Nonclassical properties of teleported optical fields in quantum teleportation of continuous variables

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When sending a quantum state which originally has nonclassical properties such as various kinds of squeezing and photon antibunching effects according to the protocol for teleportation of continuous variables [S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998)], we investigate to what extent those nonclassical properties can be preserved in the teleported field. Explicit conditions of the squeezing parameter for the second- and fourth-order quadrature-phase squeezings, the squared amplitude squeezing, and the photon antibunching effect to survive in the teleported field are obtained.

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In the rapidly developing field of quantum communication, one of the crucial problems is how to send an unknown quantum state from one place to another one. This transmission has two essential points. First, only a quantum state itself is transported and a carrier of the quantum state is kept at the original location. Second, no information about a quantum state to be sent is given to the sender prior to the transmission. Thus, in this transmission, it would seem that an unknown quantum state disappears at one place and later emerges at another one. So, this process is termed teleportation of an unknown quantum state.

Bennet *et al.* [1] first proposed a scheme for teleporting an unknown quantum state in a finite-dimensional Hilbert space via a classical information channel and a quantum channel which is based on quantum nonlocal correlation between the sender and the receiver who share the Einstein-Podolsky-Rose (EPR) state [2]. Since this proposal was raised, a lot of effort towards accomplishing the protocol has been made [3–5]. On the other hand, Vaidman [6] proposed a scheme for teleporting continuous variables. In that scheme, the perfect correlation between position and momentum of two particles in the EPR state is used as a quantum channel. It is noticed that two quadrature-phase components of a single-mode optical field are analogous to position and momentum of a particle. Braunstein and Kimble [7] proposed a quantum optical version of teleportation of continuous variables. Furusawa *et al.* [8] experimentally demonstrated the scheme for a coherent state of a single-mode optical field. The experimental success has inspired much interest in the study of quantum teleportation of continuous variables [9–19].

In the protocol for quantum teleportation of continuous variables [7], quantum correlation between quadrature-phase components of an optical field in a two-mode highly squeezed state is employed as a quantum channel. Since the squeezing degree is finite and then the quantum channel is imperfect, the fidelity of the teleportation process must be less than 1. In fact, the requirement for a complete overlap between an input state and the corresponding output one is too strict to be fulfilled because it is equivalent to requiring that the mean photon number in the squeezed vacuum state be infinite [11]. Considering this point, we now ask a question: is it possible to transmit and preserve some interesting properties of a quantum state via the teleportation process even if the transportation of a whole quantum state is not perfect? Perhaps transferring partial information about a quantum state is not the original idea of teleportation of an unknown quantum state. However, we think that it may provide us with a more effective and secure approach for sending quantum information than traditional methods. In Ref. [15], transfer of nonclassical features of a quantum state such as sub-Poissonian statistics of photons and the second-order squeezing of quadrature-phase components according to the Braunstein and Kimble teleportation protocol, in which the quantum channel is influenced by a thermal environment, was investigated by use of the quasiprobability function. It was found that the mixed two-mode squeezed vacuum state for the quantum channel may become separable in the evolution of time and then any nonclassical features of an unknown state cannot be preserved in the teleported state. In this paper, using a general expression for the density matrix of the teleported field, we study higher-order squeezing properties of the teleported field, such as the fourth-order squeezing of quadrature-phase components and the squeezing of squared amplitudes. Explicit conditions of the squeezing parameter for these nonclassical features to survive in the teleported field are derived.

Suppose that two modes A and B of an optical field are prepared in a squeezed vacuum state

$$|S\rangle_{AB}=\cosh^{-1}r\,\exp(-\coth r\,a^{\dagger}b^{\dagger})|0\rangle, \tag{1}$$

where $a^{\dagger}$ (a) and $b^{\dagger}$ (b) are bosonic creation (annihilation) operators for modes A and B, respectively. When $r\neq 0$, two quadrature-phase components of the modes are entangled. Now let mode A be sent to the sender (Alice) and simultaneously mode B to the receiver (Bob). In this way, a quantum channel between Alice and Bob is built. Let us suppose that there is also a classical channel between them, via which they may communicate information in a usual way. Now we hand over an arbitrary quantum state $|\varphi_i\rangle$ to Alice but we do not give her any information about this state. However, we ask Alice to send this state to Bob. According to the teleportation scheme [7], the task can be fulfilled by two steps. First, Alice performs a local Bell-state measurement on the subsystem which consists of the entangled mode A and the input mode. The Bell-state measurement is composed of an ideal 50/50 beam splitter and two homodyne measurement detectors for measuring eigenvalues of two commutative quadrature-phase operators for the optical fields at two output ports of the beam splitter. After the Bell-state measurement, the mode A and the input mode are entangled together and both are projected into one of the eigenstates but the mode B is separated from the entangled mode A [18]. Second, Alice sends the measured result regarding which of the eigenstates is measured to Bob via a classical information channel and then Bob performs an appropriate local unitary transformation on the mode B according to the measured result. After these two steps, Bob can obtain an approximate copy of the input state. The above teleportation scheme was originally described by use of the Wigner function [7]. In the recent publication [18], Janszky *et al.* reformulated the scheme in the coherent state representation and yielded a simple direct description of the teleportation process. In the representation of coherent states, an input state $|\varphi_i\rangle$ can be written as

$$|\varphi_i\rangle=\int d^2\alpha P(\alpha)|\alpha\rangle, \tag{2}$$

where $P(\alpha)=\langle\alpha|\varphi_i\rangle/\pi$. Using the approach proposed in Ref. [18] and completing the above teleportation steps, we can finally place the mode B in the unnormalized state

$$|\phi(x_a,p_b)\rangle=\sqrt{\frac{2}{\pi}}\cosh^{-1}r\,\exp[-2(1-\tanh r)|z|^2]\int d^2\alpha P(\alpha)\exp[-\tfrac{1}{2}|\alpha|^2+\sqrt{2}(1-\tanh r)z^*\alpha]$$
$$\times\exp\{[\alpha\tanh r+\sqrt{2}(1-\tanh r)z]\tanh b^\dagger\}|0\rangle, \tag{3}$$

where $z=x_a-ip_b$ and (x_a,p_b) are outputs of the homodyne measurement detectors. If outputs of the detectors are locked at a fixed value (x_a,p_b), the state vector (3) is the conditional teleported state in the mode B. If a series of entirely equivalent states are in sequence given to Alice in the teleportation process and the detectors are able to respond to all of the eigenvalues of the two commutative quadrature-phase operators, the mode B will be in a mixed state which is described by the density matrix

$$\hat{\rho}=\int dx_a dp_b|\phi(x_a,p_b)\rangle\langle\phi(x_a,p_b)|. \tag{4}$$

The quality of the teleportation process can be measured by the so-called fidelity, which is defined as $|\langle\varphi_i|\phi(x_a,p_b)\rangle|^2/\langle\phi(x_a,p_b)|\phi(x_a,p_b)\rangle$ [7]. Since (x_a,p_b) are continuous variables and detected in the probability density $\langle\phi(x_a,p_b)|\phi(x_a,p_b)\rangle$, an averaged fidelity is appropriate for measuring the teleportation quality and is given by

$$\begin{aligned}F_a&=\langle\varphi_i|\hat{\rho}|\varphi_i\rangle\\&=\frac{1}{2}(1+\coth r)\int d^2\alpha d^2\beta d^2\xi d^2\eta P(\alpha)P(\xi)P(\beta)^*P(\eta)^*\exp[-\tfrac{1}{2}(|\alpha|^2+|\xi|^2+|\eta|^2+|\beta|^2)]\\&\quad\times\exp\{\tfrac{1}{2}[(\alpha+\xi)(\eta^*+\beta^*)+\coth r(\alpha-\xi)(\eta^*-\beta^*)]\}.\end{aligned} \tag{5}$$

In the following part, we will use the above explicit expressions for the density matrix of the teleported field and the fidelity to investigate the preservation of nonclassical properties of an input state in the teleportation process. By use of Eq. (4), the mean photon number in the teleported field is given by

$$\langle\hat{n}\rangle_t=\langle\hat{n}\rangle_i+e^{-2r}, \tag{6}$$

where $\hat{n}=b^\dagger b$ and $\langle\rangle_{i,t}$ refers to averaging over an input state $|\varphi_i\rangle$ and the teleported field (4), respectively. It is noticed that the input field is amplified in the teleportation process but the gain of the mean photon number is independent of a quantum state to be teleported. In fact, the gain is just the amount of noise in the squeezed quadrature-phase component of an optical field in the state (1). The variance of the mean photon number in the teleported field is given by

$$\langle(\Delta\hat{n})^2\rangle_t=\langle(\Delta\hat{n})^2\rangle_i+(2\langle\hat{n}\rangle_i+1)e^{-2r}+e^{-4r}, \tag{7}$$

where $\langle(\Delta\hat{n})^2\rangle=\langle\hat{n}^2\rangle-\langle\hat{n}\rangle^2$. We see that the variance increases and the change depends on an input state. The photon antibunching effect appears when the normalized equal-time second-order correlation function is less than 1 [20], that is,

$$g^{(2)}(0)=\langle b^{+2}b^2\rangle/\langle b^+b\rangle^2<1. \tag{8}$$

From Eq. (4), we have the correlation function of the teleported field,

$$g_t^{(2)}(0)=\frac{\langle\hat{n}^2\rangle_i-(1-4e^{-2r})\langle\hat{n}\rangle_i+2e^{-4r}}{(\langle\hat{n}\rangle_i+e^{-2r})^2}. \tag{9}$$

When the teleported field exhibits the photon antibunching effect, the condition $g_t^{(2)}(0)<1$ holds. This results in the following condition of the squeezing parameter:

$$r>-\tfrac{1}{2}\ln\{\sqrt{\langle\hat{n}\rangle_i^2-[\langle(\Delta\hat{n})^2\rangle_i-\langle\hat{n}\rangle_i]}-\langle\hat{n}\rangle_i\}. \quad (10)$$

In order to investigate squeezing effects in the teleported field, we introduce two quadrature-phase operators for the mode B,

$$\hat{X}_1=\tfrac{1}{2}(b+b^\dagger), \quad (11)$$

$$\hat{X}_2=(1/2i)(b-b^\dagger). \quad (12)$$

When the condition

$$\langle(\Delta\hat{X}_{1,2})^N\rangle<(N-1)!!/2^N \quad (13)$$

is satisfied, the field is said to be in an Nth-order squeezed state [21]. For the second-order moment of the fluctuation in quadrature-phase components of the teleported field, from Eq. (4), we have

$$\langle(\Delta\hat{X}_{1,2})^2\rangle_t=\langle(\Delta\hat{X}_{1,2})^2\rangle_i+\tfrac{1}{2}e^{-2r}. \quad (14)$$

We see that half the amount of noise in the squeezed quadrature component of an optical field in the state (1) is added to the variances of two quadrature-phase components of the teleported field [9,15]. The change of the variances of two quadrature-phase components of an input field in the teleportation process is irrelevant to the input state. According to Eq. (13), we can find out that when the squeezing parameter r satisfies the condition

$$r>-\tfrac{1}{2}\ln\{2[\tfrac{1}{4}-\langle(\Delta\hat{X}_{1,2})^2\rangle_i]\}, \quad (15)$$

the second-order squeezing effect which is originally imposed on an input state can be preserved in the teleported state.

The fourth-order moment of the fluctuation in quadrature-phase components of the teleported field is given by

$$\langle(\Delta\hat{X}_{1,2})^4\rangle_t=\langle(\Delta\hat{X}_{1,2})^4\rangle_i+3\langle(\Delta\hat{X}_{1,2})^2\rangle_i e^{-2r}+\tfrac{3}{4}e^{-4r}. \quad (16)$$

According to Eq. (13), the fourth-order squeezing means $\langle(\Delta\hat{X}_{1,2})^4\rangle<\frac{3}{16}$. We see that unlike the second-order squeezing, the change of the fourth-order squeezing, in the teleportation process depends on a state to be teleported. From Eq. (16), we conclude that when the squeezing parameter r satisfies the condition

$$r>-\tfrac{1}{2}\ln\{\sqrt{\langle(\Delta\hat{X}_{1,2})^2\rangle_i^2-\tfrac{1}{3}[\tfrac{3}{16}-\langle(\Delta\hat{X}_{1,2})^4\rangle_i]}-\langle(\Delta\hat{X}_{1,2})^2\rangle_i\}, \quad (17)$$

the fourth-order squeezing effect can exist in the teleported field.

Another kind of higher-order squeezings is the so-called squared amplitude squeezing [22]. Two quadrature-phase components of the squared complex amplitude of the mode B are defined as

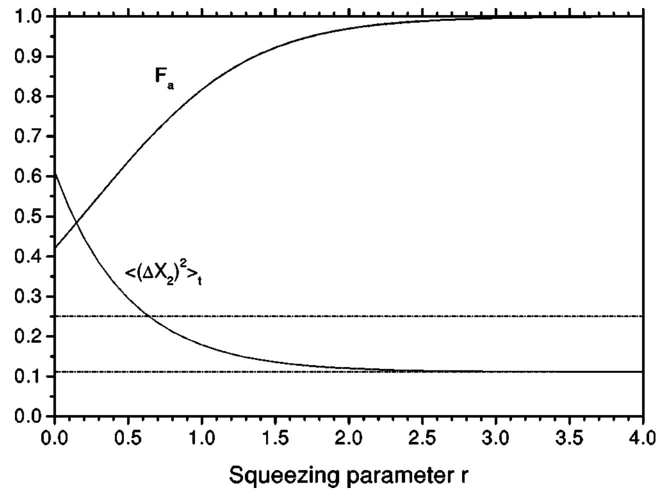

FIG. 1. Fidelity of teleportation of $|\alpha\rangle+|-\alpha\rangle$ with $\alpha=0.8$ and second-order squeezing in the teleported field vs the squeezing parameter r. The upper boundary is 0.25 and the lower one is the squeezing amount of the input state.

$$\hat{Y}_1=\tfrac{1}{2}[b^2+(b^+)^2], \quad (18)$$

$$\hat{Y}_2=(1/2i)[b^2-(b^+)^2]. \quad (19)$$

When $\langle(\Delta\hat{Y}_{1,2})^2\rangle-(\langle\hat{n}\rangle+\frac{1}{2})<0$, the field is said to be in a squared amplitude squeezing state. This is also a nonclassical effect [22]. The variance of $\hat{Y}_{1,2}$ in the teleported state (4) is given by

$$\langle(\Delta\hat{Y}_{1,2})^2\rangle_t=\langle(\Delta\hat{Y}_{1,2})^2\rangle_i+2\langle\hat{n}\rangle_i e^{-2r}+e^{-4r}. \quad (20)$$

We see that the change of the variances in the teleportation process depends on an input state. When the squared amplitude squeezing survives in the teleported field, we have the condition

$$\langle(\Delta\hat{Y}_{1,2})^2\rangle_i-(\langle n\rangle_i+\tfrac{1}{2})+2\langle n\rangle_i e^{-2r}+e^{-4r}\leqslant 0. \quad (21)$$

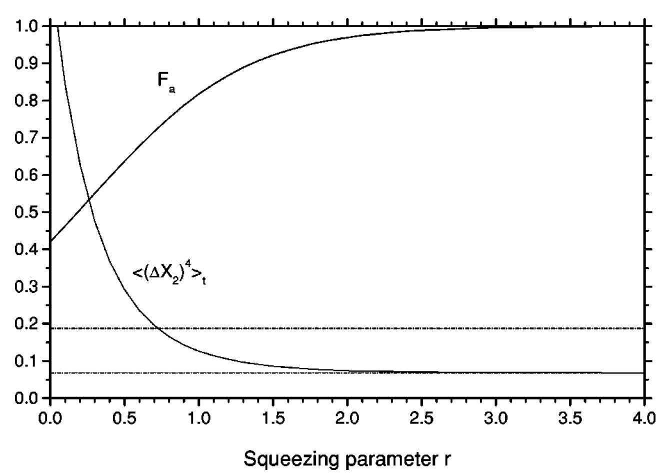

FIG. 2. Fidelity of teleportation of $|\alpha\rangle+|-\alpha\rangle$ with $\alpha=0.8$ and fourth-order squeezing in the teleported field vs the squeezing parameter r. The upper boundary is $\frac{3}{16}$ and the lower one is the squeezing amount of the input state.

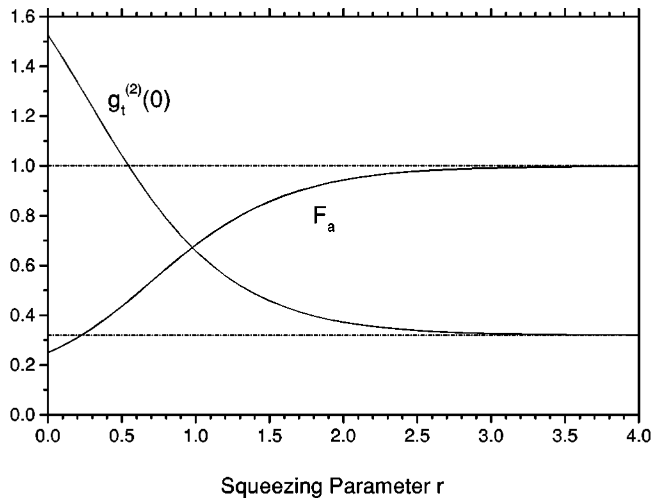

FIG. 3. Fidelity of teleportation of $|\alpha\rangle - |-\alpha\rangle$ with $\alpha = 0.8$ and second-order equal-time correlation function in the teleported field vs the squeezing parameter r. The upper boundary is 1.0 and the lower one is the value of the correlation function in the input state.

This inequality results in the requirement for the squeezing parameter r,

$$r > -\tfrac{1}{2}\ln\{\sqrt{\langle\hat{n}\rangle^2_i - [\langle(\Delta\hat{Y}_{1,2})^2\rangle_i - (\langle\hat{n}\rangle_i + \tfrac{1}{2})]} - \langle\hat{n}\rangle_i\}. \qquad (22)$$

To have a concrete idea on the transfer of nonclassical properties of a quantum state in the teleportation process, now let us consider two interesting states. As is well known, two linear superpositions of coherent states $|\alpha\rangle + |-\alpha\rangle$ and $|\alpha\rangle - |-\alpha\rangle$ can display the second-order and fourth-order squeezings and the photon antibunching effect, respectively. In Figs. 1 and 2, the second-order and fourth-order fluctuations in the out-phase quadrature component of the teleported field and the fidelity are shown against the squeezing parameter r when teleporting the state $|\alpha\rangle + |-\alpha\rangle$. In Fig. 3, the equal-time second-order correlation function and the fidelity are shown against the squeezing parameter r when sending the state $|\alpha\rangle - |-\alpha\rangle$. These figures show that when $r > 0.6, 0.65, 0.75$, the photon antibunching effect, the second-order, and fourth-order squeezings, which are originally imposed on the input states, can remain in the teleported states. In the experiment [8], r is about 0.69 and approaches the marginal region. In these figures, we also see that when $r > 1.5$, the nonclassical properties of the input states can be well preserved in the teleported one but at the same time the fidelity is around 0.85. In order to make the fidelity close to unity, the squeezing parameter r must be larger than 3.0. This means that the nonclassical properties of an input state may be satifactorily transferred from a sending station to a receiving station via the teleportation process even though a quantum state is not truly and exactly teleported.

In summary, nonclassical properties such as the second- and fourth-order quadrature-phase and squared amplitude squeezings, and the photon antibunching effect in the teleported field, are investigated. We show that these nonclassical properties which are originally imposed on an unknown quantum state to be teleported can be preserved in the teleported state if the squeezing parameter of the two-mode squeezed vacuum state which is used as an EPR source is larger than a certain value. The explicit conditions of the squeezing parameter for preserving these nonclassical properties in the teleported field are obtained. We also note that these nonclassical properties may be well transferred via the teleportation process even if the fidelity of the teleportation process is not very close to unity.

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