Entanglement characteristics of subharmonic modes reflected from a cavity for type-II second-harmonic generation

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Quantum fluctuation and quantum entanglement of the pump fields reflected from an optical cavity for type-II second-harmonic generation are theoretically analyzed. The correlation spectra of quadrature components between the reflected subharmonic fields are interpreted in terms of pump parameter, intracavity losses, and normalized frequency. High correlation of both amplitude and phase quadratures can be accessed in a triple resonant cavity before the pitchfork bifurcation occurs. The two reflected subharmonic fields are in an entangled state with quantum correlation of phase quadratures and anticorrelation of amplitude quadratures. The proposed system can be exploited as a source for generating entangled states of continuous variables.

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Introduction. Quantum information has attracted great interest in recent years. The realization of quantum teleportation [1–4] and quantum dense coding [5–9] further enhanced confidence and passion in researching and developing quantum cryptography and quantum-information processing. Quantum entanglement plays an essential role in quantum-information and, therefore, the preparation of quantum entangled states becomes the basic work in quantum-information science.

The entanglement of quantum systems with continuous spectra is closely connected with squeezed states of optical fields. The bipartite entangled states of continuous variable have been generated through the combination of two single-mode squeezed vacuum states at a beam splitter and have been successfully applied in the experimental realizations of continuous-variable quantum teleportation of arbitrary coherent states [2–4]. The light beams of Einstein-Podolsky-Rosen (EPR) entanglement have also been obtained by splitting a two-mode squeezed-state light with a polarizing beam splitter and have been used in quantum optical communication [10] and dense coding [8]. Recently, by distributing one two-mode squeezed state among three partites using linear optics device, the bright tripartite entangled light beams are generated and are utilized to achieve the controlled quantum dense coding [9]. A fully inseparable tripartite continuous-variable state has also been obtained by combining three independent squeezed vacuum states [11]. So far, in all the above experiments the entangled states of continuous variables are generated through degenerate or nondegenerate optical parametric amplifiers [2–13]. These experimental systems have to include two parts, first the second-harmonic generation (SHG) and then the parametric down-conversion. On the other hand, it has been experimentally demonstrated that the pump fields reflected from a cavity of intracavity SHG are squeezed states because of the cascaded nonlinear interaction between subharmonic and harmonic fields inside the cavity [14,15]. Ou [16] and Fabre and co-workers [15] theoretically analyzed the quantum fluctuation and squeezing characteristics of the reflected pump fields and calculated the spectra of squeezing for SHG and OPO (optical parametric oscillator), respectively. It was pointed out in Ref. [16] that for the case of type-II harmonic generation there exists a threshold that is identified as the onset of an OPO formed by a subharmonic mode with its polarization orthogonal to the input polarization (not the original modes) and that the output of the orthogonal polarization mode from the OPO exhibits phase squeezing. Jack *et al.* [17] generalized the symmetric pumping case of Ref. [16] to asymmetric case which causes large changes in the classical dynamical behavior of the system. Squeezing and entanglement of doubly resonant type-II SHG was analyzed in Ref. [18]. In Refs. [16–18], the dependences of the quantum fluctuations as well as correlation variances of the quadratures of the reflected fields on the pump parameters (p/p_{th}) and on the losses of the subharmonic modes are calculated, but the dependence on the loss of the harmonic mode is not discussed.

In the present paper, we will analyze both the classical behavior and the quantum correlations of the amplitude and phase quadratures of the two eigen-subharmonic modes reflected from a single-ended cavity of type-II second-harmonic generation under the condition of below the intracavity OPO threshold. The optical cavity is triply resonant both for the two subharmonic modes and for the harmonic mode simultaneously. The dependences of quantum correlations of the amplitude and phase quadratures on the analysis frequencies, on the pump parameter, and on the loss of the harmonic field are numerically analyzed. The results show that the two reflected subharmonic modes are in an entangled state with a phase-quadrature correlation and an amplitude-quadrature anticorrelation, which has been shown to be of great importance in quantum communication [6,8,9]. It is found that the loss of harmonic wave strongly influences the quantum correlation in the case of the triple resonance with low loss. According to the Peres-Horodecki inseparability criterion of EPR entanglement state for continuous variables proposed by Duan [19], the inseparability between the two reflected subharmonic modes is confirmed by numerical calculations. The proposed system can be exploited to be a different type of entanglement sources for continuous variables

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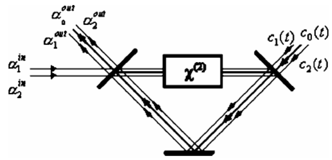

FIG. 1. Sketch of experimental setup.

with relatively simple configuration and high quantum correlation. The given numerical calculations may provide useful references for the design of the entanglement sources.

Fluctuation and correlation spectra of reflected pump modes. The sketch of SHG is shown in Fig. 1. We consider the SHG process in a triply resonating optical cavity with a nonlinear $\chi^{(2)}$ crystal cut for type-II phase matching. The triple resonance means that the two subharmonic pump modes and the harmonic mode simultaneously resonate in the cavity. Under the ideal case with perfect phase matching and without any detuning, the equations of motion for a single-ended cavity with one mirror used for input and output coupler can be expressed as [20]

$$\tau\dot{\alpha}_0(t) = -\gamma_0\alpha_0(t) - \chi\alpha_1(t)\alpha_2(t) + \sqrt{2\gamma_0}c_0(t), \tag{1a}$$

$$\tau\dot{\alpha}_1(t) = -\gamma_1\alpha_1(t) + \chi\alpha_2^*(t)\alpha_0(t) + \sqrt{2\gamma_{b1}}\alpha_1^{in}e^{i\phi_1} + \sqrt{2\gamma_{c1}}(t), \tag{1b}$$

$$\tau\dot{\alpha}_2(t) = -\gamma_2\alpha_2(t) + \chi\alpha_1^*(t)\alpha_0(t) + \sqrt{2\gamma_{b2}}\alpha_2^{in}e^{i\phi_2} + \sqrt{2\gamma_{c2}}c_2(t). \tag{1c}$$

Here α_0, α_1, and α_2 are the amplitudes of harmonic field and two pump fields inside the cavity, respectively. The round-trip time of all modes in the cavity is assumed to be same. The single pass loss parameters γ_{bi} and γ_{ci} $(i=0,1,2)$ stand for the transmission loss through the input-output coupler and all other extra losses, respectively. $\gamma_i=\gamma_{bi}+\gamma_{ci}$ denotes the total loss coefficient, α_1^{in} and α_2^{in} denote the amplitudes of two input pump fields outside the coupler. In the case of $\gamma_{bi}\ll 1$, γ_{bi} is related to the amplitude reflection coefficients r_i and transmission coefficients t_i of the coupler by the following formula: $r_i=1-\gamma_{bi}$, $t_i=\sqrt{2\gamma_{bi}}$. Assuming the two pump modes have the same positive real amplitude β, zero initial phase, and balanced losses in the cavity, we have

$$\gamma_1 = \gamma_2 = \gamma, \tag{2a}$$

$$\gamma_{b1} = \gamma_{b2} = \gamma_b, \tag{2b}$$

$$\gamma_{c1} = \gamma_{c2} = \gamma_c. \tag{2c}$$

The steady-state solutions of Eqs. (1) are obtained to be

$$\overline{\alpha_0} = -k\overline{\alpha_1}\,\overline{\alpha_2}/\gamma_0, \tag{3a}$$

$$[-\gamma - (\chi^2/\gamma_0)|\overline{\alpha_2}|^2]\overline{\alpha_1} + \sqrt{2\gamma_b}\beta = 0, \tag{3b}$$

$$[-\gamma - (\chi^2/\gamma_0)|\overline{\alpha_1}|^2]\overline{\alpha_2} + \sqrt{2\gamma_b}\beta = 0, \tag{3c}$$

where $\overline{\alpha_0}$, $\overline{\alpha_1}$, $\overline{\alpha_2}$ are the steady-state amplitudes of the three intracavity modes α_0, α_1, and α_2. Equations (3b) and (3c) show that both $\overline{\alpha_1}$ and $\overline{\alpha_2}$ are real numbers. The oscillation threshold $\beta^{th}$and pump parameter σ are expressed by

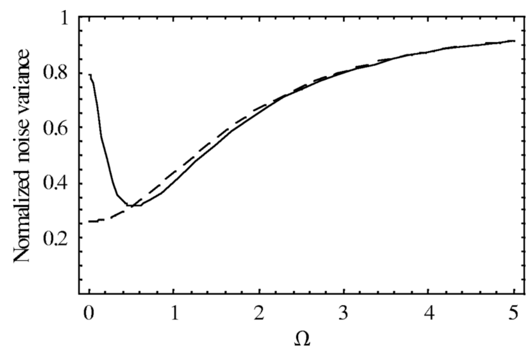

FIG. 2. The quantum noise spectra of $S^{out}_{X_1+X_2}$ and $S^{out}_{Y_1-Y_2}$ vs normalized frequency Ω with $\gamma=0.02$, $\gamma_b=0.015$, $\gamma_0=0.002$, and $\sigma=0.8$.

$$\beta^{th} = \sqrt{2\gamma^3\gamma_0/\chi^2\gamma_b}, \tag{4a}$$

$$\sigma = \beta/\beta^{th}. \tag{4b}$$

Solving Eqs. (3b) and (3c) the steady-state solutions of three the modes above the threshold $(\sigma\geqslant 1)$ are given by

$$\overline{\alpha_1} = \frac{\sqrt{\gamma\gamma_0}}{\chi}\sigma - \frac{\sqrt{\gamma\gamma_0(\sigma^2-1)}}{\chi}, \tag{5a}$$

$$\overline{\alpha_2} = \frac{\sqrt{\gamma\gamma_0}}{\chi}\sigma + \frac{\sqrt{\gamma\gamma_0(\sigma^2-1)}}{\chi}, \tag{5b}$$

$$\overline{\alpha_0} = -\gamma/\chi, \tag{5c}$$

and below the threshold $(\sigma\leqslant 1)$ by

$$\overline{\alpha_1} = \overline{\alpha_2} = \alpha, \tag{6a}$$

$$\alpha = (\sqrt{\gamma\gamma_0}/\chi)\sigma', \tag{6b}$$

$$\overline{\alpha_0} = -\gamma\sigma'^2/\chi, \tag{6c}$$

$$\sigma' = \left(\sigma + \sqrt{\sigma^2 + \tfrac{1}{27}}\right)^{1/3} - \tfrac{1}{3}\left(\sigma + \sqrt{\sigma^2 + \tfrac{1}{27}}\right)^{-1/3}.$$

Entanglement characteristics between the two subharmonic modes reflected from the coupler are denoted by the correlations of quantum fluctuations of their amplitude quadrature X and phase quadrature Y. We just consider the case of below the threshold. The dynamics of the quantum fluctuations can be described by linearizing the classical equations of motion around the stationary state. Setting $\alpha_i=\overline{\alpha_i}+\delta\alpha_i$, $\alpha_i^{in}=\beta_i+\delta b_i$ $(i=0,1,2)$, and using Eq. (1), we have

$$\tau\delta\dot{\alpha}_0(t) = -\gamma_0\delta\alpha_0(t) - \chi[\overline{\alpha_2}\delta\alpha_1(t) + \overline{\alpha_1}\delta\alpha_2(t)] + \sqrt{2\gamma_0}c_0(t), \tag{7a}$$

$$\tau\delta\dot{\alpha}_1(t) = -\gamma\delta\alpha_1(t) + \chi[\overline{\alpha_0}\delta\alpha_2^*(t) + \overline{\alpha_2}\delta\alpha_0(t)] + \sqrt{2\gamma_b}\delta b_1(t) + \sqrt{2\gamma_c}c_1(t), \tag{7b}$$

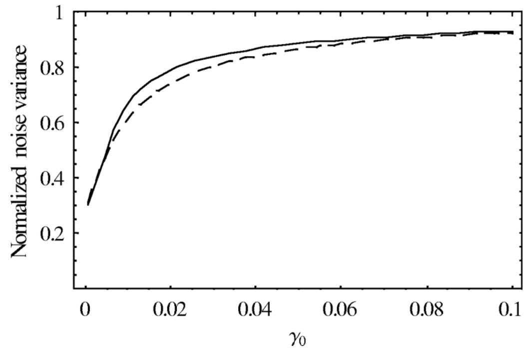

FIG. 3. The quantum noise spectra of $S^{out}_{X_1+X_2}$ and $S^{out}_{Y_1-Y_2}$ vs harmonic loss γ_0 with $\gamma=0.02$, $\gamma_b=0.015$, $\Omega=0.6$, and $\chi\beta=0.001$.

$$\tau\delta\dot{\alpha}_2(t)=-\gamma\delta\alpha_2(t)+\chi[\overline{\alpha_0}\delta\alpha_1^*(t)+\overline{\alpha_1}\delta\alpha_0(t)]+\sqrt{2\gamma_b}\delta b_2(t)+\sqrt{2\gamma_c}c_2(t). \qquad (7c)$$

The optical modes O can be expressed through amplitude quadratures X and phase quadratures Y as $O=\frac{1}{2}(X+iY)$, with $O=[\alpha_0,\alpha_1,\alpha_2,b_0,c_0,b_1,c_1,b_2,c_2]$, $X=[X_0,X_1,X_2,X_{b0},X_{c0},X_{b1},X_{c1},X_{b2},X_{c2}]$, and $Y=[Y_0,Y_1,Y_2,Y_{b0},Y_{c0},Y_{b1},Y_{c1},Y_{b2},Y_{c2}]$.

Substituting these into Eqs. (7a)–(7c), we have

$$\tau\delta\dot{X}_0(t)=-\gamma_0\delta X_0(t)-\chi[\overline{\alpha_2}\delta X_1(t)+\overline{\alpha_1}\delta X_2(t)]+\sqrt{2\gamma_0}\delta X_{c0}(t), \qquad (8a)$$

$$\tau\delta\dot{X}_1(t)=-\gamma\delta X_1(t)+\chi[\overline{\alpha_0}\delta X_2(t)+\overline{\alpha}\delta X_0(t)]+\sqrt{2\gamma_b}\delta X_{b1}(t)+\sqrt{2\gamma_c}\delta X_{c1}(t), \qquad (8b)$$

$$\tau\delta\dot{X}_2(t)=-\gamma\delta X_2(t)+\chi[\overline{\alpha_0}\delta X_1(t)+\overline{\alpha}\delta X_0(t)]+\sqrt{2\gamma_b}\delta X_{b2}(t)+\sqrt{2\gamma_c}\delta X_{c2}(t), \qquad (8c)$$

and

$$\tau\delta\dot{Y}_0(t)=-\gamma_0\delta Y_0(t)-\chi[\overline{\alpha_2}\delta Y_1(t)+\overline{\alpha_1}\delta Y_2(t)]+\sqrt{2\gamma_0}\delta Y_{c0}(t), \qquad (9a)$$

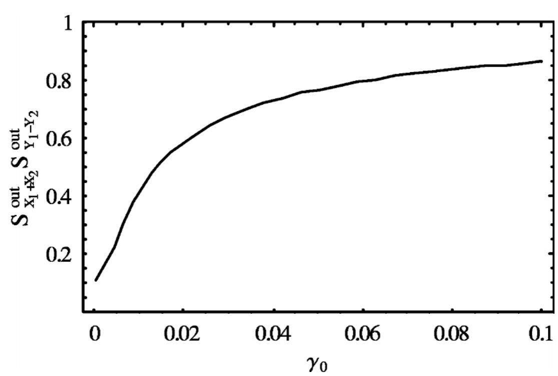

FIG. 4. The quantum noise spectrum of $S^{out}_{X_1+X_2}(\Omega)S^{out}_{Y_1-Y_2}(\Omega)$ vs harmonic loss γ_0 with $\Omega=0.6$, $\gamma=0.02$, $\gamma_b=0.015$, and $\chi\beta=0.001$.

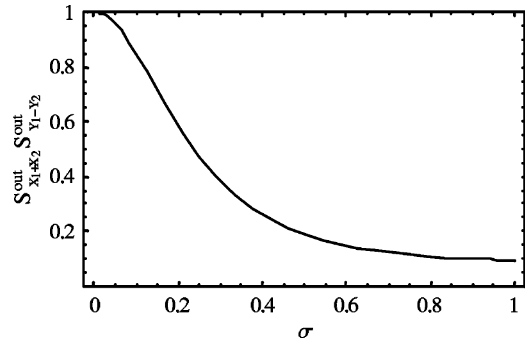

FIG. 5. The quantum noise spectrum of $S^{out}_{X_1+X_2}(\Omega)S^{out}_{Y_1-Y_2}(\Omega)$ vs pump parameter σ with $\gamma=0.02$, $\gamma_b=0.015$, $\gamma_0=0.002$, and $\Omega=0.6$.

$$\tau\delta\dot{Y}_1(t)=-\gamma\delta Y_1(t)-\chi[\overline{\alpha_0}\delta Y_2(t)-\overline{\alpha}\delta Y_0(t)]+\sqrt{2\gamma_b}\delta Y_{b1}(t)+\sqrt{2\gamma_c}\delta Y_{c1}(t), \qquad (9b)$$

$$\tau\delta\dot{Y}_2(t)=-\gamma\delta Y_2(t)-\chi[\overline{\alpha_0}\delta Y_1(t)-\overline{\alpha}\delta Y_0(t)]+\sqrt{2\gamma_b}\delta Y_{b2}(t)+\sqrt{2\gamma_c}\delta Y_{c2}(t). \qquad (9c)$$

Under the condition of below threshold, combining steady-state solution expressions (6a) and (6b), the correlation spectra $\delta X_1(\omega)+\delta X_2(\omega)$ and $\delta Y_1(\omega)-\delta Y_2(\omega)$ (Fourier transformation at ω) are given by

$$\delta X_1(\omega)+\delta X_2(\omega)=\frac{2\sqrt{\gamma\gamma_0}\sigma' Q_{x0}+(Q_{x1}+Q_{x2})(\gamma_0+i\omega\tau)}{D}, \qquad (10a)$$

$$\delta Y_1(\omega)-\delta Y_2(\omega)=(Q_{y1}-Q_{y2})/(\gamma+i\omega\tau+\gamma\sigma'^2), \qquad (10b)$$

where

$$D=(\gamma_0+i\omega\tau)(\gamma+i\omega\tau+\gamma\sigma'^2)+2\gamma\gamma_0\sigma'^2, \qquad (11)$$

$$Q_{x(y)0}=\sqrt{2\gamma_0}\delta X(Y)_{c0}, \qquad (12a)$$

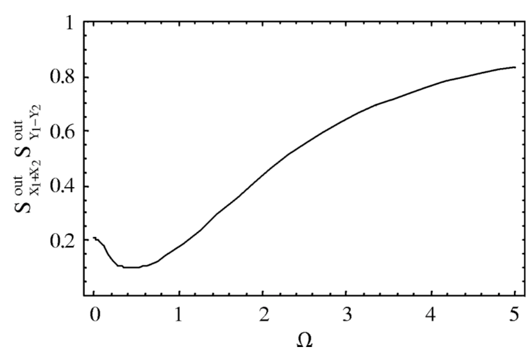

FIG. 6. The quantum noise spectrum of $S^{out}_{X_1+X_2}(\Omega)S^{out}_{Y_1-Y_2}(\Omega)$ vs normalized frequency Ω with $\gamma=0.02$, $\gamma_b=0.015$, $\gamma_0=0.002$, and $\sigma=0.8$.

$$Q_{x(y)1} = \sqrt{2\gamma_b}\delta X(Y)_{b1} + \sqrt{2\gamma_c}\delta X(Y)_{c1}, \quad (12b)$$

$$Q_{x(y)2} = \sqrt{2\gamma_b}\delta X(Y)_{b2} + \sqrt{2\gamma_c}\delta X(Y)_{c2}. \quad (12c)$$

Using the input-output relations of cavity,

$$\delta X_1^{out}(\omega) + \delta X_2^{out}(\omega) = \sqrt{2\gamma_b}[\delta X_1(\omega) + \delta X_2(\omega)] - [\delta X_{b1}(\omega) + \delta X_{b2}(\omega)], \quad (13a)$$

$$\delta Y_1^{out}(\omega) - \delta Y_2^{out}(\omega) = \sqrt{2\gamma_b}[\delta Y_1(\omega) - \delta Y_2(\omega)] - [\delta Y_{b1}(\omega) - \delta Y_{b2}(\omega)], \quad (13b)$$

the correlation spectra of the sum of amplitude quadratures and the difference of phase quadratures of outgoing subharmonic fields are obtained:

$$S_{X_1+X_2}^{out}(\Omega) = |\delta X_1^{out}(\omega) + \delta X_2^{out}(\omega)|^2 = \frac{8}{|D|^2}[2|\gamma_0\sigma'\sqrt{\gamma_b\gamma}|^2 + |\gamma_b(\gamma_0 + i\Omega\gamma) - D/2|^2 + |\sqrt{\gamma_b\gamma_c}(\gamma_0 + i\Omega\gamma)|^2], \quad (14a)$$

$$S_{Y_1-Y_2}^{out}(\Omega) = |\delta Y_1^{out}(\omega) - \delta Y_2^{out}(\omega)|^2 = 2\left|\frac{2\gamma_b}{\gamma + i\Omega\gamma + \gamma\sigma'^2} - 1\right|^2 + 2\left|\frac{2\sqrt{\gamma_b\gamma_c}}{\gamma + i\Omega\gamma + \gamma\sigma'^2}\right|^2, \quad (14b)$$

where $\Omega = \omega\tau/\gamma$ is the normalized frequency.

Figure 2 shows the normalized correlation spectra of $S_{X_1+X_2}^{out}$ and $S_{Y_1-Y_2}^{out}$, both of which are smaller than the standard quantum limit (SQL, normalized to 1). Figure 3 shows the correlation variances of amplitude and phase quadrature vs harmonic loss for given pump power $\chi\beta$ and normalized frequency Ω. Both $S_{X_1+X_2}^{out}$ and $S_{Y_1-Y_2}^{out}$ promptly increase when γ_0 increases from 0 to 0.02, then they increase smoothly to SQL. With enough small γ_0, say $\gamma_0 \leq 0.002$, the reflected subharmonic modes from a triply resonating cavity for type-II SHG are in a strongly entangled state with an anticorrelation of amplitude quadratures and a correlation of phase quadratures [8,9].

Inseparability of amplitude and phase quadratures. The inseparability criterion of EPR entanglement state for continuous variables proposed by Duan [18] is

$$S_{X_1+X_2}^{out}(\Omega) + S_{Y_1-Y_2}^{out}(\Omega) < 2, \quad (15)$$

which is suitable for entangled beams with equal quadrature fluctuations. For the unequal case, asymmetrization procedure has to be carried out, and the criterion becomes [21]

$$S_{X_1+X_2}^{out}(\Omega) S_{Y_1-Y_2}^{out}(\Omega) < 1. \quad (16)$$

Based on Eqs. (14) we numerically analyzed the dependences of inseparability $S_{X_1+X_2}^{out}(\Omega)S_{Y_1-Y_2}^{out}(\Omega)$on the pump parameters σ, normalized frequency Ω, and harmonic loss γ_0, respectively. Figure 4 shows the inseparability of two subharmonic modes as a function of γ_0 at $\Omega=0.6$, $\chi\beta=0.001$, $\gamma=0.02$, and $\gamma_b=0.015$. In Figs. 5 and 6 the correlation spectra $S_{X_1+X_2}^{out}(\Omega)S_{Y_1-Y_2}^{out}(\Omega)$ are plotted vs σ and Ω, respectively, in which the loss parameters are chosen to be $\gamma=0.02$, $\gamma_b=0.015$, and $\gamma_0=0.002$, with $\Omega=0.6$ (Fig. 5) and $\sigma=0.8$ (Fig. 6). It is obvious that the inseparability criterion is satisfied in a wide frequency range with reasonable loss parameters. As is the same in general OPO, the perfect continuous-variable entangled state can be obtained in ideal limit.

In summary, we analyzed the continuous-variable entanglement characteristic of the reflected subharmonic field in type-II SHG using semiclassical approaches. Dependence of correlation variances on the parameters are calculated. Compared to parametric down-conversion usually used for the entangled-state generation, this scheme is relatively simple. The reflected pump field from the SHG cavity is an entangled state with an anticorrelated amplitude quadrature and a correlated phase quadrature, which is very useful in the quantum information [6,8,9].

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[1] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
[2] A. Furusawa *et al.*, Science **282**, 706 (1998).
[3] T. C. Zhang *et al.*, Phys. Rev. A **67**, 023802 (2003).
[4] W. P. Bowen *et al.*, Phys. Rev. A **67**, 032302 (2003).
[5] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[6] J. Zhang and K. C. Peng, Phys. Rev. A **62**, 064302 (2000).
[7] S. L. Braunstein and H. J. Kimble, Phys. Rev. A **61**, 042302 (2000).
[8] X. Y. Li *et al.*, Phys. Rev. Lett. **88**, 047904 (2002).
[9] J. T. Jing *et al.*, Phys. Rev. Lett. **90**, 167903 (2003).
[10] S. F. Pereira *et al.*, Phys. Rev. A **62**, 042311 (2000).
[11] Takao Aoki *et al.*, Phys. Rev. Lett. **91**, 080404 (2003).
[12] Y. Zhang *et al.*, Phys. Rev. A **62**, 023813 (2000).
[13] Z. Y. Ou *et al.*, Phys. Rev. Lett. **68**, 3663 (1992).
[14] S. F. Pereira *et al.*, Phys. Rev. A **38**, 4931 (1988).
[15] K. Kasai *et al.*, Europhys. Lett. **40**, 25 (1997).
[16] Z. Y. Ou, Phys. Rev. A **49**, 4902 (1994).
[17] M. W. Jack *et al.*, Phys. Rev. A **53**, 1801 (1996).
[18] Ulrik L. Andersen and Preben Buchhave, J. Opt. Soc. Am. B **20**, 1947 (2003).
[19] L. M. Duan *et al.*, Phys. Rev. Lett. **84**, 2722 (2000).
[20] C. Fabre *et al.*, J. Phys. (France) **50**, 1209 (1989).
[21] W. P. Bowen *et al.*, J. Mod. Opt. **50**, 801 (2003).