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Influence of random deviation of domain length in quasi-phase-matched crystals on degenerate optical parametric amplification

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Abstract

During the fabrication of quasi-phase-matched (QPM) devices, errors of periodic structure are usually inevitable. The errors result in the deviation of the actual periodic domain length from the theoretical value. In this paper we numerically analyze the influence of the errors on the degenerate optical parametric amplifier consisting of QPM devices. It is shown that in this case the gains of signal photon number and normalized photon number variance are decreased with respect to those of an ideal QPM device. However, there are no extra noises to be introduced, the output signal-noise-ratio is equal to the input signal-noise-ratio if the small linear absorption in QPM devices is ignored. It has been also proved that the noise figure of the noiseless amplifier does not vary with the pump power and the propagation length of the signal light in the QPM crystal.

Keywords: Quasi-phase-matched; Errors of periodic structure; Degenerate optical parametric amplifier; Noiseless amplification

1. Introduction

In recent years a new type of non-linear crystals called quasi-phase-matched (QPM) crystals has appeared. In a QPM crystal the non-linear susceptibility is periodically inverted to compensate for phase–velocity mismatch between the interaction waves. High parametric conversion efficiency and long utilizable interaction length are the significant advantages of QPM crystals. Due to the periodically inverted construction the largest effective non-linear coefficient of crystals can be exploited and the non-critical phase matching can be achieved within a very large wavelength range by tuning the temperature of QPM [1]. The favorable features of the QPM materials have made it being widely used in non-linear optical frequency conversion [2–4]. In 1995 [5] analyzed the quantum noise reduction and the noiseless optical parametric amplification using QPM crystal for the first time. The calculated results in this paper showed that a QPM

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parametric amplifier could achieve noiseless amplification in one of quadratures of light field and squeezing in other conjugate quadrature with an equivalent gain. Comparing with the traditional birefringent materials, the quantum noise reduction and noiseless amplification can be demonstrated at a relatively low pump power when QPM devices are used. More and more interests in the theoretical and experimental investigation of QPM crystal are growing up [6]. The generations of the squeezed state light and noiseless amplification have been experimentally accomplished with QPM devices [7–13].

In a practical QPM device, there are inevitable deviations between the real domain length and the designed length due to the imperfect fabrication technology. Ref. [14,15] have considered the effect of the deviations on the conversion efficiency of the second harmonic generation (SHG) and [16–18] have analyzed the effect on squeezing of light fields produced from the process of SHG and degenerate optical parametric amplification. In this paper we numerically calculated the influence of the errors in domain length on the properties of degenerate optical parametric amplification.

2. Model of periodic errors

At present, the best way to fabricate a QPM device is the technique of electric-field poling reversal, that is a high electric field is momentarily applied on the grating electrodes of QPM crystal formed by lithography to produce permanent inversion patterns. This technique is very effective, and in fact, the production and quality have been significantly improved. But during the fabrication process the errors of the periodic domain length are inevitable. For QPM materials fabricated by the above mentioned method, the main periodic error is the random duty cycle error as shown in Fig. 1 [14,15]. In a QPM device with this error each domain boundary is randomly shifted from the ideal position while the average domain period length remains constant.

In the calculation we assume this deviation obeys a Gaussian distribution [14]

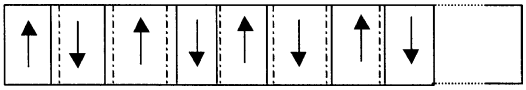

Fig. 1. Model of random duty-cycle error. Up arrows and down arrows indicate non-inverted and inverted polarization, respectively, the dash dot line and the solid line are ideal boundary and actual boundary, respectively.

$$P(\varepsilon_k) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\varepsilon_k^2}{2\sigma^2}\right], \tag{1}$$

where $\varepsilon_k = (z_k - k(\Lambda/2))/(\Lambda/2)$, z_k is the distance from the beginning of the first domain to the end of the k th domain, Λ is ideal period length, σ is standard deviation.

3. Model of calculation

The system that we consider consists of signal field, idler field, strong pump field and a QPM non-linear medium with its bulk effective non-linear coefficient $d^{(2)}$. We assume that the pump field is undepleted and described by a c-number (classical treatment of the pump field) and ignoring optical losses, the equations of coupled waves of the non-linear interaction are written [5]

$$\begin{aligned} \frac{\mathrm{d}a(z)}{\mathrm{d}z} &= g\exp[\mathrm{i}(\Delta kz + \phi_{\mathrm{pump}})]b^{+}(z), \\ \frac{\mathrm{d}b(z)}{\mathrm{d}z} &= g\exp[\mathrm{i}(\Delta kz + \phi_{\mathrm{pump}})]a^{+}(z), \end{aligned} \tag{2}$$

where $a(z)$ and $b(z)$ stand for operators of signal and idler fields, ϕ_{pump} is the phase of pump. $\Delta k = k_{\mathrm{pump}} - k_{\mathrm{idler}} - k_{\mathrm{signal}}$ is the wave vector mismatch,

$$g = m\left[\frac{2\omega_{\mathrm{signal}}\omega_{\mathrm{idler}}|d^{(2)}|^2 I_{\mathrm{pump}}}{n_{\mathrm{signal}}n_{\mathrm{idler}}n_{\mathrm{pump}}\varepsilon_0 c^3}\right]^{1/2}$$

is the non-linear coupling coefficient, the parameter m is 1 for non-inverted domains and -1 for inverted domains. $d^{(2)}$ is the second order effective non-linear coefficient, c is the speed of light in vacuum, I_{pump} is the pump power, ε_0 is the dielectric constant of vacuum, n_{signal}, n_{idler} and n_{pump} are the refractive indices for signal, idler and pump fields, respectively. For a degenerate frequency down-conversion with identical signal and idler fields Eq. (2) are simplified to

$$\frac{\mathrm{d}a(z)}{\mathrm{d}z} = g\exp[\mathrm{i}(\Delta kz + \phi_{\text{pump}})]a^{+}(z). \tag{3}$$

According to the method proposed by [5] we introduce two variables

$$\begin{aligned} p(z) &= \frac{1}{2}\left\{\exp\left[-\mathrm{i}\left(\frac{\Delta k}{2}z + \frac{\phi_{\text{pump}}}{2}\right)\right]a(z) \right. \\ &\quad \left. + \exp\left[\mathrm{i}\left(\frac{\Delta k}{2}z + \frac{\phi_{\text{pump}}}{2}\right)\right]a^{+}(z)\right\}, \\ q(z) &= \frac{-\mathrm{i}}{2}\left\{\exp\left[-\mathrm{i}\left(\frac{\Delta k}{2}z + \frac{\phi_{\text{pump}}}{2}\right)\right]a(z) \right. \\ &\quad \left. - \exp\left[\mathrm{i}\left(\frac{\Delta k}{2}z + \frac{\phi_{\text{pump}}}{2}\right)\right]a^{+}(z)\right\}. \end{aligned} \tag{4}$$

Combining Eqs. (3) and (4), the equations of motion of $p(z)$ and $q(z)$ are obtained

$$\begin{aligned} \frac{\mathrm{d}p(z)}{\mathrm{d}z} &= gp(z) + \frac{\Delta k}{2}q(z), \\ \frac{\mathrm{d}q(z)}{\mathrm{d}z} &= -gq(z) - \frac{\Delta k}{2}p(z). \end{aligned} \tag{5}$$

Taking $K = [(\Delta k/2)^2 - g^2]^{1/2}$ and assuming $(\Delta k/2)^2 > g^2$ (which is usually satisfied in the practical system), we introduce the dimensionless variables

$$\Delta = \Delta k/2K, \quad G = g/K,$$

$$C = \cos(Kz), \quad S = \sin(Kz).$$

Equation (4) can be simplified to

$$\begin{bmatrix} p(z) \\ q(z) \end{bmatrix} = \begin{bmatrix} C + GS & \Delta S \\ -\Delta S & C - GS \end{bmatrix}\begin{bmatrix} p(0) \\ q(0) \end{bmatrix}. \tag{6}$$

For a first order QPM device, we have $\Lambda = \pi/K$. At the end of the first domain, we have

$$C_1 = \cos\left[\frac{\pi}{2}(1 + \varepsilon_1)\right]$$

and

$$S_1 = \sin\left[\frac{\pi}{2}(1 + \varepsilon_1)\right],$$

thus Eq. (6) becomes

$$\begin{bmatrix} p(z_1) \\ q(z_1) \end{bmatrix} = \begin{bmatrix} C_1 + GS_1 & \Delta S_1 \\ -\Delta S_1 & C_1 - GS_1 \end{bmatrix}\begin{bmatrix} p(0) \\ q(0) \end{bmatrix}. \tag{7}$$

At the end of the second domain we have

$$\begin{aligned} \begin{bmatrix} p(z_2) \\ q(z_2) \end{bmatrix} &= \begin{bmatrix} C_2 + GS_2 & \Delta S_2 \\ -\Delta S_2 & C_2 - GS_2 \end{bmatrix} \\ &\quad \times \begin{bmatrix} C_1 + GS_1 & \Delta S_1 \\ -\Delta S_1 & C_1 - GS_1 \end{bmatrix}\begin{bmatrix} p(0) \\ q(0) \end{bmatrix}. \end{aligned} \tag{8}$$

After propagating through n periods, we obtain

$$\begin{bmatrix} p(z_{2n}) \\ q(z_{2n}) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} p(0) \\ q(0) \end{bmatrix}, \tag{9}$$

where matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

is the product of $2n$ matrices

$$\begin{bmatrix} C_i + GS_i & \Delta S_i \\ -\Delta S_i & C_i - GS_i \end{bmatrix}$$

describing the propagation of the signal wave.

Combining Eqs. (4) and (9) we have

$$\begin{aligned} a(z) = \frac{\exp[\mathrm{i}(\Delta kz/2 + \phi_p/2)]}{2}&\big[a(0)\exp(-\mathrm{i}\phi_p/2)\alpha \\ &+ \mathrm{i}a^{+}(0)\exp(\mathrm{i}\phi_p/2)\beta\big], \end{aligned} \tag{10}$$

where $\alpha = (A + D) + \mathrm{i}(C - B)$, $\beta = (B + C) + \mathrm{i}(D - A)$.

We assume that the original signal is in a coherent state $|\alpha_s\rangle$, after some calculations the mean number of photon and the photon number variance at the output of the amplifier are given by

$$\langle\hat{n}\rangle = |\alpha_s|^2\left(|\alpha|^2 + |\beta|^2 + \alpha^{*}\beta\mathrm{e}^{\mathrm{i}(\Phi+\pi/2)} + \alpha\beta^{*}\mathrm{e}^{-\mathrm{i}(\Phi+\pi/2)}\right), \tag{11}$$

$$\begin{aligned} \langle(\Delta\hat{n})^2\rangle = |\alpha_s|^2\Big\{&(|\alpha|^2 + |\beta|^2)^2 + 4|\alpha|^2|\beta|^2 \\ &+ 2\left(|\alpha|^2 + |\beta|^2\right)\left(\alpha^{*}\beta\mathrm{e}^{\mathrm{i}(\Phi+\pi/2)}\right. \\ &\left.+ \alpha\beta^{*}\mathrm{e}^{-\mathrm{i}(\Phi+\pi/2)}\right)\Big\}, \end{aligned} \tag{12}$$

where $\Phi = \phi_{\text{pump}} - 2\phi_{\text{signal}}$ is the relative phase between the pump and the signal fields, ϕ_{signal} is the phase of signal field.

It is useful to introduce the noise figure *NF* defined as the ratio between the input and output signal-to-noise ratio [19]

$$NF = \frac{S_{\rm in}/N_{\rm in}}{S_{\rm out}/N_{\rm out}} = \Big(\big(|\alpha|^2+|\beta|^2\big)^2 + 4|\alpha|^2|\beta|^2 + 2\big(|\alpha|^2+|\beta|^2\big)\big(\alpha^*\beta e^{i(\Phi+\pi/2)} + \alpha\beta^* e^{-i(\Phi+\pi/2)}\big)\Big) \Big/ \Big(|\alpha|^2+|\beta|^2+\alpha^*\beta e^{i(\Phi+\pi/2)} + \alpha\beta^* e^{-i(\Phi+\pi/2)}\Big)^2, \quad (13)$$

where $S_{\rm out}$ and $S_{\rm in}$ are the output and input signal powers, $N_{\rm out}$ and $N_{\rm in}$ are the output and input noise powers. In the case of $NF = 1$, the amplifier is noiseless, i.e., the input signal-to-noise ratio is preserved and no extra noise is introduced during the amplification.

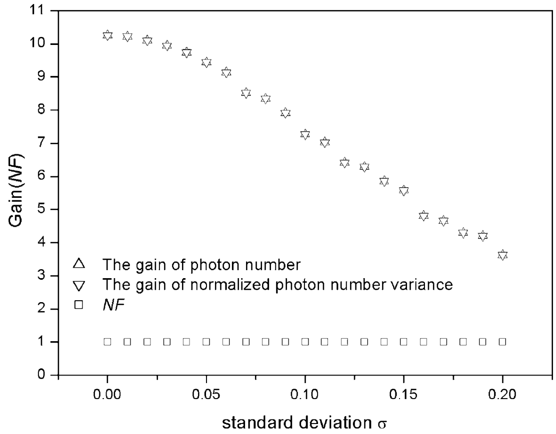

Fig. 2. The gains of photon number and normalized photon number variance, noise figure NF as the functions of standard deviation, $I_{\rm pump} = 2.55$ MW/cm^2, $L_{\rm ppln} = 2$ cm.

4. Numerical analysis

As an example, we consider the first-order QPM lithium niobate, the d_{33} of which is used for performing the parametric down conversion from 1064 to 2128 nm. The corresponding parameters used in the calculation are as follows: $\lambda_{\rm pump} = 1.064$ μm, $\lambda_{\rm signal} = \lambda_{\rm idler} = 2.128$ μm, the non-linear coefficient $d^{(2)} = 22$ pm/V, and the period length $\Lambda = 31$ μm.

The maximum gains of mean photon number and normalized photon variance, and the noise figure NF as the functions of standard deviation σ are plotted in Fig. 2. As σ increases, the gains of mean photon number and normalized photon number variance decrease. The two curves of gains overlap at any point of σ, that means they are always equal. However, NF remains the constant of 1, i.e., noiseless amplification. Although the period errors degrade the gain of the amplifier, the noise figure is not changed, so the amplifier is still noiseless. In Fig. 3 we show the gains of mean photon number and normalized photon number variance, and the noise figure NF as the functions of the relative phase between the pump and the signal light with $\sigma = 0$. We can see the gains of photon number and normalized photon number variance are the same for the two cases of amplification (highest point) and deamplification (lowest point). The curve of NF goes to 1 at the maximum and minimum gain points, it shows that only when the amplifier operates at the maximum amplification and deamplificaton the amplifier is exactly noiseless. Fig. 4 is similar to Fig. 3 but with $\sigma = 0.1$. The gains of mean photon number and normalized photon number variance are less than that of $\sigma = 0$ due to the existence of errors. We should pay attention on that there are totally similar function relationships between Figs. 3 and 4. The two gains are still same at the maximum

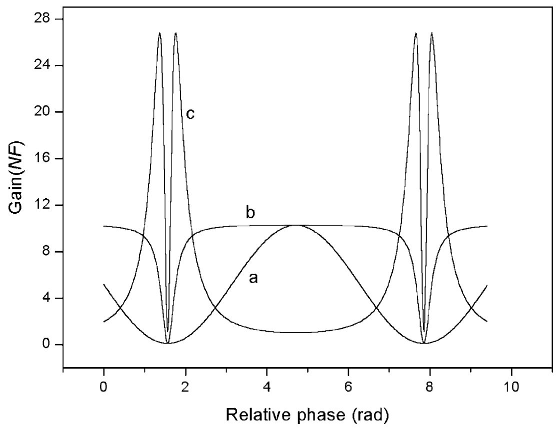

Fig. 3. The gains of photon number (a) and normalized photon number variance (b), noise figure NF (c) as the functions of relative phase between pump and signal light for $\sigma = 0$, $I_{\rm pump} = 2.55$ MW/cm^2, $L_{\rm ppln} = 2$ cm.

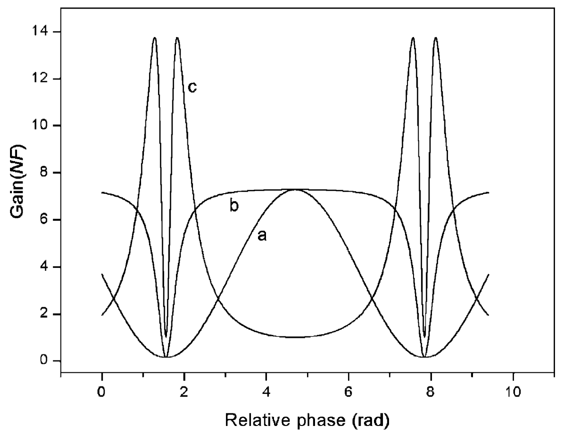

Fig. 4. The gains of photon number (a) and normalized photon number variance (b), noise figure *NF* (c) as the functions of relative phase for $\sigma = 0.1$, $I_{pump} = 2.55$ MW/cm^2, $L_{ppln} = 2$ cm.

amplification and deamplification points and where *NF* equal to 1 also. Thus when there are errors of domain length in QPM materials the noiseless parametric amplification and deamplification can be still be accomplished.

The maximum gain of amplifier and corresponding noise figure *NF* as the functions of propagation length for three different standard deviation are shown in Fig. 5. It is clear the gain increases along with the increase of the propagation length for all given σ, but when the error σ is

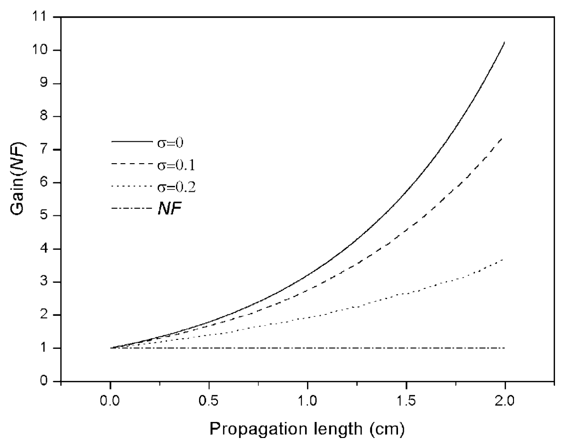

Fig. 5. The gains of photon number and normalized photon number variance, noise figure *NF* as the functions of propagation length for different standard deviation, $I_{pump} = 2.55$ MW/cm^2.

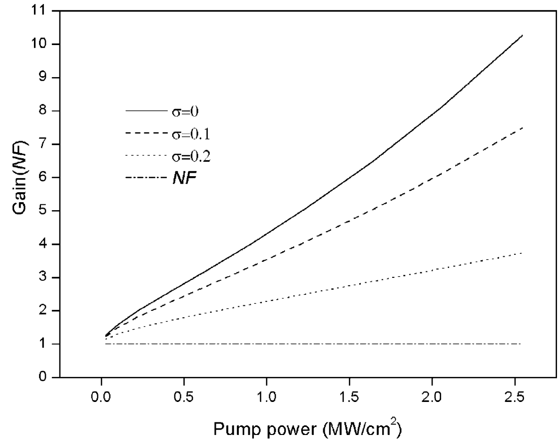

Fig. 6. The gains of photon number and normalized photon number variance, noise figure *NF* as the functions of pump power for different standard deviations, $L_{ppln} = 2$ cm.

larger the gain becomes lower. For example, at the propagation length of 2 cm, the gains are 10.3, 7.5 and 3.7, respectively, with respect to $\sigma = 0$, $\sigma = 0.1$ and $\sigma = 0.2$. Whatever the noise figure *NF* always equals 1 as mentioned above.

The maximum gain of amplifier and corresponding noise figure *NF* as the functions of pump power for three different standard deviations are shown in Fig. 6. Similar to Fig. 5, the gain in all cases increases when the pump power is increased. Existence of errors only reduces the slope of curves. The larger σ is, the smaller the slope of the gain curves is, while *NF* still remains constant.

5. Conclusion

In this paper we numerically analyze the influence of random boundary error in QPM crystals which are formed in the fabrication process on the degenerate optical parametric amplification. It is shown that when the errors increase the gains of the signal photon number and normalized photon number variance degrade synchronously, but the noise figure remains constant and thus the QPM amplifier is still noiseless. We also prove that the noise figure does not vary with the propagation length of signal light in QPM material and pump power when the amplifier operates at the maximum parametric amplification and deamplification. A QPM device with the random boundary errors do not result in the extra noises and only suffer from the degradation of the parametric gain relative to an ideal QPM amplifier. However, the gains of the present QPM amplifiers with random boundary errors are still much higher than that of normal birefringent crystals. Therefore, QPM devices can be widely used not only in the field of non-linear optics but also in the field of quantum optics, such as the squeezing of quantum noise and noiseless amplifier etc.

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