemes of teleportation and dense coding for continuous variables using the bright amplitude-squeezed state light from the injection-locked laser diodes. The mature technic of producing coherent amplitude-squeezed beams from the injection-locked laser diodes and the simplicity of direct measurement make the scheme valuable for performing experiments.

Acknowledgements

This research is supported by the National Natural Science Foundation of China (Approval No. 60178012, 69837010), the Excellent Young Teacher Foundation from Education Department of China and the Shanxi Province Youth Science Foundation.

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