Controlled dense coding for continuous variables using three-particle entangled states

Jing Zhang, Changde Xie, and Kunchi Peng*
The State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan 030006, People's Republic of China

(Received 20 February 2002; revised manuscript received 22 May 2002; published 26 September 2002)

A simple scheme to realize quantum controlled dense coding with a bright tripartite entangled state light generated from nondegenerate optical parametric amplifiers is proposed in this paper. The quantum channel between Alice and Bob is controlled by Claire. As a local oscillator and balanced homodyne detector are not needed, the proposed protocol is easy to be realized experimentally.

DOI: 10.1103/PhysRevA.66.032318 PACS number(s): 03.67.Hk, 42.50.Dv, 03.65.Ta

In the development of theoretical and experimental studies of quantum information, the quantum teleportation that is the disembodied transport of an unknown quantum state from a sender to a remote receiver [1] and the dense coding in which the single bit sent from a sender to a receiver can successfully carry two bits of classical information [2], have attracted extensive interests. The nonlocal quantum entanglement plays a determinant role in the quantum information processing. Towards possible applications in quantum communication, both theoretical and experimental investigations increasingly focus on quantum states of continuous variables in an infinite-dimensional Hilbert space, since the Einstein-Podolsky-Rosen (EPR) entangled state can be efficiently generated using squeezed light and beam splitters, for instance, the entangled EPR pairs resulting from two-mode squeezed vacuum state have been successfully employed in demonstrating unconditional quantum teleportation [3]. Later, the schemes realizing highly efficient dense coding for continuous variables are theoretically proposed, in which the two-mode squeezed-state entanglement is utilized to achieve unconditional signal transmission [4–6]. The bright EPR beams have been experimentally produced with a nondegenerate optical parametric amplifier (NOPA) [7] and the dense coding for continuous variables based on bright EPR beam has been demonstrated initially [8]. Loock and Braunstein have given that the superposition of more than independently squeezed states can yield multipartite entanglement for continuous variables and presented the scheme of quantum teleportation using entangled three-mode state [9]. The fidelity in this scheme depends on the measurement of the third particle. Controlled dense coding for discrete variables was proposed recently using the Greenberger-Horne-Zeilinger state (GHZ) [10]. Inspired by the similarity and difference between dense coding and quantum teleportation, in this paper we study dense coding using the tripartite entangled state. It is shown that when using the tripartite entangled state, the information transmission capacity of dense coding is controlled by the measurement of the third particle. We introduce a simple, experimentally realizable, controlled dense coding protocol for continuous variables by exploring nondegenerate optical parametric amplifier. Due to adopting the bright EPR beams and the simple direct measurement for Bell state, the controlled dense coding is within the reach of current technology and significantly simplify the implementation.

The schematic diagram for phase-sensitive NOPA is shown in Fig. 1. Two coherent input signals $a_{\updownarrow}$ and $a_{\leftrightarrow}$ with same frequency ω_0 and orthogonal polarization are injected into a NOPA. For simplification, the polarizations of the injected signal and idler field are orientated along the vertical and horizontal directions, and their intensities and original phases before NOPA are considered to be identical. The amplifier is pumped with the second-harmonic wave of $\omega_p = 2\omega_0$ and amplitude of pump field $a_p \gg a_{\updownarrow}, a_{\leftrightarrow}$; in this case the pump field can be considered as a classical field without depletion during the amplification process. The output signal and idler fields polarized along the vertical and horizontal directions are rotated by a half-wave plate at angle $\vartheta/2$, then pass a polarizing beam splitter with the output fields $b_{\updownarrow}$ and $b_{\leftrightarrow}$. We define the operators of the light fields at the center frequency ω_0 in the rotating frame,

$$\hat{O}(t) = \hat{o}(t) e^{i\omega_0 t}, \tag{1}$$

where $\hat{O} = [\hat{a}_{\updownarrow}, \hat{a}_{\leftrightarrow}, \hat{b}_{\updownarrow}, \hat{b}_{\leftrightarrow}]$ are the field envelope operators and $\hat{o} = [\hat{A}_{\updownarrow}, \hat{A}_{\leftrightarrow}, \hat{B}_{\updownarrow}, \hat{B}_{\leftrightarrow}]$ are the field operators corresponding to input and output signal and idler fields. By the Fourier transformation, we have

$$\hat{O}(\Omega) = \frac{1}{\sqrt{2\pi}} \int dt \hat{O}(t) e^{-i\Omega t}. \tag{2}$$

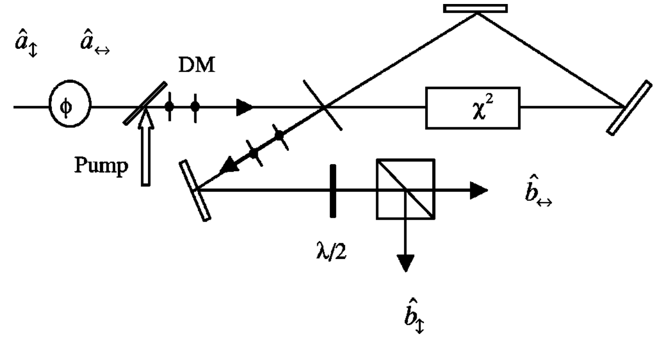

FIG. 1. The schematic for phase-sensitive NOPA. DM represents dichroic mirror.

*FAX: +86-351-7011500.
Email address: kcpeng@mail.sxu.edu.cn

Here, the fields are described as functions of the modulation frequency Ω with commutation relation $[\hat{O}(\Omega),\hat{O}^{+}(\Omega')]=\delta(\Omega-\Omega')$. A practical light field can be decomposed to a carrier $\hat{O}(0)$ oscillating at the center frequency ω_0 with an average amplitude (O_{ss}) that equals to the amplitude of its steady-state field, and surrounded by "noise sidebands" $\hat{O}(\Omega)$ oscillating at frequency $\omega_0\pm\Omega$ with zero average amplitude [4]

$$\langle\hat{O}(\Omega=0)\rangle=O_{ss};\quad \langle\hat{O}(\Omega\neq 0)\rangle=0. \tag{3}$$

The noise spectral component at frequency Ω is the heterodyne mixing of the carrier and the noise sidebands. The amplitude and phase quadrature are expressed by

$$\hat{X}_O(\Omega)=\hat{O}(\Omega)+\hat{O}^{+}(-\Omega);$$

$$\hat{Y}_O(\Omega)=\frac{1}{i}[\hat{O}(\Omega)-\hat{O}^{+}(-\Omega)], \tag{4}$$

with

$$[\hat{X}_O(\Omega),\hat{Y}_O(\Omega')]=2i\,\delta(\Omega+\Omega'). \tag{5}$$

The input-output Heisenberg evolutions of the field modes of the NOPA are given by [11,12]

$$\hat{b}_{0\updownarrow}=\sin\vartheta(\mu\hat{a}_{0\updownarrow}+\nu\hat{a}^{+}_{0\leftrightarrow})+\cos\vartheta(\mu\hat{a}_{0\leftrightarrow}+\nu\hat{a}^{+}_{0\updownarrow}),$$

$$\hat{b}_{0\leftrightarrow}=\cos\vartheta(\mu\hat{a}_{0\updownarrow}+\nu\hat{a}^{+}_{0\leftrightarrow})-\sin\vartheta(\mu\hat{a}_{0\leftrightarrow}+\nu\hat{a}^{+}_{0\updownarrow}),$$

$$\hat{b}_{+\updownarrow}=\sin\vartheta(\mu\hat{a}_{+\updownarrow}+\nu\hat{a}^{+}_{+\leftrightarrow})+\cos\vartheta(\mu\hat{a}_{+\leftrightarrow}+\nu\hat{a}^{+}_{+\updownarrow}),$$

$$\hat{b}_{+\leftrightarrow}=\cos\vartheta(\mu\hat{a}_{+\updownarrow}+\nu\hat{a}^{+}_{+\leftrightarrow})-\sin\vartheta(\mu\hat{a}_{+\leftrightarrow}+\nu\hat{a}^{+}_{+\updownarrow}),$$

$$\hat{b}_{-\updownarrow}=\sin\vartheta(\mu\hat{a}_{-\updownarrow}+\nu\hat{a}^{+}_{-\leftrightarrow})+\cos\vartheta(\mu\hat{a}_{-\leftrightarrow}+\nu\hat{a}^{+}_{-\updownarrow}),$$

$$\hat{b}_{-\leftrightarrow}=\cos\vartheta(\mu\hat{a}_{-\updownarrow}+\nu\hat{a}^{+}_{-\leftrightarrow})-\sin\vartheta(\mu\hat{a}_{-\leftrightarrow}+\nu\hat{a}^{+}_{-\updownarrow}), \tag{6}$$

where $\hat{a},\hat{a}^{+}$, and $\hat{b},\hat{b}^{+}$ denote the annihilation and creation operators of the input and the output modes. The subindices 0 and $\pm$ stand for the central mode at frequency ω_0 and the sidebands at frequency $\omega_0\pm\Omega$, respectively. The parameters $\mu=\cosh r$ and $\nu=e^{i\theta_p}\sinh r$ are the function of the squeezing factor r ($r\propto L\chi^2|a_p|$, L is the nonlinear crystal length, χ^2 is the effective second-order susceptibility of the nonlinear crystal in NOPA, and a_p is the amplitude of pump field) and the phase θ_p of the pump field. In the following calculation, the phase θ_p is set to zero as the reference of relative phase of all other light fields. For bright optical field, the quadratures of the output orthogonal polarization modes at a certain rotated phase θ are expressed by

$$\hat{X}_{\hat{b}_\updownarrow}(\theta)=\frac{b^{*}_{0\updownarrow}\hat{b}_{+\updownarrow}e^{-i\theta}+b_{0\updownarrow}\hat{b}^{+}_{-\updownarrow}e^{i\theta}}{|b_{0\updownarrow}|}$$

$$=\hat{b}_{+\updownarrow}e^{-i(\theta+\varphi)}+\hat{b}^{+}_{-\updownarrow}e^{i(\theta+\varphi)},$$

$$\hat{X}_{\hat{b}_\leftrightarrow}(\theta)=\hat{b}_{+\leftrightarrow}e^{-i(\theta+\varphi)}+\hat{b}^{+}_{-\leftrightarrow}e^{i(\theta+\varphi)}, \tag{7}$$

where $\varphi=\arg(b_{0\updownarrow})=\arg(b_{0\leftrightarrow})=\arg(e^{i\Phi}+e^{-i\Phi}\tanh r)$ is the phase of the modes $\hat{b}_{0\updownarrow},\hat{b}_{0\leftrightarrow}$ relative to θ_p, and Φ is the phase of the modes $\hat{a}_{0\updownarrow},\hat{a}_{0\leftrightarrow}$ relative to θ_p. Taking $\theta=0$ and $\theta=\pi/2$ in Eq. (7), the amplitude and phase quadrature of the output field are obtained,

$$\hat{X}_{\hat{b}_\updownarrow}=\hat{X}_{\hat{b}_\updownarrow}(0)=\hat{b}_{+\updownarrow}e^{-i\varphi}+\hat{b}^{+}_{-\updownarrow}e^{i\varphi},$$

$$\hat{X}_{\hat{b}_\leftrightarrow}=\hat{X}_{\hat{b}_\leftrightarrow}(0)=\hat{b}_{+\leftrightarrow}e^{-i\varphi}+\hat{b}^{+}_{-\leftrightarrow}e^{i\varphi},$$

$$\hat{Y}_{\hat{b}_\updownarrow}=\hat{X}_{\hat{b}_\updownarrow}\left(\frac{\pi}{2}\right)=-i(\hat{b}_{+\updownarrow}e^{-i\varphi}-\hat{b}^{+}_{-\updownarrow}e^{i\varphi}),$$

$$\hat{Y}_{\hat{b}_\leftrightarrow}=\hat{X}_{\hat{b}_\leftrightarrow}\left(\frac{\pi}{2}\right)=-i(\hat{b}_{+\leftrightarrow}e^{-i\varphi}-\hat{b}^{+}_{-\leftrightarrow}e^{i\varphi}). \tag{8}$$

When the injected subharmonic signal and harmonic pump field are in phase ($\Phi=\varphi=0$), maximum parametric amplification is achieved [7]. The difference of the amplitude quadratures and the sum of the phase quadratures between two orthogonal polarization modes at $\vartheta=0$ are

$$\hat{X}_{\hat{b}_\updownarrow}-\hat{X}_{\hat{b}_\leftrightarrow}=e^{-r}\hat{X}_{\hat{a}_\updownarrow}-e^{-r}\hat{X}_{\hat{a}_\leftrightarrow},$$

$$\hat{Y}_{\hat{b}_\updownarrow}+\hat{Y}_{\hat{b}_\leftrightarrow}=e^{-r}\hat{Y}_{\hat{a}_\updownarrow}+e^{-r}\hat{Y}_{\hat{a}_\leftrightarrow}. \tag{9}$$

Under the limit $r\rightarrow\infty$, the output orthogonal polarization modes are the perfect EPR beams (bipartite entanglement) with quadrature amplitude correlation and quadrature phase anticorrelation [7]. When the injected subharmonic signal and harmonic pump field are out of phase, i.e., $\Phi=\varphi=\pi/2$, NOPA operates at parametric deamplification [8,13]. Therefore the sum of the amplitude quadratures and the difference of the phase quadratures of the orthogonal polarization modes at $\vartheta=0$ are as follows:

$$\hat{X}_{\hat{b}_\updownarrow}+\hat{X}_{\hat{b}_\leftrightarrow}=e^{-r}\hat{Y}_{\hat{a}_\updownarrow}+e^{-r}\hat{Y}_{\hat{a}_\leftrightarrow}$$

$$\hat{Y}_{\hat{b}_\updownarrow}-\hat{Y}_{\hat{b}_\leftrightarrow}=e^{-r}\hat{X}_{\hat{a}_\updownarrow}-e^{-r}\hat{X}_{\hat{a}_\leftrightarrow}. \tag{10}$$

Obviously, the EPR beams with the quadrature amplitude anticorrelation and quadrature phase correlation are obtained for $r>0$. Recently, the dense coding for continuous variables demonstrated experimentally [8] is just based on bright EPR beam from NOPA operating at parametric deamplification.

The proposed scheme is shown in Fig. 2. We generate tripartite entangled state using two NOPAs that can yield four-particle entangled state (discard a squeezed mode). We assume that the two NOPAs operating at parametric deam-

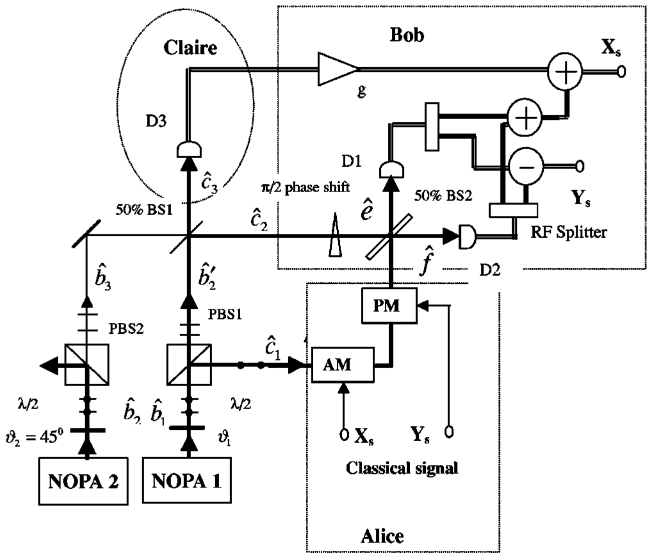

FIG. 2. Schematic for controlled dense coding using NOPA.

plification have the squeezing factors r_1 and r_2, respectively. The polarizations of two output modes from NOPA1 are rotated with $\vartheta_1=\arcsin[(\sqrt{2}-1)/\sqrt{6}]$ by a half-wave plate and the polarizations of two output modes from NOPA2 are rotated with $\vartheta_2=45°$ by a half-wave plate, then the beams are mixed respectively on polarizing beam splitters (PBS1 and PBS2). The resulting output beams are given by

$$\hat{X}_{\hat{c}_1}=\frac{1}{\sqrt{6}}\hat{Y}_{\hat{a}_1}(e^{-r_1}+\sqrt{2}e^{r_1})+\frac{1}{\sqrt{6}}\hat{Y}_{\hat{a}_2}(e^{-r_1}-\sqrt{2}e^{r_1}),$$

$$\hat{Y}_{\hat{c}_1}=\frac{1}{\sqrt{6}}\hat{X}_{\hat{a}_1}(e^{r_1}+\sqrt{2}e^{-r_1})+\frac{1}{\sqrt{6}}\hat{X}_{\hat{a}_2}(e^{r_1}-\sqrt{2}e^{-r_1}),$$

$$\hat{X}_{\hat{b}_2'}=\frac{1}{\sqrt{6}}\hat{Y}_{\hat{a}_1}(\sqrt{2}e^{-r_1}-e^{r_1})+\frac{1}{\sqrt{6}}\hat{Y}_{\hat{a}_2}(\sqrt{2}e^{-r_1}+e^{r_1}),$$

$$\hat{Y}_{\hat{b}_2'}=\frac{1}{\sqrt{6}}\hat{X}_{\hat{a}_1}(\sqrt{2}e^{r_1}-e^{-r_1})+\frac{1}{\sqrt{6}}\hat{X}_{\hat{a}_2}(\sqrt{2}e^{r_1}+e^{-r_1}),$$

$$\hat{X}_{\hat{b}_3}=\frac{1}{\sqrt{2}}(\hat{Y}_{\hat{a}_3}e^{r_2}-\hat{Y}_{\hat{a}_4}e^{r_2}),$$

$$\hat{Y}_{\hat{b}_3}=\frac{1}{\sqrt{2}}(\hat{X}_{\hat{a}_3}e^{-r_2}-\hat{X}_{\hat{a}_4}e^{-r_2}), \tag{11}$$

where $\langle\hat{b}_2'\rangle=\sqrt{2}\langle\hat{c}_1\rangle\gg\langle\hat{b}_3\rangle$. The beams $\hat{b}_2'$ and $\hat{b}_3$ then are mixed on a 50% beam splitter (BS1). Finally three output modes $\hat{c}_1$, $\hat{c}_2$, and $\hat{c}_3$ obviously exhibit tripartite entanglement,

$$\hat{X}_{\hat{c}_1}=\frac{1}{\sqrt{6}}\hat{Y}_{\hat{a}_1}(e^{-r_1}+\sqrt{2}e^{r_1})+\frac{1}{\sqrt{6}}\hat{Y}_{\hat{a}_2}(e^{-r_1}-\sqrt{2}e^{r_1}),$$

$$\hat{Y}_{\hat{c}_1}=\frac{1}{\sqrt{6}}\hat{X}_{\hat{a}_1}(e^{r_1}+\sqrt{2}e^{-r_1})+\frac{1}{\sqrt{6}}\hat{X}_{\hat{a}_2}(e^{r_1}-\sqrt{2}e^{-r_1}),$$

$$\hat{X}_{\hat{c}_2}=\frac{1}{\sqrt{12}}\hat{Y}_{\hat{a}_1}(\sqrt{2}e^{-r_1}-e^{r_1})+\frac{1}{\sqrt{12}}\hat{Y}_{\hat{a}_2}(\sqrt{2}e^{-r_1}+e^{r_1})$$
$$+\frac{1}{2}(\hat{Y}_{\hat{a}_3}e^{r_2}-\hat{Y}_{\hat{a}_4}e^{r_2}),$$

$$\hat{Y}_{\hat{c}_2}=\frac{1}{\sqrt{12}}\hat{X}_{\hat{a}_1}(\sqrt{2}e^{r_1}-e^{-r_1})+\frac{1}{\sqrt{12}}\hat{X}_{\hat{a}_2}(\sqrt{2}e^{r_1}+e^{-r_1})$$
$$+\frac{1}{2}(\hat{X}_{\hat{a}_3}e^{-r_2}-\hat{X}_{\hat{a}_4}e^{-r_2}),$$

$$\hat{X}_{\hat{c}_3}=\frac{1}{\sqrt{12}}\hat{Y}_{\hat{a}_1}(\sqrt{2}e^{-r_1}-e^{r_1})+\frac{1}{\sqrt{12}}\hat{Y}_{\hat{a}_2}(\sqrt{2}e^{-r_1}+e^{r_1})$$
$$-\frac{1}{2}(\hat{Y}_{\hat{a}_3}e^{r_2}-\hat{Y}_{\hat{a}_4}e^{r_2}),$$

$$\hat{Y}_{\hat{c}_3}=\frac{1}{\sqrt{12}}\hat{X}_{\hat{a}_1}(\sqrt{2}e^{r_1}-e^{-r_1})+\frac{1}{\sqrt{12}}\hat{X}_{\hat{a}_2}(\sqrt{2}e^{r_1}+e^{-r_1})$$
$$-\frac{1}{2}(\hat{X}_{\hat{a}_3}e^{-r_2}-\hat{X}_{\hat{a}_4}e^{-r_2}), \tag{12}$$

where $\langle\hat{c}_1\rangle=\langle\hat{c}_2\rangle=\langle\hat{c}_3\rangle\gg 0$. The outgoing bright "GHZ-like" state is a "three-mode position eigenstate" with total position $\hat{X}_{\hat{c}_1}+\hat{X}_{\hat{c}_2}+\hat{X}_{\hat{c}_3}\to 0$ and relative momenta $\hat{Y}_{\hat{c}_i}-\hat{Y}_{\hat{c}_j}\to 0$ $(i,j=1,2,3)$. It corresponds to a three-mode squeezed state, obtained by superimposing one bright amplitude-quadrature-squeezed state and two vacuum phase-quadrature-squeezed states. Now we construct controlled dense coding protocol using this tripartite entanglement state and involving three participants Alice, Bob, and Claire. Let us send the three modes of Eqs. (12) to Alice, Bob, and Claire, respectively. We assume that Alice wants to send classical information to Bob, while Claire supervises and controls the transmission through his measurement. To send the information to Bob, Alice modulates classical amplitude and phase signals on two quadratures of her mode $\hat{c}_1$ by amplitude and phase modulators, which lead to a displacement of a_s

$$\hat{c}_1'=\hat{c}_1+a_s, \tag{13}$$

where $a_s=X_s+iY_s$ is the sent signal via the quantum channel. From Eqs. (12), we know the amplitude and phase quadrature of EPR beams have large noise, $\langle \delta(\hat{X}_{\hat{c}_1})^2\rangle \to\infty$, $\langle \delta(\hat{Y}_{\hat{c}_1})^2\rangle\to\infty$ for $r_1,r_2\to\infty$. The signal-noise ratios are given by

$$\mathcal{R}_X=\frac{\langle \delta(X_s)^2\rangle}{\langle \delta(\hat{X}_{\hat{c}_1})^2\rangle}\to 0,\quad \mathcal{R}_Y=\frac{\langle \delta(Y_s)^2\rangle}{\langle \delta(\hat{Y}_{\hat{c}_1})^2\rangle}\to 0. \tag{14}$$

No one other than Bob and Claire can gain any signal information from the modulated EPR beam in the ideal condition because the signal is submerged in large noises. Then Alice sends the beam $\hat{c}_1'$ to Bob. Now Bob demodulates the transmitted signal from the beam $\hat{c}_1'$. He combines her mode $\hat{c}_2$ with $\hat{c}_1'$ on another 50% beam splitter BS2 and before combination a $\pi/2$ phase shift is imposed between them. The two bright output beams are directly detected by D_1 and D_2. Each photocurrent of D_1 and D_2 is divided into two parts through the power splitter. The sum and difference of the divided photocurrents are expressed by [6]

$$\begin{aligned}\hat{i}_+&=\frac{1}{\sqrt{2}}(\hat{X}_{\hat{c}_1'}+\hat{X}_{\hat{c}_2})\\&=\frac{1}{\sqrt{2}}\Bigg[\left(\frac{2}{\sqrt{6}}e^{-r_1}+\frac{1}{\sqrt{12}}e^{r_1}\right)\hat{Y}_{\hat{a}_1}\\&\quad+\left(\frac{2}{\sqrt{6}}e^{-r_1}-\frac{1}{\sqrt{12}}e^{r_1}\right)\hat{Y}_{\hat{a}_2}\\&\quad+\frac{1}{2}(\hat{Y}_{\hat{a}_3}e^{r_2}-\hat{Y}_{\hat{a}_4}e^{r_2})\Bigg]+\frac{1}{\sqrt{2}}X_s,\end{aligned} \tag{15}$$

$$\begin{aligned}\hat{i}_-&=\frac{1}{\sqrt{2}}(\hat{Y}_{\hat{c}_1'}-\hat{Y}_{\hat{c}_2})=\frac{1}{\sqrt{2}}\Bigg(\frac{3}{\sqrt{12}}\hat{X}_{\hat{a}_1}e^{-r_1}-\frac{3}{\sqrt{12}}\hat{X}_{\hat{a}_2}e^{-r_1}\\&\quad+\frac{1}{2}\hat{X}_{\hat{a}_3}e^{-r_2}-\frac{1}{2}\hat{X}_{\hat{a}_4}e^{-r_2}\Bigg)+\frac{1}{\sqrt{2}}Y_s.\end{aligned} \tag{16}$$

Assuming $r_1=r_2=r$, we obtain the power spectra of photocurrents,

$$\langle \delta(\hat{i}_+)^2\rangle=\frac{2}{3}e^{-2r}+\frac{1}{3}e^{2r}+\frac{1}{2}V_{X_s},$$

$$\langle \delta(\hat{i}_-)^2\rangle=e^{-2r}+\frac{1}{2}V_{Y_s}. \tag{17}$$

Thus if $r\to\infty$, Bob only can gain the phase signal with high accuracy, however, he cannot gain the amplitude signal that is submerged in large noise. Bob wants to extract the amplitude signal, so he must need the Claire's result of the amplitude-quadrature detection. Claire detects the amplitude quadrature of her mode $\hat{c}_3$ and sends the result to Bob. Bob displaces the Claire's result on the sum photocurrent

$$\begin{aligned}\hat{i}_+'&=\frac{1}{\sqrt{2}}\Bigg[\left(\frac{2+g\sqrt{2}}{\sqrt{6}}e^{-r_1}+\frac{1-g\sqrt{2}}{\sqrt{12}}e^{r_1}\right)\hat{Y}_{\hat{a}_1}\\&\quad+\left(\frac{2+g\sqrt{2}}{\sqrt{6}}e^{-r_1}-\frac{1-g\sqrt{2}}{\sqrt{12}}e^{r_1}\right)\hat{Y}_{\hat{a}_2}\\&\quad+\frac{1-g\sqrt{2}}{2}(\hat{Y}_{\hat{a}_3}e^{r_2}-\hat{Y}_{\hat{a}_4}e^{r_2})\Bigg]+\frac{1}{\sqrt{2}}X_s,\end{aligned} \tag{18}$$

where g describes gain at Bob for the transformation from Claire's photocurrent to his sum photocurrent. Assuming $r_1=r_2=r$ and $g=1/\sqrt{2}$, we obtain the power spectra of sum photocurrent,

$$\langle \delta(\hat{i}_+')^2\rangle=\frac{3}{2}e^{-2r}+\frac{1}{2}V_{X_s}. \tag{19}$$

Thus Bob also gains amplitude signal with the help of Claire, at this time the coding capacity reaches twice. Therefore, Claire can control the information transmission capacity of dense coding by entangling with the other two parties.

We consider the general condition for finite squeezing. There is an optimum gain for the maximum squeezing of $\hat{i}_+'$, which one can easily find by minimizing $V_{\hat{i}_+'}$

$$g_{opt}=\frac{e^{2r_1}+3e^{2r_2}-4e^{-2r_1}}{\sqrt{2}(e^{2r_1}+3e^{2r_2}+2e^{-2r_1})}. \tag{20}$$

Assuming $r_1=r_2=r$, the optimum gain and the power spectra of photocurrent are given by

$$g_{opt}=\frac{2-2s^2}{\sqrt{2}(2+s^2)},$$

$$\langle \delta(\hat{i}_-)^2\rangle=s+\frac{1}{2}V_{Y_s},$$

$$\langle \delta(\hat{i}_+)^2\rangle=\frac{2s^2+1}{3s}+\frac{1}{2}V_{X_s},$$

$$\langle \delta(\hat{i}_+')^2\rangle_{opt}=\frac{3s}{2+s^2}+\frac{1}{2}V_{X_s}, \tag{21}$$

where $s=e^{-2r}$. Figure 3 shows the noise floor of phase signal, amplitude signal without Claire's help, and amplitude signal with Claire's help for $r_1=r_2=r$. In this case, the noise floor of phase and amplitude signal with Claire's help are below the quantum noise limit (QNL) when $r>0$. The noise floor of amplitude signal without Claire's help is below the QNL with $1>s>0.5$ and above the QNL only with $s<0.5$ (3-dB squeezing in each mode). However, the noise floor of amplitude signal with Claire's help is consistently below that without Claire's help. This shows Claire is entangled with Bob. The GHZ state generated from three beams of equal squeezing $r_1=r_2=r$ is not maximal, as discussed in Ref. [14], because the correlations between the

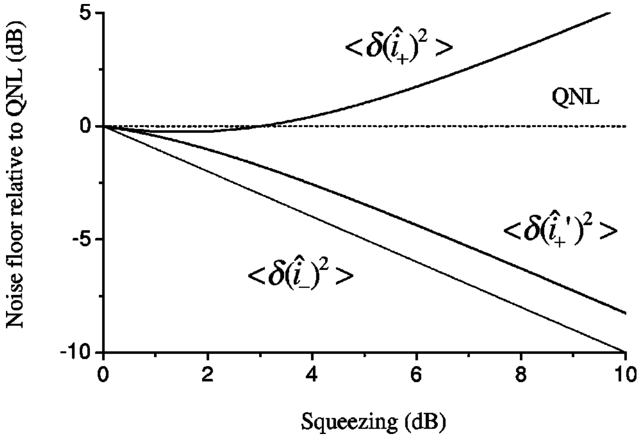

FIG. 3. Noise floor of amplitude and phase signals for $r_1=r_2=r$.

beams are biased in amplitude and phase quadratures. As shown in Eq. (21), the noise floor of amplitude signal is not equal to that of phase signal. One reason is that the nonmaximal GHZ state is used, the other reason is that decoding amplitude signal must have the aid of Claire's classical information and phase signal only needs the joint measurement. For $r_1=r$ and $r_2=0$, the optimum gain and the power spectra of photocurrent are given by

$$g_{opt}=\frac{1+3s-4s^2}{\sqrt{2}(1+3s+2s^2)},$$

$$\langle\delta(\hat{i}_-)^2\rangle=\frac{3s+1}{4}+\frac{1}{2}V_{Y_s},$$

$$\langle\delta(\hat{i}_+)^2\rangle=\frac{8s^2+3s+1}{12s}+\frac{1}{2}V_{X_s},$$

$$\langle\delta(\hat{i}'_+)^2\rangle_{opt}=\frac{3s(1+3s)}{2(1+3s+2s^2)}+\frac{1}{2}V_{X_s}. \tag{22}$$

Figure 4 shows the noise floor of phase signal, amplitude signal without Claire's help, and amplitude signal with Claire's help for $r_2=0$. In this case, the noise floor of phase and amplitude signal with Claire's help can be also below the QNL when $r>0$. The noise floor of phase signal can only reach 0.25 (6-dB squeezing) for $r\to\infty$. The noise floor of amplitude signal without Claire's help is above the QNL only with $s<1/8$ (roughly 9-dB squeezing).

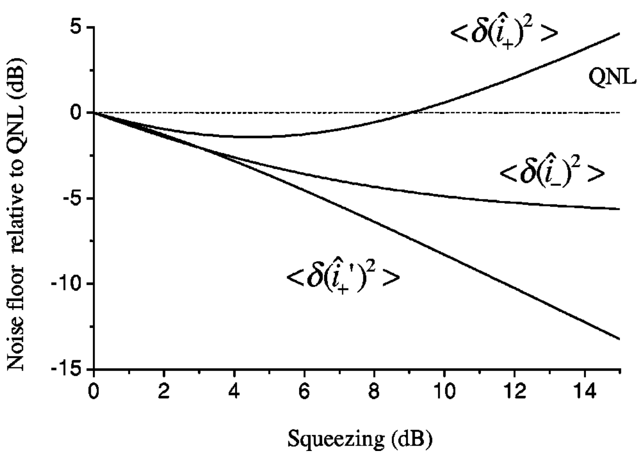

FIG. 4. Noise floor of amplitude and phase signals for $r_2=0$.

The quantum channel capacity for dense coding has recently been obtained in Ref. [5] by sharing a two-particle entangled state. In the following, we briefly give the channel capacity of controlled dense coding for $r_1=r_2=r$. We assume that the original signal is subject to the Gaussian distribution [5]

$$P_{in}(a)=\frac{1}{\pi\sigma^2}\exp\left[-\frac{|a|^2}{\sigma^2}\right], \tag{23}$$

where the parameter σ^2 is the average value of the signal photon number. The information carrying capacity by sharing a three-particle entangled state is given from Eq. (21),

$$I_{n-c}^{dense}=\frac{1}{2}\ln\left(1+\frac{\sigma^2}{s}\right)+\frac{1}{2}\ln\left(1+\frac{3s\sigma^2}{2s^2+1}\right),$$

$$I_{c}^{dense}=\frac{1}{2}\ln\left(1+\frac{\sigma^2}{s}\right)+\frac{1}{2}\ln\left(1+\frac{\sigma^2(s^2+2)}{3s}\right), \tag{24}$$

where I_{n-c}^{dense} and I_c^{dense} are the Shannon mutual information of dense coding without and with Claire's help, respectively. Suppose that the communication system is supplied with the average photon number $\overline{n}$ per mode. The photon numbers supplied to the communication system are used for the signal and squeezing, and thus the following equality should be satisfied:

$$\overline{n}=\sigma^2+\sinh^2 r. \tag{25}$$

For simplification, we only maximize the mutual information of the phase quadrature under the constraint, Eq. (25). When $\overline{n}=e^r\sinh r$ and $\sigma^2=\sinh r\cosh r$, we obtain the approximate optimum channel capacities

$$C_{n-c}^{dense}=\frac{1}{2}\ln(1+\overline{n}+\overline{n}^2)+\frac{1}{2}\ln\left(1+\frac{3(\overline{n}^2+\overline{n})}{2+(2\overline{n}+1)^2}\right),$$

$$C_{c}^{dense}=\frac{1}{2}\ln(1+\overline{n}+\overline{n}^2)+\frac{1}{2}\ln\left(1+\frac{2}{3}(\overline{n}+\overline{n}^2)+\frac{\overline{n}+\overline{n}^2}{3(2\overline{n}+1)^2}\right). \tag{26}$$

where C_{n-c}^{dense} and C_c^{dense} are optimum channel capacities of dense coding without and with Claire's help, respectively. The channel capacity for dense coding has recently been obtained by sharing a two-particle entangled state [5],

$$C_{EPR}^{dense}=\ln(1+\overline{n}+\overline{n}^2). \tag{27}$$

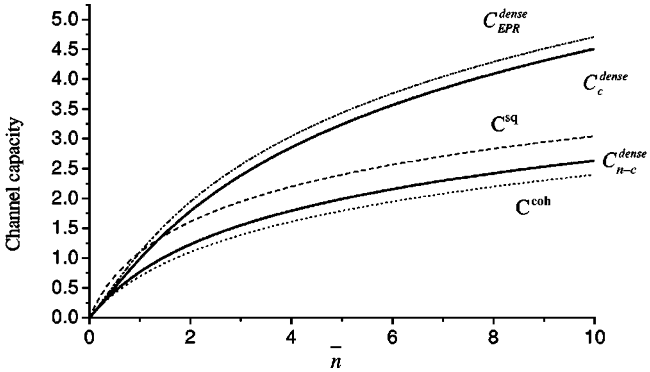

FIG. 5. Comparison of the channel capacity.

A fairer comparison is against single-mode coherent-state communication with heterodyne detection. Here the channel capacity is well known [15] for the mean photon number constraint to be

$$C^{coh} = \ln(1+\overline{n}), \tag{28}$$

which is always beaten by the optimal controlled dense coding scheme described by Eq. (26). An improvement on coherent-state communication is squeezed state communication with a single mode. The channel capacity of this channel has been calculated [15] to be

$$C^{sq} = \ln(1+2\overline{n}). \tag{29}$$

The channel capacity for the different quantum channels are shown in Fig. 5 as the functions of the supplied average photon number. The transmitted information with Claire's help is twice of that without Claire's help for the large squeezing r.

In conclusion, we propose an experimental scheme of the quantum controlled dense coding with bright tripartite entangled state light. The bright tripartite entangled state light that is a ''three-mode position eigenstate'' with total position $\hat{X}_1+\hat{X}_2+\hat{X}_3 \to 0$ and relative momenta $\hat{Y}_i-\hat{Y}_j \to 0$ $(i,j=1,2,3)$ generates from two NOPAs operating in the state of deamplification. Due to exploiting the bright entangled beams generated from NOPA and the directly measuring technique of the ''Bell state,'' the trouble to meet high efficiency of mode matching in experiment is eliminated. The mature technique of producing entangled beams from NOPA and the simplicity of direct measurement make this scheme valuable for performing experiments.

This research was supported by the National Fundamental Research Program (Grant No. 2001CB309304), the National Natural Science Foundation of China (Grant Nos. 60178012 and 69837010), and the Shanxi Province Young Science Foundation (Grant No. 20021014).

[1] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[2] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992); K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, *ibid.* **76**, 4656 (1996).
[3] A. Furusawa, J. L. Sorensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282**, 706 (1998).
[4] M. Ban, J. Opt. B: Quantum Semiclassical Opt. **1**, L9-L11 (1999).
[5] S. L. Braunstein and H. J. Kimble, Phys. Rev. A **61**, 042302 (2000).
[6] J. Zhang and K. C. Peng, Phys. Rev. A **62**, 064302 (2000).
[7] Y. Zhang, H. Wang, X. Y. Li, J. T. Jing, C. D. Xie, and K. C. Peng, Phys. Rev. A **62**, 023813 (2000).
[8] X. Y. Li, Q. Pan, J. T. Jing, J. Zhang, C. D. Xie, and K. C. Peng, Phys. Rev. Lett. **88**, 047904 (2002).
[9] P. V. Loock and S. L. Braunstein, e-print quant-ph/9906021; Phys. Rev. Lett. **84**, 3482 (2000).
[10] J. C. Hao, C. F. Li, and G. C. Guo, Phys. Rev. A **63**, 054301 (2001).
[11] G. M. D'Ariano, M. Vasilyev, and P. Kumar, Phys. Rev. A **58**, 636 (1998).
[12] J. Zhang, C. D. Xie, and K. C. Peng, Phys. Lett. A **287**, 7 (2001).
[13] K. Schneider, R. Bruckmeier, H. Hansen, S. Schiller, and J. Mlynek, Opt. Lett. **21**, 1396 (1996).
[14] W. P. Bowen, P. K. Lam, and T. C. Ralph, e-print quant-ph/0104108.
[15] Y. Yamamoto and H. A. Haus, Rev. Mod. Phys. **58**, 1001 (1986).