Generation of the four-photon *W* state and other multiphoton entangled states using parametric down-conversion

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We propose two interesting methods of generating the four-photon *W* state. These methods use parametric down-conversion processes, linear optical elements, and commercial photon detectors, which are readily feasible under current technology. They can also be used to generate the three-photon *W* state, the three-photon Greenberger-Horne-Zeilinger state, and the three-photon maximally entangled photon-number state (a typical photon-number entanglement state) by simply changing some experimental components or their parameters. Moreover, assuming we have photon number-resolving detectors, these methods can develop into methods that generate a general *n*-photon *W* state. They are expected to become powerful tools for experimental investigations of multipartite entanglement and its applications to quantum information processing.

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I. INTRODUCTION

Entanglement has a key role in not only fundamental quantum physics [1,2] but also various quantum information processing, including quantum cryptography [3–6], dense coding [7], teleportation [8,9], and quantum computation [10]. Quite recently, multipartite entanglement has been well studied theoretically and experimentally. Dür *et al.* [11] showed that a genuine three-qubit pure entangled state is either equivalent to the maximally entangled Greenberger-Horne-Zeilinger (GHZ) state,

$$|\mathrm{GHZ}_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), \tag{1}$$

or to the *W* state,

$$|W_3\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle), \tag{2}$$

under stochastic local operations and classical communications (SLOCC). In the same way, pure four-qubit systems can be classified into nine classes under SLOCC [12]. The four-qubit GHZ state,

$$|\mathrm{GHZ}_4\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle), \tag{3}$$

and the four-qubit *W* state,

$$|W_4\rangle = \frac{1}{2}(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle), \tag{4}$$

belong to different classes from each other, as in the three-qubit case.

Multipartite entanglement has significant importance in its fundamental aspects, such as violation of nonlocality [13–15]. Differences between the GHZ state and the *W* state have also been discussed [16–19]. There are also many proposals for applications of multipartite entangled states. For GHZ-class states, quantum teleportation [20], dense coding [21], quantum cloning [22], quantum secret sharing [23,24], and quantum key distribution [25,26] have been proposed. For *W*-class states, quantum teleportation [27–32], dense coding [33], quantum telecloning [34,35], quantum key distribution [36], and generation of the universal entangled state [37] have been proposed.

There are quite a few experimental investigations of multipartite entanglement. The three-qubit GHZ state and the three-qubit *W* state have been demonstrated using photons [38–40], atoms [41], NMR [42,43], and ions [44]. The four-qubit GHZ state has also been demonstrated using photons [45,46] and ions [47]. Also another four-qubit state has been demonstrated and proven its nonlocality [48], which belongs to the GHZ class. However, there is no experimental realization of four-qubit entangled states which does not belong to the GHZ class, especially, the four-qubit *W* state.

There is another type of entanglement in photonic qubit region, namely, photon-number entanglement. A typical photon-number entanglement is the maximally entangled photon-number (NOON) state, which has the form

$$|\mathrm{NOON}\rangle = \frac{1}{\sqrt{2}}(|N\rangle|0\rangle + |0\rangle|N\rangle), \tag{5}$$

where N and 0 denote the photon number in each mode [49]. The NOON state is useful for high-resolution phase measurement at the Heisenberg limit [50] or quantum lithography [51]. Experimental demonstrations have been performed for $N=2$ [52] and 3 [53].

In this paper, we show two experimental proposals for generating the four-photon *W* state. They are composed of parametric down-conversion (PDC) processes, linear optical elements, and commercial photon detectors (which cannot resolve the photon number of detection). Qubits are encoded in polarization of photons typically as $|0\rangle \rightarrow |H\rangle$, $|1\rangle \rightarrow |V\rangle$. They do not require single photon sources or photon number-resolving detectors as in Refs. [54,55], thus are readily feasible under current technology. Simultaneously they can be used to generate the three-photon *W* state, the three-photon GHZ state, and the three-photon NOON state only by chang-

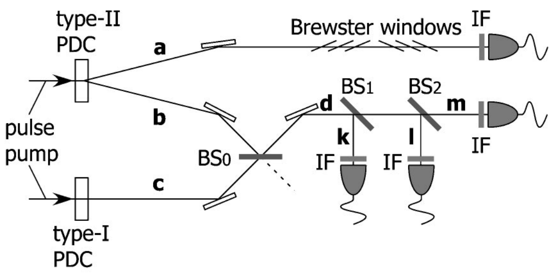

FIG. 1. The schematic of proposal 1; BS_0, BS_1, BS_2, nonpolarizing beam splitter; IF, narrow-band interference filter.

ing a few experimental components or their parameters. Especially, one of them has the highest success probability of generating the polarization three-photon W state within former proposals. Generation of general n-qubit W state assuming photon number-resolving detectors is also discussed.

II. PROPOSAL 1

A. Four-photon W state and three-photon W state

The schematic of the first experimental proposal is shown in Fig. 1. We utilize well-known parametric down-conversion (PDC) processes, which generate two-photon pairs. They occur in nonlinear crystals in the presence of a strong pump light field which is now assumed to be from a femtosecond pulse laser. Especially in this scheme degenerate noncollinear type-II [56] and collinear type-I [57–59] configurations are used. In their processes, for the first-order term, a polarization entangled state and a two-photon state with definite polarization is produced, respectively. Now we focus on the case where each nonlinear crystal generates one photon pair. By setting the polarization of the pump beams properly, such a state can be written as

$$(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b)|HH\rangle_c, \tag{6}$$

where subscripts a, b, and c denote three different spatial modes, and H (V) denotes one photon with horizontal (vertical) polarization. The normalization coefficient was omitted for simplicity, which holds for all the following descriptions. Note that this description is valid only for the photons transmitted through narrow-band interference filters before detectors, whose bandwidths are narrower than that of the pump field, as in Fig. 1 [60]. The same discussion is applied to all the following parts of this paper. The photons in modes b and c are mixed by the nonpolarizing beam splitter (BS_0). We select the case where all the injected photons to BS_0 go out to one output mode named d. Then the total state becomes

$$|H\rangle_a|HHV\rangle_d + \sqrt{3}|V\rangle_a|HHH\rangle_d. \tag{7}$$

The three photons on mode d are split by the nonpolarizing beam splitters BS_1 and BS_2. Here we again select the case where one photon exists in each of the output modes k, l, and m. Hence the state (7) is converted into

$$|H\rangle_a|H\rangle_k|H\rangle_l|V\rangle_m + |H\rangle_a|H\rangle_k|V\rangle_l|H\rangle_m + |H\rangle_a|V\rangle_k|H\rangle_l|H\rangle_m + 3|V\rangle_a|H\rangle_k|H\rangle_l|H\rangle_m. \tag{8}$$

In order to set all the coefficients in Eq. (8) to be equal, Brewster windows are inserted in mode a. Their total transmission coefficient for horizontal (vertical) light, t_H (t_V), is set to be $t_V/t_H=1/3$. Thus we obtain the four-photon W state:

$$|H\rangle_a|H\rangle_k|H\rangle_l|V\rangle_m + |H\rangle_a|H\rangle_k|V\rangle_l|H\rangle_m + |H\rangle_a|V\rangle_k|H\rangle_l|H\rangle_m + |V\rangle_a|H\rangle_k|H\rangle_l|H\rangle_m. \tag{9}$$

In this state, the first three terms represent the three-photon W state with respect to modes k, l, and m. Thus if we recognize the detector in mode a as a trigger and select the case where it detects a horizontally polarized photon, the three-photon W state can also be observed.

B. Three-photon GHZ state and three-photon NOON state

Next let us see how to generate the three-photon GHZ state and the three-photon NOON state using the same setup as the former one. To observe them, another type-I crystal should be inserted just after the first one and the axis is directed so that it generates vertically polarized photon pairs collinearly [58,59]. The two type-I crystals are pumped by a pulsed laser with a certain linear polarization. The detector in mode a works as a trigger and we select the case where the trigger detects a horizontally polarized photon. Then the four-photon generation process which can be detected by coincidence events of four detectors is described as

$$|H\rangle_a|V\rangle_b(\cos 2\theta|HH\rangle_c + \sin 2\theta|VV\rangle_c), \tag{10}$$

where θ is the angle of polarization direction of the pump with respect to the horizontal line. Again we select the case where the three photons in modes b and c appear in mode d after the beam splitter BS_0. Then the state is transformed into

$$|H\rangle_a(\cos 2\theta|HHV\rangle_d + \sqrt{3}\sin 2\theta|VVV\rangle_d). \tag{11}$$

Here we change the basis in mode d into that of $+45°/-45°$ polarization, where the eigenvectors are $|P\rangle=(1/\sqrt{2})(|H\rangle+|V\rangle)$ and $|M\rangle=(1/\sqrt{2})(-|H\rangle+|V\rangle)$. When θ is set so that $\cos 2\theta=3\sin 2\theta$, i.e., $\theta=9.22°$, the state (11) is equivalent to

$$|H\rangle_a(|PPP\rangle_d + |MMM\rangle_d). \tag{12}$$

This is the three-photon NOON state with respect to mode d. Again the three photons in mode d are split by BS_1 and BS_2. If we select the case where one photon goes into each 5 detector in modes k, l, and m, the state (12) is converted to

$$|H\rangle_a(|P\rangle_k|P\rangle_l|P\rangle_m + |M\rangle_k|M\rangle_l|M\rangle_m), \tag{13}$$

where the three-photon GHZ state is generated in modes k, l, and m.

C. Discussion and further improvements

In this section several characteristics of the proposed scheme are described. First let us discuss the detection efficiency of the scheme above. The success probability of detecting the four-photon W state can be represented by the transmission coefficients of the beam splitters BS_i, t_i, as $4t_0^2(1-t_0)t_1^2(1-t_1)t_2(1-t_2)$. This has the maximum value $\frac{16}{729}\approx 0.0219$ when $t_0=t_1=\frac{2}{3}, t_2=\frac{1}{2}$ [61]. This value is comparable with that of the previous experimental work of generating the three-photon W state, $\frac{1}{36}\approx 0.0278$ [40]. This fact implies that the proposed scheme has enough feasibility with respect to measurement time. Moreover, as shown below, this value can be greatly improved.

The success probabilities of the three-photon W state, the three-photon GHZ state, and the three-photon NOON state are also calculated. Their values are $3(1-t_0)t_0^2(1-t_1)t_1^2(1-t_2)t_2$, $\frac{18}{5}(1-t_0)t_0^2(1-t_1)t_1^2(1-t_2)t_2$, and $\frac{3}{5}(1-t_0)t_0^2$, respectively. When $t_0=t_1=\frac{2}{3}$, $t_2=\frac{1}{2}$, they also have maximum values $\frac{4}{243}\approx 0.0165$, $\frac{8}{405}\approx 0.0198$, and $\frac{4}{45}\approx 0.0889$, respectively.

Next we will discuss possible errors in real experiment of the proposed scheme. Consider the other four-photon generation processes. The probabilities that four photons are generated from one crystal are of the same order as that considered in the scheme above. However, one can easily see that such unwanted processes are never detected by fourfold coincidence events of the detectors, and thus is negligible.[1] On the other hand, higher-order processes in which more than four photons are generated also exist. In real experiment, the efficiency of generating two photons per pulse from PDC process is typically $\gamma\sim 10^{-4}$. Then the efficiency of four- and six-photon generation from PDC is $\gamma^2\sim 10^{-8}$ and $\gamma^3\sim 10^{-12}$, respectively. Therefore the six-photon generation rate is $\sim 10^{-4}$ times lower than that of four-photon generation and is almost negligible [61]. Another error is the dark count of detectors, but that of current detectors is quite small so that they are negligible in multiphoton coincidence experiments. From the discussions above, we find that no error prevents our scheme.

For real implementation of the proposed scheme, it is important to adjust the timing of the injection between two inputs into the beam splitter BS_0. At least it should be within the coherence time of the photons, which is determined by the bandwidth of the interference filters used. If we suppose that the above condition is fulfilled and a little change of timing occurs, a certain phase factor δ appears in one of the injection modes, e.g., $|H\rangle_c \to e^{i\delta}|H\rangle_c$. Then the initial four-photon state (6) becomes

$$(|H\rangle_a|V\rangle_b + |V\rangle_a|H\rangle_b)e^{i\delta}|HH\rangle_c. \tag{14}$$

However, this phase shift modifies only the global phase factor on the final four-photon W state:

$$e^{i\delta}(|H\rangle_a|H\rangle_k|H\rangle_l|V\rangle_m + |H\rangle_a|H\rangle_k|V\rangle_l|H\rangle_m + |H\rangle_a|V\rangle_k|H\rangle_l|H\rangle_m + |V\rangle_a|H\rangle_k|H\rangle_l|H\rangle_m), \tag{15}$$

i.e., the state does not change at all as long as we see only these modes. In other words, the position stability along the beam, which determines the injection timing of photons into BS_0, is required to be only within the coherence length. Such a condition has been satisfied in many multiphoton experiments [9,38,40], and is easily achieved. For the case of the three-photon W state, the three-photon GHZ state, and the three-photon NOON state, the same required condition can be applied.

[1] In Refs. [54,55], if the single photons are replaced by outputs from down-conversion, other unwanted terms always exist. Thus their schemes cannot be carried out by only using down-conversion processes but require single photon sources.

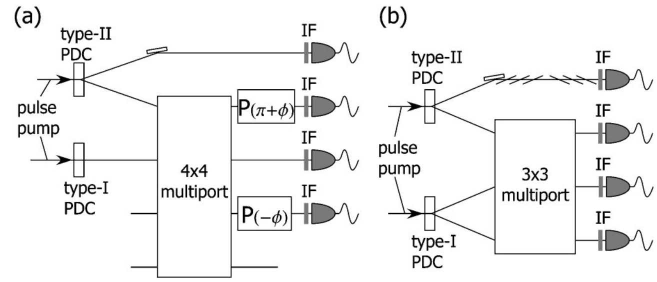

FIG. 2. The improved schemes of the first proposal using multiport elements; $P(\pi+\phi)$, $P(-\phi)$, birefringent phase shifter which induces relative phase $e^{i(\pi+\phi)}$, $e^{-i\phi}$, respectively, in each vertically polarized photon.

Next we will describe the improvement of the proposal. For the improvement of the success probabilities, we utilize $n\times n$ optical multiport elements [62,63]. The transformation matrices of symmetric ones for $n=3$ and 4 are described respectively as

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2\pi i}{3}} & e^{\frac{4\pi i}{3}} \\ 1 & e^{\frac{4\pi i}{3}} & e^{\frac{2\pi i}{3}} \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{i\phi} & -1 & -e^{i\phi} \\ 1 & -1 & 1 & -1 \\ 1 & -e^{i\phi} & -1 & e^{i\phi} \end{pmatrix}, \tag{16}$$

where the internal phase ϕ can have an arbitrary value. They act on vectors whose elements are annihilation operators of the input modes. When we utilize a 4×4 multiport element, modes b and c are injected to two of the input modes as shown in Fig. 2(a). In addition, the Brewster windows are removed and birefringent phase shifters $P(x)$ $(x=\pi+\phi,-\phi)$ are inserted in two of the output modes, which cause relative phase shift x on vertical polarization as $|H\rangle\to|H\rangle$, $|V\rangle\to e^{ix}|V\rangle$. Then the four-photon W state (9) can be obtained in mode a and the three of the output modes of the multiport element, with the success probability of $\frac{1}{8}=0.125$. This value is more than five times higher than that of the scheme before improvement.

When we utilize a 3×3 multiport element, the collinear type-I PDC is replaced by the noncollinear one [64–67] as shown in Fig. 2(b). This process emits photon pairs in different spatial modes from each other and all emitted photons have one definite polarization (which now we assume to be horizontal). Its two spatial modes and one mode of the noncollinear type-II PDC are each injected into three different input ports of a 3×3 multiport unit. We detect each one photon in mode a and the three output port of the multiport unit. Then the four-photon W state (9) is obtained in these modes. The success probability of this scheme is $\frac{2}{27}\approx 0.0741$, which is more than 3.3 times higher than that of the scheme before improvement. In real experiment, fused fiber multiport couplers are useful as multiport elements. Especially for the case of $n=3$, the experimental investigations

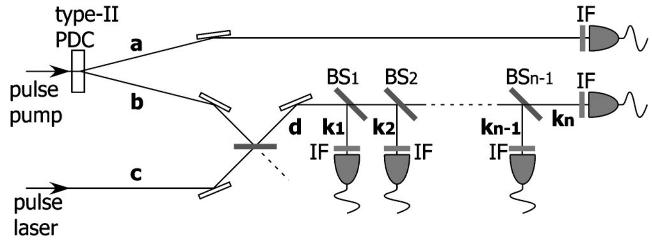

FIG. 3. The developed scheme of the first proposal for generating the general n-photon W state. Detectors are photon number-resolving ones.

have been performed into the validation of the use of a fiber multiport coupler as an ideal multiport element [68,69]. Thus they are surely available for our use.

Changing the amplitude or the relative phase of each term of the W state or the GHZ state is easily accomplished by inserting Brewster windows (for changing the amplitude) or birefringent phase shifters (for changing the relative phase) into modes to be detected. As a result, starting from the W states, we can also generate the states as

$$\sqrt{\frac{2}{3}}|H\rangle_k|H\rangle_l|V\rangle_m - \frac{1}{\sqrt{6}}|H\rangle_k|V\rangle_l|H\rangle_m - \frac{1}{\sqrt{6}}|V\rangle_k|H\rangle_l|V\rangle_m \tag{17}$$

or

$$c_1|V\rangle|H\rangle\cdots|H\rangle + c_2|H\rangle|V\rangle\cdots|H\rangle + \cdots + c_n|H\rangle|H\rangle\cdots|V\rangle, \tag{18}$$

where the coefficients c_i satisfy both $\sum_{i=1}^n c_i=0$ and $\sum_{i=1}^n |c_i|^2=1$. The former state is used as a resource of telecloning protocol [34,35]. The latter one is called the zero sum amplitude state, which isused to generate the universal entangled state [37]. Thus our scheme also has the possibility of contributing to various quantum information processing. This discussion holds for proposal 2, which will be described in the following section.

D. Generating general *n*-photon *W* states

The proposed scheme can develop into that of generating general n-photon W states as in Fig. 3. For the development, we assume the detectors to be photon number-resolving ones [70,71]. In addition, type-I PDC can be replaced by generation of a coherent state (laser output). Here we select the case where there is one horizontally polarized photon in mode a and n photons in mode d. Such a state is

$$|H\rangle_a|\underbrace{H\cdots H}_{n-1}V\rangle_d, \tag{19}$$

where the $n-1$ horizontally polarized photons in mode d originate from the coherent state. The photons in mode d are split by the beam splitters BS_i $(i=1,\ldots,n-1)$ and we again select the case where there is one photon in each of output modes k_i $(i=1,\ldots,n)$. Then the state (19) is converted into the general n-qubit W state with respect to modes k_i:

$$|H\rangle_a(|V\rangle_{k_1}|H\rangle_{k_2}\cdots|H\rangle_{k_n} + |H\rangle_{k_1}|V\rangle_{k_2}\cdots|H\rangle_{k_n} + \cdots + |H\rangle_{k_1}|H\rangle_{k_2}\cdots|V\rangle_{k_n}). \tag{20}$$

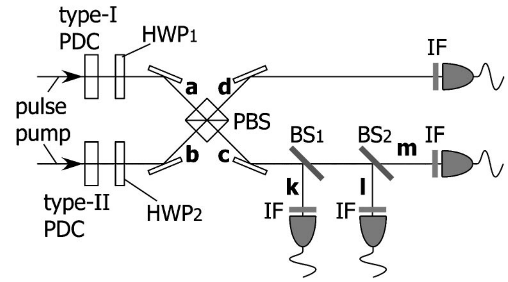

FIG. 4. The schematic of proposal 2; HWP_1, HWP_2, half wave plate for down-converted photons; PBS, polarizing beam splitter.

III. PROPOSAL 2

A. Four-photon *W* state and three-photon *W* state

The schematic of the second experimental proposal is shown in Fig. 4. As in proposal 1, two crystals are pumped by pulse lasers and generate photon pairs via PDC processes. Here the collinear type-I and the collinear type-II [72,73] configurations are used. The collinear type-II PDC process is the one by which photon pairs are generated in one spatial mode and photons in one pair have orthogonal polarization with each other (which now we assume to be horizontal and vertical). There are different types of four-photon generation process, i.e., (i) four photons from the type-II PDC, (ii) two photons from both the type-I and the type-II processes, (iii) four photons from the type-I PDC. Their processes are described as

$$|HHHH\rangle_a|0\rangle_b + |HH\rangle_a|HV\rangle_b + |0\rangle_a|HHVV\rangle_b, \tag{21}$$

where we assumed that the type-I PDC and the type-II PDC have the same efficiency [58]. The photons in modes a and b pass through the half wave plates HWP_1 and HWP_2, respectively. Their corresponding fast-axis angles with respect to the horizontal line are denoted by θ_1 and θ_2. Now, for generating the four-photon W state and the three-photon W state, θ_2 is set to zero. Therefore the photons in mode b undergo phase shift through HWP_1 as $|H\rangle\to|H\rangle$, $|V\rangle\to-|V\rangle$. On the other hand, polarization of the photons in mode a are rotated via HWP_2 as $|H\rangle\to\cos 2\theta_1|H\rangle+\sin 2\theta_1|V\rangle$, $|V\rangle\to\sin 2\theta_1|H\rangle-\cos 2\theta_1|V\rangle$. Then the photons are injected into the polarizing beam splitter (PBS), which transmits horizontally polrized photons and reflects vertically polarized photons. The output modes denoted by c and d are the counterparts of modes a and b, respectively. We select the case where there are three photons in mode c and is one photon in mode d. The state of such a process before the PBS is

$$\sin 4\theta_1|HHHV\rangle_a|0\rangle_b - |HH\rangle_a|HV\rangle_b. \tag{22}$$

After the PBS, this state is converted into

$$\sin 4\theta_1|HHH\rangle_c|V\rangle_d - |HHV\rangle_c|H\rangle_d. \quad (23)$$

The photons in mode c are split by BS_1 and BS_2. Again we consider the case where there is one photon in each of output modes k, l, and m. Then the state which is detected by coincidence events of four detectors in modes k, l, m, and d becomes

$$\sqrt{3}\sin 4\theta_1|H\rangle_k|H\rangle_l|H\rangle_m|V\rangle_d - |H\rangle_k|H\rangle_l|V\rangle_m|H\rangle_d - |H\rangle_k|V\rangle_l|H\rangle_m|H\rangle_d - |V\rangle_k|H\rangle_l|H\rangle_m|H\rangle_d. \quad (24)$$

When θ_1 is set as $\sin 4\theta_1 = -1/\sqrt{3}$, i.e., $\theta_1 = -8.82°$, the state (24) becomes the four-photon W state:

$$|H\rangle_k|H\rangle_l|H\rangle_m|V\rangle_d + |H\rangle_k|H\rangle_l|V\rangle_m|H\rangle_d + |H\rangle_k|V\rangle_l|H\rangle_m|H\rangle_d + |V\rangle_k|H\rangle_l|H\rangle_m|H\rangle_d. \quad (25)$$

When $\theta_1 = 0$, the state (24) becomes the three-photon W state with respect to modes k, l, m:

$$(|V\rangle_k|H\rangle_l|H\rangle_m + |H\rangle_k|V\rangle_l|H\rangle_m + |H\rangle_k|H\rangle_l|V\rangle_m)|H\rangle_d. \quad (26)$$

B. Three-photon GHZ state and the three-photon NOON state

Next we will show how to generate the three-photon GHZ state and the three-photon NOON state. The difference from the generation of the W states is in the values of θ_1, θ_2. Now we set $\theta_1 = 0$. Then the state which contains one photon in mode d and three photons in mode c becomes

$$|HHV\rangle_c|H\rangle_d - \frac{\sqrt{6}}{2}\sin 4\theta_2|VVV\rangle_c|H\rangle_d. \quad (27)$$

When we set θ_2 as $4\theta_2 = -\sqrt{2}/3$, i.e., $\theta_2 = -7.03°$, the state (27) becomes the three-photon NOON state along the $+45°/-45°$ polarization basis in mode c:

$$(|PPP\rangle_c + |MMM\rangle_c)|H\rangle_d. \quad (28)$$

The photons in mode c are split by the beam splitters BS_1 and BS_2. If we select the case where one photon go out to each of three output modes k, l, and m, the state (28) is converted into the three photon GHZ state:

$$(|P\rangle_k|P\rangle_l|P\rangle_m + |M\rangle_k|M\rangle_l|M\rangle_m)|H\rangle_d. \quad (29)$$

C. Discussion and further improvements

In the second scheme, the success probabilities of generating the four-photon W state, the three-photon W state, the three-photon GHZ state, and the three-photon NOON state are calculated as $\frac{5}{162} + \frac{\sqrt{6}}{81} \approx 0.0611$, $\frac{2}{9} \approx 0.222$, $\frac{28}{243} \approx 0.115$, and $\frac{14}{27} \approx 0.519$, respectively. These values are higher than those of the proposal 1. Especially, the value for the three-photon W state, $\frac{2}{9}$, is the highest one compared with previous works of generating the polarization three-photon W state including proposals [40,54,55,61,74].

Consider the injection timing of the two input modes into the PBS. Suppose that the timing is within the coherence-time order and changes a little. Then a phase factor appears in each photon of one mode. Hence the initial state becomes

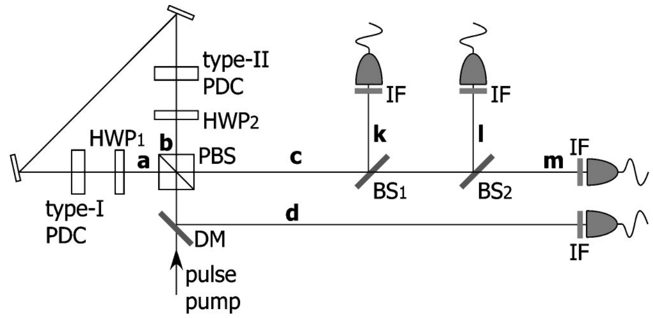

FIG. 5. The improved scheme of the second proposal using a Sagnac interferometer; PBS, polarizing beam splitter which works for both pump and down-converted light; HWP_1, HWP_2, half wave plate for down-converted light, which do nothing for pump light; DM, dichroic mirror which reflects down-converted light and transmits pump light.

$$e^{4i\delta}|HHHH\rangle_a|0\rangle_b + e^{2i\delta}|HH\rangle_a|HV\rangle_b + |0\rangle_a|HHVV\rangle_b. \quad (30)$$

Here different phase factors appears in each term. For the four-photon W state, the three-photon GHZ state, and the three-photon NOON state, two terms of the initial state are required, thus the final states change with respect to phase factors of each term. This fact means that the position stability along the beam, which determines the injection timing of photons into the PBS, needs to be within the wavelength order. Such a situation is more difficult to be realized than the case of coherence-length order stability as in proposal 1. However, there are several experimental studies under such conditions [75,76], thus they are considered to be sufficiently feasible. Moreover, our new scheme can utilize a Sagnac interferometer [77] as shown in Fig. 5, which has excellently high stability. On the other hand, for the three-photon W state, only the second term in Eq. (30) contributes to the final state, hence the phase factor becomes merely a global one on the final state and the total state does not change. Therefore the stability needs to be only within the coherence length as in proposal 1.

D. Generating general *n*-photon *W* state

The second scheme can also be extended to that of generating a general n-photon W state as in Fig. 6. Again we assume photon number-resolving detectors and replace the type-I PDC by generation of a coherent state. Then in the same manner as in proposal 1, we can obtain the general n-qubit W state in modes k_i $(i=1,\ldots,n)$:

$$|H\rangle_d(|V\rangle_{k_1}|H\rangle_{k_2}\cdots|H\rangle_{k_n} + |H\rangle_{k_1}|V\rangle_{k_2}\cdots|H\rangle_{k_n} + \cdots + |H\rangle_{k_1}|H\rangle_{k_2}\cdots|V\rangle_{k_n}). \quad (31)$$

Compared with proposal 1, this scheme has a higher success probability. This is because all photons from the coherent light pass through the PBS and there is no loss of photon.

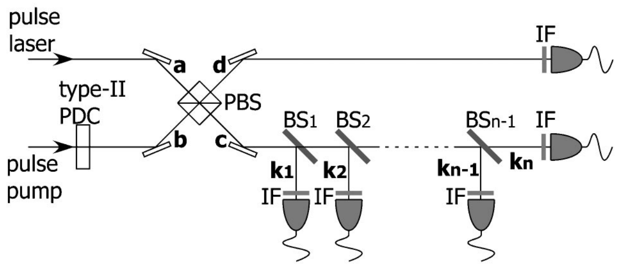

FIG. 6. The developed scheme of the second proposal for the generation of the general n-photon W state. Detectors are photon number-resolving ones.

IV. CONCLUSION

We have proposed two experimental schemes for generating the four-photon W state. They can also be used as sources of the three-photon W state, the three-photon GHZ state, and the three-photon NOON state only by changing some experimental components or their parameters. Especially using the second proposal, the highest success probability of generating the polarization three-photon W state can be achieved. We have discussed some problems for experimental realization of the schemes and shown that they can be improved, thus they are strictly feasible under current technology. Using photon number-resolving detectors, they can develop into those of generating general n-photon W state.

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