Nonlinear Optics, Quantum Optics, Vol. 30, pp. 89-102
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Tripartite Entanglement Swapping of Bright Light Beams

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We propose a new and experimentally realizable continuous variable quantum communication protocol named tripartite entanglement swapping. Three optical modes, which are not in a tripartite entangled state initially, can be entangled by means of the direct detections of Bell-states, the amplitude and phase modulators, and the beamsplitters. The initial bipartite and tripartite entangled states are produced with the bright single mode quadrature squeezed state of light and linear optics. We analystically proved that tripartite entanglement swapping can be accomplished with two schemes in which a tripartite entangled state or two bipartite entangled states serve as quantum channel respectively. The direct detection of Bell-state is applied in the proposed schemes, thus the needs for local beams used in homodyne detectors and the trouble for mode matching are significantly avoided.

Keywords: EPR, tripartite entanglement, entanglement swapping, Bell state measurement, quantum teleportation, quantum information.

INTRODUCTION

Quantum entanglement is a fundamental resource for the quantum information processing. In 1935, Einstein, Podolsky and Rosen [1] proposed a gedanken experiment to demonstrate the incompleteness of quantum mechanics description for physical reality, which was named EPR paradox. It means that the nonlocal correlation would exist between the subsystems of a composite system. In recent years, the entanglement has been exten-

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sively utilized in quantum information processing systems [2-11].

Based on bipartite entanglement, quantum teleportation and quantum dense coding for discrete and continuous variables have been demonstrated experimentally [4-11]. Tripartite discrete-variable entanglement in which the entanglement is shared by three particles, called Greenberger-Horne-Zeilinger(GHZ) states, has been experimentally accomplished for single photon polarization state[12] and using nuclear magnetic resonance[13]. The tripartite entanglement for continuous variables has been experimentally demonstrated and successfully applied to the controlled quantum dense coding by distributing a bipartite entangled beam among three parties using linear optics, very recently [14]. The multipartite entanglement protocol for continuous variables using beamsplitters and single-mode squeezed states has been theoretically discussed [15]. In the references [16]-[18] the bipartite entanglement swapping for continuous variables, the teleportation of entanglement, have been proposed. It has been pointed out that two independent quantum systems can be entangled without directly interacting with each other only by means of entanglement swapping [19]. The experiment of entanglement swapping for discrete variables has already been demonstrated using single photons [20]. The practical applications of entanglement swapping have also been suggested [21-24].

For constructing a quantum information network using continuous light field, it is interesting to investigate tripartite entanglement swapping of continuous variables, i.e. exploiting entanglement swapping to entangle three electromagnetic field states which never share entanglement originally and accomplish unconditional teleportation of entanglement among three space-seperate stations. In Ref. [21], S. Bose et.al. gave a multipartite generalization of entanglement swapping and their schemes can be regarded as a method of generating entangled states of many particles. Based on references [14]-[16], in this paper, we propose a protocol of experimentally realizable tripartite entanglement swapping using squeezed state entanglement and linear optics. It has been theoretically proved by Loock and Braunstein in Ref.[15], that a nonclassical teleportation fidelity can serve as sufficient criterion for the presence of N-partite entanglement and the criterion of the optimum fidelity larger than 0.5 for quantum teleportation of a coherent-state input can be expanded to the case for N-partite entanglement. In the proposed scheme, one bright quardrature amplitude squeezed state and two bright quardrature phase squeezed states are used to generate tripartite entanglement state. The Bell-state direct detection [11] is utilized in correlation measurements between amplitude-phase quadratures of entangled electromagnetic field modes instead of the usual balanced-homodyne detection, so the trouble from mode-matching in experiments can be significantly avoided. This paper is organized as follows: firstly, discussing the generation of bright tripartite entanglement state, successively, describing two protocols demonstrating tripartite entanglement swapping and confirming the existence of entanglement with fidelity criterion expanded in Ref.[15], then comparing the fidelities before and after tripartite entanglement swapping and at last concluding briefly.

GENERATION OF BRIGHT TRIPARTITE ENTANGLEMENT STATE

In Reference [15] Loock and Braunstein proposed a scheme to produce the tripartite entanglement state using a phase-squeezed (momentum-squeezed) and two amplitude-squeezed (position-squeezed) vacuum modes. For designing an easily realized generation system of tripartite entanglement state of continuous optical mode, we follow the configuration of Reference [15] but use a bright amplitude-squeezed and two bright phase-squeezed states instead of the squeezed vacuum modes. The annihilation operators $a_{1(\mathrm{r})}$, $a_{2(\mathrm{r})}$ and $a_{3(\mathrm{r})}$ of the three single mode squeezed states are expressed:

$$\begin{aligned} a_{1(\mathrm{r})} &= e^{-r} X_{1(0)} + ie^{r} Y_{1(0)} \\ a_{2(\mathrm{r})} &= e^{r} X_{2(0)} + ie^{-r} Y_{2(0)} \\ a_{3(\mathrm{r})} &= e^{r} X_{3(0)} + ie^{-r} Y_{3(0)} \end{aligned} \tag{1}$$

$X_{i(0)}$ and $Y_{i(0)}$ ($i=1, 2, 3$) are the amplitude quadratures and phase quadratures of coherent state light with a nonzero average intensity. We suppose, for simplification, the three bright squeezed states of light have same average amplitude, $\langle a_{1(\mathrm{r})}\rangle = \langle a_{2(\mathrm{r})}\rangle = \langle a_{3(\mathrm{r})}\rangle = \langle a_{(\mathrm{r})}\rangle > 0$, and same squeezing parameters r. Figure 1 is the schematic for generation of the tripartite entanglement state. $a_{1(\mathrm{r})}$, $a_{2(\mathrm{r})}$ and $a_{3(\mathrm{r})}$ are the three input squeezed states of light on the beamsplitter BS1 and BS2, a_1, a_2 and a_3 are the output beams. If BS1 and BS2 are characterized with the matrices:

$$\begin{pmatrix} \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \\ \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{3}} \end{pmatrix} \text{ and } \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \end{pmatrix} \tag{2}$$

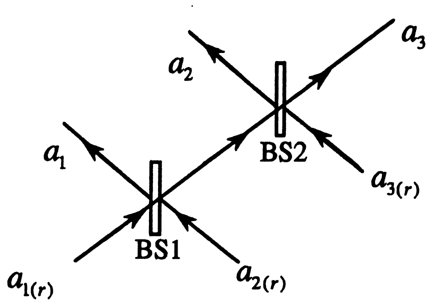

FIGURE 1
The schematic for the generation of bright tripartite entanglement state. $a_{1(r)}$ is a bright single-mode quadrature amplitude squeezed state and $a_{2(r)}$, $a_{3(r)}$ are two bright single-mode quadrature phase squeezed states. BS1 and BS2 are two beamsplitters. (a_1, a_2, a_3) is the produced bright tripartite entanglement state.

We can easily get:

$$\begin{aligned} a_1 &= \sqrt{\tfrac{1}{3}}a_{1(r)} + \sqrt{\tfrac{2}{3}}a_{2(r)} \\ a_2 &= \sqrt{\tfrac{1}{3}}a_{1(r)} - \sqrt{\tfrac{1}{6}}a_{2(r)} + \sqrt{\tfrac{1}{2}}a_{3(r)} \\ a_3 &= \sqrt{\tfrac{1}{3}}a_{1(r)} - \sqrt{\tfrac{1}{6}}a_{2(r)} - \sqrt{\tfrac{1}{2}}a_{3(r)} \end{aligned} \tag{3}$$

From Eqs.(1) and (3), we obtain:

$$\begin{aligned} &a_1 + a_2 + a_3 = \sqrt{3}a_{1(r)}, a_1 - a_2 = \sqrt{\tfrac{3}{2}}a_{2(r)} - \sqrt{\tfrac{1}{2}}a_{3(r)} \\ &a_1 - a_3 = \sqrt{\tfrac{3}{2}}a_{2(r)} + \sqrt{\tfrac{1}{2}}a_{3(r)}, a_2 - a_3 = \sqrt{2}a_{3(r)} \end{aligned} \tag{4}$$

and

$$\begin{aligned} &X_{a1} + X_{a2} + X_{a3} = \sqrt{3}e^{-r}X_{a1}^{(0)}, Y_{a1} - Y_{a2} = \sqrt{\tfrac{3}{2}}e^{-r}Y_{a2}^{(0)} - \sqrt{\tfrac{1}{2}}e^{-r}Y_{a3}^{(0)} \\ &Y_{a1} - Y_{a3} = \sqrt{\tfrac{3}{2}}e^{-r}Y_{a2}^{(0)} + \sqrt{\tfrac{1}{2}}e^{-r}Y_{a3}^{(0)}, Y_{a2} - Y_{a3} = \sqrt{2}e^{-r}Y_{a3}^{(0)} \end{aligned} \tag{5}$$

when $r \to \infty$, the variances $\langle \delta^2(X_{a1} + X_{a2} + X_{a3})\rangle$, $\langle \delta^2(Y_{a1} - Y_{a2})\rangle$, $\langle \delta^2(Y_{a1} - Y_{a3})\rangle$, and $\langle \delta^2(Y_{a2} - Y_{a3})\rangle$ tend to zero. It means that a_1, a_2 and a_3 become a perfect tripartite entanglement state. For the finite squeezing case, Loock and Braunstein have strictly proved that using linear optics suffices to produce a truly N-partite entangled state with any nonzero squeezing [15]. The

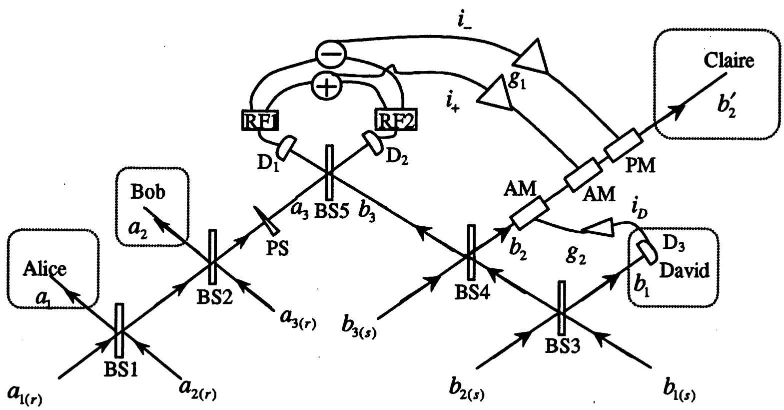

FIGURE 2
The schematic of tripartite entanglement swapping using a tripartite entangled state as the quantum channel. (a_1, a_2, a_3) and (b_1, b_2, b_3) are two independent tripartite entanglement states. PS is a phase shifter. BS is a 50:50 beamsplitter. D_1, D_2, D_3 are three photodiode detectors. AM is amplitude modulator, PM is phase modulator. RF1, RF2 are two radio frequency power splitters.

demonstration and conclusion can be applied to the present protocol almost in totally same schemes, so the repetitive calculations are omitted.

TRIPARTITE ENTANGLEMENT SWAPPING OF BRIGHT LIGHT BEAMS

We propose two protocols of tripartite entanglement swapping according to the different choice for quantum channel. In one of the protocols, a tripartite entanglement state serves as the quantum channel, and in other one two bipartite entangled state (EPR) states are applied. The exploited entanglement resources in the two schemes are different, that is, a tripartite entangled state or two bipartite entangled states are used respectively. For implementing experiments, we can choose either one according to possessed entanglement resources.

Tripartite entanglement swapping using a tripartite entanglement state as quantum channel

Supposing (a_1, a_2, a_3) and (b_1, b_2, b_3) are two independent tripartite entanglement states with same average amplitude, $\langle a_i \rangle = \langle b_i \rangle = \langle a \rangle (i = 1,2,3)$, which are characterized by Eq.(3):

$$\begin{aligned} a_1 &= \sqrt{\tfrac{1}{3}}a_{1(r)} + \sqrt{\tfrac{2}{3}}a_{2(r)} & b_1 &= \sqrt{\tfrac{1}{3}}b_{1(s)} + \sqrt{\tfrac{2}{3}}b_{2(s)} \\ a_2 &= \sqrt{\tfrac{1}{3}}a_{1(r)} - \sqrt{\tfrac{1}{6}}a_{2(r)} + \sqrt{\tfrac{1}{2}}a_{3(r)}, & b_2 &= \sqrt{\tfrac{1}{3}}b_{1(s)} - \sqrt{\tfrac{1}{6}}b_{2(s)} + \sqrt{\tfrac{1}{2}}b_{3(s)} \\ a_3 &= \sqrt{\tfrac{1}{3}}a_{1(r)} - \sqrt{\tfrac{1}{6}}a_{2(r)} - \sqrt{\tfrac{1}{2}}a_{3(r)} & b_3 &= \sqrt{\tfrac{1}{3}}b_{1(s)} - \sqrt{\tfrac{1}{6}}b_{2(s)} - \sqrt{\tfrac{1}{2}}b_{3(s)} \end{aligned} \tag{6}$$

There, $a_{i(r)}$, $b_{j(s)}$ ($i,j=1,2,3$) are the six single mode squeezed states which are used for generating two tripartite entanglement states (a_1, a_2, a_3) and (b_1, b_2, b_3) according to the description in the Eq.1 and their squeezing parameters are r and s respectively. Our aim is to entangle (a_1, a_2, b_1) or (a_1, a_2, b_2), which are not tripartite-entangled originally, by means of tripartite entanglement swapping described in figure 2. Firstly, we implement directly a joint Bell-state measurement of a_3 and b_3 with the detection system consisting of the photodetector D_1, D_2 and radio frequency beamsplitters RF1, RF2. A phase difference of $\pi/2$ is added between a_3 and b_3 by a phase shifter (PS) before they are mixed on the beamsplitter (BS5). According to the same calculation about the direct detection of Bell-State described in reference [25], we obtain the sum (i_+) and difference (i_-) photocurrents of a_3 and b_3:

$$\begin{aligned} i_+ &= \tfrac{\langle a\rangle}{\sqrt{2}}(X_{a3} + X_{b3}) \\ i_- &= \tfrac{\langle a\rangle}{\sqrt{2}}(Y_{a3} - Y_{b3}) \end{aligned} \tag{7}$$

Then we modulate the output beam b_2 from BS4 using the measured classical photocurrent signals i_+ and i_- by means of the amplitude modulator (AM) and phase modulator (PM) with gain g_1. Beams a_1, a_2, b_1, b_2 are distributed to Alice, Bob, David and Claire respectively. For accomplishing the entanglement of any three among the four members, for example, to entangle Alice, Bob and Claire, the rest one, David, has to cooperate. David detects the amplitude of beam b_1, and then modulate the beam b_2 using the measured photocurrent $i_D \langle a\rangle X_{b1}$ with gain g_2. The modulated optical beam becomes b'_2

$$\begin{aligned} b'_2 &= b_2 + g_1\tfrac{\langle a\rangle}{\sqrt{2}}(X_{a3} + X_{b3}) + ig_1\tfrac{\langle a\rangle}{\sqrt{2}}(Y_{a3} - Y_{b3}) + g_2\langle a\rangle X_{b1} \\ &= g_1\tfrac{\langle a\rangle}{\sqrt{2}}(X_{a3} + iY_{a3}) + \langle a\rangle(g_2X_{b1} + X_{b2} + \tfrac{g_1}{\sqrt{2}}X_{a3}) + i\langle a\rangle(Y_{b2} - \tfrac{g_1}{\sqrt{2}}Y_{b3}) \end{aligned} \tag{8}$$

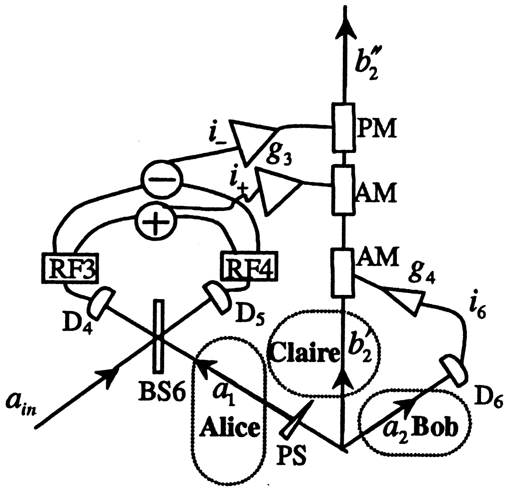

FIGURE 3
The schematic for proving the existence of entanglement among Alice, Bob and Claire after entanglement swapping using a tripartite entanglement state as quantum channel. PS is a phase shifter. BS is a 50:50 beamsplitter. D_4, D_5, D_6 are three photodiode detectors. AM is amplitude modulator, PM is phase modulator. RF3, RF4 are two radio frequency power splitters.

when $g_1 = \sqrt{2}$, $g_2 = 1$,

$$b_2' = a_3 + \langle a\rangle\sqrt{3}e^{-s}X_{b1}^{(0)} + i\langle a\rangle\sqrt{2}e^{-s}Y_{b3}^{(0)} \tag{9}$$

For infinite squeezing ($s \to \infty$), we can get:

$$b'_2 = a_3 \tag{10}$$

It means that, (a_1, a_2, b'_2) is a tripartite entanglement state just as the original (a_1, a_2, a_3). But the infinite squeezing is not physical, so we must consider the case of finite squeezing. For demonstrating that Alice, Bob and Claire still share entanglement, we use the fidelity criterion of quantum teleportation for an unknown coherent state [15]. As shown in figure 3, a coherent state a_{in} is sent to Claire from Alice under the help of Bob. A phase difference of $\pi/2$ is added between a_{in} and a_1 by PS placed in a_1 according to the requirement of direct measurement of Bell-state [25]. Firstly, we implement a joint Bell-state measurement of a_{in} and a_1 with the detection system

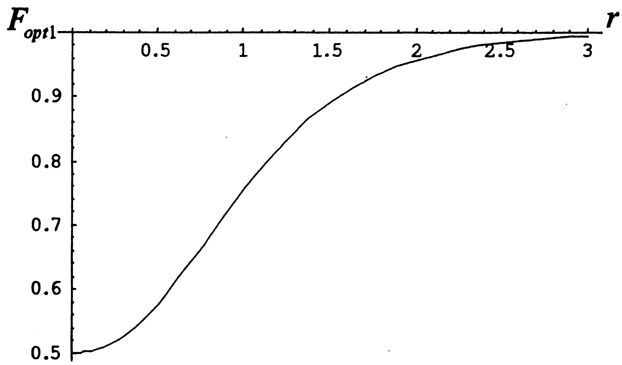

FIGURE 4
The optimum fidelity F_{opt1} vs squeezing parameter r for the scheme using a tripartite entangled state as the quantum channel.

consisting of the photodetector D_5, D_4 and radio frequency beamsplitters RF3, RF4. After the beam b'_2 is amplitude-modulated by the photocurrents i_+ from D_4 and D_5 as well i_6 from D_6 and also is phase-modulated by the photocurrent i_- from D_4 and D_5, the resultant beam b''_2 becomes:

$$b''_2 = a_{in} + \langle a \rangle \left(\tfrac{g_3}{\sqrt{2}} X_{a1} + g_4 X_{a2} + \tfrac{g_1}{\sqrt{2}} X_{a3} + g_2 X_{b1} + X_{b2} + \tfrac{g_1}{\sqrt{2}} X_{b3}\right) + i\langle a \rangle \left(\tfrac{g_1}{\sqrt{2}} Y_{a3} - \tfrac{g_3}{\sqrt{2}} Y_{a1} + Y_{b2} - \tfrac{g_1}{\sqrt{2}} Y_{b3}\right) \tag{11}$$

There g_3 and g_4 are the gains of (i_+, i_-) and i_6. If the fidelity of quantum teleportation is larger than the fidelity without entanglement which equals 0.5 [7], we say that Alice, Bob and Claire share entanglement. There are four gain parameters g_i ($i=1,2,3,4$) in above equations, for simplification of calculation, we take $g_3 = \sqrt{2}$ and $r = s$, then the fidelity F of the input state a_{in} is related to the Q function of the teleported mode [$F = \pi Q_{tel}(\alpha_{in})$][15]

$$F = \tfrac{1}{6}\{[6 + 3\cosh(2r)(2 + g_1^2) - \sinh(2r)(2 + 4\sqrt{2}g_1 + g_1^2)] [6 + 3\cosh(2r)(2 + g_1^2 + g_2^2 + g_4^2) - \sinh(2r)(4\sqrt{2}g_1 + 4g_2 + 4g_4 - g_1^2 - g_2^2 - g_4^2 + 2\sqrt{2}g_1 g_2 + 2\sqrt{2}g_1 g_4 - 2)]\}^{-\frac{1}{2}} \tag{12}$$

Differentiating F with respect to g_1, g_2 and g_4 and solving simultaneously the resultant equations, the optimum fidelity F_{opt1}

$$F_{opt1} = \left[\frac{e^{-2r}(1+e^{2r})^2(2+e^{2r})(1+e^{2r}+e^{4r})}{(2+e^{4r})^2}\right]^{-\frac{1}{2}} \tag{13}$$

is obtained at $g_1 = \frac{\sqrt{2}(e^{4r}-1)}{2+e^{4r}}$, $g_2 = g_4 = \frac{(e^{4r}-1)}{2+e^{4r}}$. Figure 4 shows the function of the F_{opt1} versus the squeezing parameter r. It is obvious, for any nonzero squeezing ($r>0$) the optimum fidelity F_{opt1} is larger than the classical limit without squeezed-state entanglement ($r=0$, $F_{opt1}(0)=0.5$). In other words, Alice, Bob and Claire share entanglement for any squeezing parameters of $r>0$, and when $r \to \infty$, the optimum fidelity $F_{opt1}(0)\to 1$ which means perfect entanglement.

Tripartite entanglement swapping with two EPR states as quantum channel

Supposing we have a tripartite entanglement state (a_1, a_2, a_3) described in Eq.(3) and two EPR states (b_1, b_2), (c_1, c_2) each one of which is produced by combining two single mode squeezed states ($b_{1(s)}$ and $b_{2(s)}$) or ($c_{1(t)}$ and $c_{2(t)}$) on a beamsplitter [7].

$$\begin{aligned} b_1 &= \tfrac{1}{\sqrt{2}} b_{1(s)} + \tfrac{1}{\sqrt{2}} b_{2(s)} \quad & c_1 &= \tfrac{1}{\sqrt{2}} c_{1(t)} + \tfrac{1}{\sqrt{2}} c_{2(t)} \\ b_2 &= \tfrac{1}{\sqrt{2}} b_{1(s)} - \tfrac{1}{\sqrt{2}} b_{2(s)} \quad & c_2 &= \tfrac{1}{\sqrt{2}} c_{1(t)} - \tfrac{1}{\sqrt{2}} c_{2(t)} \end{aligned} \tag{14}$$

For enabling to implement the direct detection of Bell-state [25] we require that $b_{1(s)}$, $c_{1(t)}$ are two bright quadrature amplitude squeezing states, and $b_{2(s)}$, $c_{2(t)}$, are two bright quadrature phase squeezing states (s, t are the squeezing parameters). As an example we accomplish the tripartite entanglement of three independent optical mode (a_2, b_2, c_2) which are not entangled initially. The schematic is shown in figure 5. Firstly, we implement joint Bell-state measurements of (a_1, c_1) and (a_3, b_1) with two direct detection systems consisting of the photodetectors D1, D2 and D3, D4 as well as radio frequency beamsplitters RF1, RF2, RF3, RF4. The phase shifter PS1(PS2) adds a phase difference of $\pi/2$ between a_3 and b_1 (a_1 and c_1) before mixing on the beamsplitter BS1(BS2) [25]. Then, the beam b_2 (c_2) is modulated with the sum and difference photocurrents of D3 and D4 (D1 and D2) by means of the amplitude and phase modulators, AM and PM with the gains g_1 and g_2 respectively. The modulated beams b'_2 and c'_2 are kept by Alice

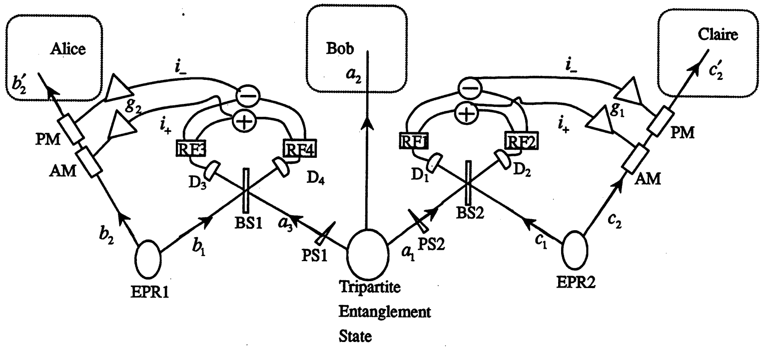

FIGURE 5
The schematic of tripartite entanglement swapping with two EPR states as quantum channel. (b_1, b_2), (c_1, c_2) are two bright EPR states, (a_1, a_2, a_3) is a bright tripartite entanglement state. The detection system consists of the photodetectors D_1, D_2, D_3, D_4 and radio frequency beam-splitters RF1, RF2, RF3, RF4. Two phase shifters PS1(PS2) are used for adding a $\pi/2$ phase difference between a_1 and c_1 (a_3 and b_1).

and Claire. Bob holds the beam a_2. When the squeezing exists we can prove that a_2, b'_2 and c'_2 are a triparticle entanglement state. It is obvious, b'_2 and c'_2 equal to:

$$
\begin{aligned}
b_2' &= b_2 + \tfrac{g_1}{\sqrt{2}}\langle a\rangle(X_{a3} + X_{b1}) + i\tfrac{g_1}{\sqrt{2}}\langle a\rangle(Y_{a3} - Y_{b1}) \\
c_2' &= c_2 + \tfrac{g_1}{\sqrt{2}}\langle a\rangle(X_{a1} + X_{c1}) + i\tfrac{g_2}{\sqrt{2}}\langle a\rangle(Y_{a1} - Y_{c1})
\end{aligned}
\tag{15}
$$

There we have supposed $\langle a_i\rangle = \langle b_j\rangle = \langle c_k\rangle = \langle a\rangle$ ($i=1,2,3; j,k=1,2$) for simplification. Using the same schematic and analysis as figure 3 we get:

$$
\begin{aligned}
b_2'' = {} & a_{in} + \langle a\rangle\left(\tfrac{g_2 g_4}{\sqrt{2}} X_{a1} + X_{a2} + \tfrac{g_1}{\sqrt{2}} X_{a3} + \tfrac{g_1}{\sqrt{2}} X_{b1} + X_{b2} + \tfrac{g_2 g_4}{\sqrt{2}} X_{c1} + g_4 X_{c2}\right) \\
& + i\langle a\rangle\left(\tfrac{g_1}{\sqrt{2}} Y_{a3} - Y_{a1} + Y_{b2} - \tfrac{g_1}{\sqrt{2}} Y_{b1}\right)
\end{aligned}
\tag{16}
$$

In Eq.(16) there are four gain parameters ($i=1,2,3,4$), taking $g_3 = \sqrt{2}$, $r = s = t$, we get the fidelity of teleporting an unknown coherent state:

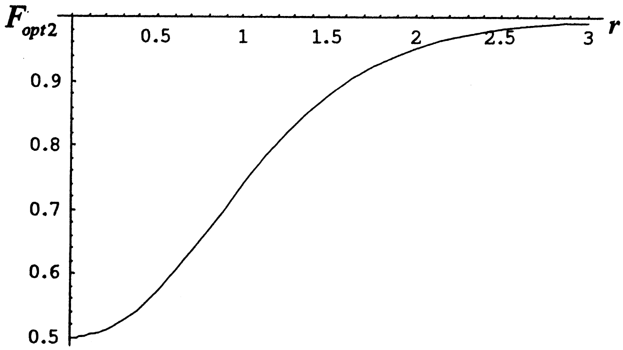

FIGURE 6
The optimum fidelity F_{opt2} vs squeezing parameter r for the scheme using two bipartite entangled states as quantum channel.

$$F = \frac{1}{12}\left\{\left[12 + 6\cosh(2r)(2+g_1^2) - \sinh(2r)(2+10\sqrt{2}g_1 + g_1^2)\right]\left[\begin{array}{l} 12 - \sinh(2r)(-2+10\sqrt{2}g_1 - g_1^2 + 4\sqrt{2}g_2g_4 + 4g_1g_2g_4 \\ +6\sqrt{2}g_2g_4^2 - g_2^2g_4^2) + 6\cosh(2r)(2+g_1^2+g_4^2+g_2^2g_4^2) \end{array}\right]\right\}^{-\frac{1}{2}} \tag{17}$$

When $g_1 = \frac{\sqrt{2}(5e^{6r}-5e^{2r})}{7e^{2r}+5e^{6r}}$, $g_2 = \frac{\sqrt{2}(1+e^{4r})}{e^{4r}-1}$, $g_4 = \frac{2e^{2r}(e^{4r}-1)(1+5e^{4r})}{(7e^{2r}+5e^{6r})(1+9e^{4r}+2e^{8r})}$, the optimum fidelity F_{opt2} is obtained:

$$F_{opt2} = \left[\begin{array}{l} \frac{e^{-4r}(1+e^{2r})^2(2+5e^{2r}+5e^{4r})}{(7+5e^{4r})^3(1+9e^{4r}+2e^{8r})}(1+48e^{2r}+50e^{4r}+461e^{6r} \\ +379e^{8r}+374e^{10r}+240e^{12r}+125e^{14r}+50e^{16r}) \end{array}\right]^{-\frac{1}{2}} \tag{18}$$

Figure 6 shows the function of F_{opt2} versus the squeezing parameter r. We see, when $r=0$, $F_{opt2}=0.5$ is the classical limit of fidelity [7,15] and when $r>0$, $F_{opt2}>0.5$ is in the region of quantum teleportation, which confirms that Alice, Bob and Claire share a tripartite entanglement.

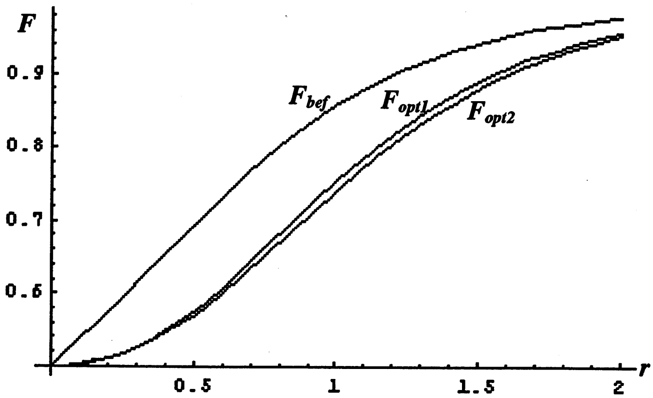

FIGURE 7
The functions of F_{bef}, F_{opt1}, F_{opt2} versus the squeezing parameter r.

COMPARISION OF FIDELITIES BEFORE AND AFTER TRIPARTITE ENTANGLEMENT SWAPPING

The fidelity teleporting a coherent state is similarly used to quantify the tripartite entanglement. For simplification and without loss of generality, we suppose that all squeezing parameters r, s and t are equal. The fidelity of the teleportation of the tripartite entanglement state before the tripartite entanglement swapping is calculated from the Q function [15]:

$$F_{bef} = \left[(1+e^{-2r})(1+\tfrac{3}{2e^{2r}+e^{-2r}})\right]^{-\frac{1}{2}} \tag{19}$$

This result is same with that obtained in reference [15]. The three functions of fidelities, F_{bef} (Eq.19), F_{opt1} (Eq.13) and F_{opt2} (Eq.18) versus squeezing parameter r are drawn in figure 7. The fidelities, F_{opt1} and F_{opt2}, after the tripartite entanglement swapping are less than F_{bef} before swapping for a given and limited squeezing. The higher the squeezing, the smaller the difference between before and after swapping. For perfect squeezing ($r \to \infty$), F_{bef} , F_{opt1} and F_{opt2} are equal. The quantum entanglement is partly lost in the swapping process using imperfect squeezing and thus the fidelities are reduced. However, if the perfect squeezing ($r \to \infty$) were utilized the entanglement and the fidelities would be retained. Here, F_{opt1} for the scheme using tripartite entanglement is slightly higher than F_{opt2} for other scheme using two bipartite entangled states, but the difference is not significant. Thus, we can choose any one of them to implement the tripartite entanglement swapping according to our convenience of entanglement resources.

CONCLUSION

In summary, we have proposed a new quantum communication protocol of continuous variables named tripartite entanglement swapping that enables us to entangle three systems which do not share tripartite entanglement originally and to implement teleportation of entanglement among subsystems more than two with a fidelity higher than that of the classical limit (0.5). The existence of entanglement among Alice, Bob and Claire is clarified with the fidelity criterion of quantum teleportation for a coherent state. Two schemes accomplishing tripartite entanglement swapping are discussed analytically. In the proposed protocols the directly detecting technic of Bell-states [25] is applied, thus the local beam used in the usual balanced homodyne detection is not needed and the experimental system is simplified. The proposed schemes are useful for developing the quantum network and the quantum communication between stations more than two.

Acknowledgments

This research is supported by the National Fundamental Reaserch Program (No.2001CB309304) and the National Nature Science Foundation of China(No.60238010).

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