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Multi-photon entangled states from two-crystal geometry parametric down-conversion and their application in quantum teleportation

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Abstract

In this paper, we show that two kinds of W-type entangled states can be generated from a non-degenerate two-crystal geometry parametric down-conversion and they are very useful for quantum teleportation and quantum cloning. We also show that quantum teleportation of unknown one-particle state from a sender to either one of three receivers can be realized probabilistically by using the four-photon entangled state directly from a degenerate two-crystal geometry parametric down-conversion.

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1. Introduction

Entanglement plays an important role in quantum information theory. Especially an entangled state of more than two particles can be used for improved tests of local hidden variable theory and also key ingredient for multiparty quantum communication. To date, spontaneous parametric down-conversion (SPDC) has proven to be the best method to generate polarization-entangled photons. Recently, Kwiat et al. [1] presented a way for preparation of Bell states using cw laser pumped two spatially separate non-collinear type-I SPDC, because all pairs of a given color are entangled, this source is more than ten times brighter than the previous sources. Later, Kim et al. showed that both degenerate [2] and non-degenerate SPDC [3] created from two

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spatially separate collinear type-I crystals pumped by femtosecond laser pulse exhibit high-visibility quantum interference, unlike cw pumped two-crystal cases, the temporal compensation is needed here. Different from the degenerate SPDC [1,2], the signal and idler wavelengths are distinguishable (two-color) by using non-degenerate SPDC [3], which is very useful for quantum communications. Quite recently, we presented that four-photon entangled W state can be generated from the degenerate two-crystal geometry SPDC [4]. In this paper, we show that non-degenerate two-crystal geometry SPDC can be utilized to generate "two-color" W-type entangled states conveniently [3]. Comparing with other known schemes [5–7], the features in our scheme include: (i) the generated W states have two different wavelengths which are very useful in quantum teleportation; (ii) this scheme need not the sophisticated and fragile interferometric setups and thus more stable; (iii) the intensity is much higher because of the two-crystal geometry SPDC [1–3]. Then, we show that quantum teleportation of unknown one-particle state from a sender to either one of three receivers can be realized probabilistically by using the four-photon entangled state directly from a degenerate two-crystal geometry parametric down-conversion. Comparing with the teleportation scheme by using the four-photon GHZ state [9], this four-photon entangled state is more easily generated and has much higher intensity.

2. Multi-photon entangled states from a non-degenerate two-crystal geometry parametric down-conversion

The proposed setup is shown in Fig. 1. A pulsed laser is used to pump two identically cut type-I crystals with their optic axes aligned in mutually perpendicular planes (collinear non-degenerate SPDC). When narrow band filters are used (before photon detectors) to make the coherence time of the down-converted photons much longer than the pump pulse duration, the simplified Hamiltonian of the non-degenerate SPDC can be given by

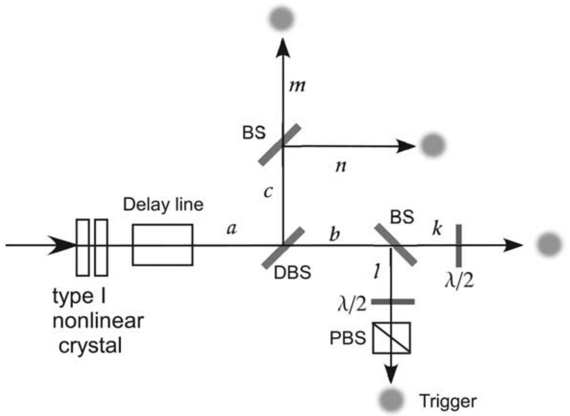

Fig. 1. The schematic digram of the setup. BS, 50–50 non-polarization beam splitter; DBS, dichroic beam splitter; PBS, polarization beam splitter.

$$H_1 = k_1 \hat{a}^{\dagger}_{s,H} \hat{a}^{\dagger}_{i,H} + H.c., \quad H_2 = k_2 \hat{a}^{\dagger}_{s,V} \hat{a}^{\dagger}_{i,V} + H.c., \tag{1}$$

where H_1 and H_2 are the Hamiltonians of the first and the second crystal SPDC, respectively; k_1 and k_2 are coupling constant which depend on the nonlinear coefficients of the crystals and on the intensity of the pump pulse. After a delay line [3], one can erase the information that down-conversion photons are coming from the first crystal or coming from the second crystal. At this stage the state of down-conversion fields is

$$|\Psi\rangle = e^{-iH_1 t/\hbar} e^{-iH_2 t/\hbar} |0\rangle \approx [1 - iHt/\hbar + (-iHt/\hbar)^2/2]|0\rangle, \tag{2}$$

where, $H = H_1 + H_2$, and the higher-order terms have been dropped in the last step. Here, we are interested in only four-photon state, from Eq. (2) the four-photon term can be given by

$$\frac{1}{2}\left[\eta_1^2 \left(\hat{a}^{\dagger}_{s,H}\right)^2 \left(\hat{a}^{\dagger}_{i,H}\right)^2 + \eta_2^2 \left(\hat{a}^{\dagger}_{s,V}\right)^2 \left(\hat{a}^{\dagger}_{i,V}\right)^2 + 2\eta_1\eta_2 \hat{a}^{\dagger}_{s,H} \hat{a}^{\dagger}_{s,V} \hat{a}^{\dagger}_{i,H} \hat{a}^{\dagger}_{i,V}\right] |0\rangle, \tag{3}$$

where $\eta_1 = k_1 t/\hbar$, $\eta_2 = k_2 t/\hbar$. A dichroic beam splitter is used to separate signal beam and idler beam, i.e., it directs signal beam to mode b and reflects idler beam to mode c, then Eq. (3) becomes

$$\frac{1}{2}\left[\eta_1^2\left(\hat{b}_{s,H}^{\dagger}\right)^2\left(\hat{c}_{i,H}^{\dagger}\right)^2+\eta_2^2\left(\hat{b}_{s,V}^{\dagger}\right)^2\left(\hat{c}_{i,V}^{\dagger}\right)^2 +2\eta_1\eta_2\hat{b}_{s,H}^{\dagger}\hat{b}_{s,V}^{\dagger}\hat{c}_{i,H}^{\dagger}\hat{c}_{i,V}^{\dagger}\right]|0\rangle. \quad (4)$$

Next, mode b and mode c are directed to 50–50 non-polarization beam splitter, respectively. We assume that at the beam splitter $\hat{b}$ is transformed into $(1/\sqrt{2})(\hat{k}+\hat{l})$ and $\hat{c}$ is transformed into $(1/\sqrt{2})(\hat{m}+\hat{n})$, where $\hat{k}$ and $\hat{m}$, $\hat{l}$ and $\hat{n}$ denote the transmitted mode and reflected mode, respectively. After this transformation, the state can be given by

$$\frac{1}{2}\left[\eta_1^2\hat{k}_{s,H}^{\dagger}\hat{l}_{s,H}^{\dagger}\hat{m}_{i,H}^{\dagger}\hat{n}_{i,H}^{\dagger}+\eta_2^2\hat{k}_{s,V}^{\dagger}\hat{l}_{s,V}^{\dagger}\hat{m}_{i,V}^{\dagger}\hat{n}_{i,V}^{\dagger} +\frac{1}{2}\eta_1\eta_2\left(\hat{k}_{s,H}^{\dagger}\hat{l}_{s,V}^{\dagger}+\hat{k}_{s,V}^{\dagger}\hat{l}_{s,H}^{\dagger}\right) \times\left(\hat{m}_{i,H}^{\dagger}\hat{n}_{i,V}^{\dagger}+\hat{m}_{i,V}^{\dagger}\hat{n}_{i,H}^{\dagger}\right)\right]|0\rangle. \quad (5)$$

Then, each of modes k and l passes through a half wave plate, which rotates their polarization by 90°, namely, $\hat{k}_H\to\hat{k}_V,\ \hat{k}_V\to\hat{k}_H;\ \hat{l}_H\to\hat{l}_V,\ \hat{l}_V\to\hat{l}_H$. Now the state can be given by

$$\frac{1}{2}\left[\eta_1^2\hat{k}_{s,V}^{\dagger}\hat{l}_{s,V}^{\dagger}\hat{m}_{i,H}^{\dagger}\hat{n}_{i,H}^{\dagger}+\eta_2^2\hat{k}_{s,H}^{\dagger}\hat{l}_{s,H}^{\dagger}\hat{m}_{i,V}^{\dagger}\hat{n}_{i,V}^{\dagger} +\frac{1}{2}\eta_1\eta_2\left(\hat{k}_{s,V}^{\dagger}\hat{l}_{s,H}^{\dagger}+\hat{k}_{s,H}^{\dagger}\hat{l}_{s,V}^{\dagger}\right) \times\left(\hat{m}_{i,H}^{\dagger}\hat{n}_{i,V}^{\dagger}+\hat{m}_{i,V}^{\dagger}\hat{n}_{i,H}^{\dagger}\right)\right]|0\rangle. \quad (6)$$

If one of the four photons (assume it is photon l) is measured in the basis $\{|H\rangle,|V\rangle\}$ and the measurement result is $|V\rangle$, then the entangled state is projected to

$$\frac{1}{2}\left[\eta_1^2\hat{k}_{s,V}^{\dagger}\hat{m}_{i,H}^{\dagger}\hat{n}_{i,H}^{\dagger}+\frac{1}{2}\eta_1\eta_2\hat{k}_{s,H}^{\dagger}\left(\hat{m}_{i,H}^{\dagger}\hat{n}_{i,V}^{\dagger}+\hat{m}_{i,V}^{\dagger}\hat{n}_{i,H}^{\dagger}\right)\right]|0\rangle. \quad (7)$$

After the normalization, the above state can be rewritten as

$$|\Phi\rangle=\frac{1}{\sqrt{2+4\eta_1^2/\eta_2^2}} \times\left(\frac{2\eta_1}{\eta_2}|VHH\rangle_{sii}+|HHV\rangle_{sii}+|HVH\rangle_{sii}\right). \quad (8)$$

By setting $\eta_2 = 2\eta_1$ (in real experiment, this can be achieved by adjusting the polarization of the pump beam [1]), we have

$$|\Phi\rangle=\tfrac{1}{\sqrt{3}}(|VHH\rangle_{sii}+|HHV\rangle_{sii}+|HVH\rangle_{sii}). \quad (9)$$

The above state is exactly a "two-color" W state [10]. One of the potential application for this "two-color" W state is quantum teleportation [11,12], the wavelength of the signal photon is selected around visible light to obtain the maximum detection efficiency [13], and the wavelength of two idler photons is selected around 1550 nm to optimize the telecom optical fiber transmission, thus a high efficiency quantum teleportation can be achieved.

From Eq. (8), we can see that more general W states can be generated by adjusting the value of $2\eta_1/\eta_2$. For example, by selecting $\eta_1 = -\eta_2$, we have

$$|\Phi\rangle=\sqrt{\tfrac{2}{3}}|VHH\rangle_{sii}-\tfrac{1}{\sqrt{6}}|HHV\rangle_{sii}-\tfrac{1}{\sqrt{6}}|HVH\rangle_{sii}. \quad (10)$$

This particular three-particle state can results precisely in a Bužek–Hillery cloning from one sender to two receivers, provided the results are averaged over the four possible measurement outcomes [14,15]. Taking advantage of this freedom ($2\eta_1/\eta_2$), it is possible to generate other useful two-color multi-photon entangled states based on the proposed setup.

3. Quantum teleportation by using the four-photon entangled state from a degenerate two-crystal geometry parametric down-conversion

In quantum teleportation [11,16–19], an arbitrary qubit can be transmitted from a sender to receiver at a distant location via a quantum channel (two-particle maximally entangled state) with the aid of some classical information. Recently, some multi-particle entangled state, e.g., GHZ state [9,20] and W state [12] are used to realize the teleportation in such a way that, either one of two receivers can fully reconstruct the quantum state conditioned on the measurement outcome of the other with 100% probability (GHZ state) or probabilistically (W state). In this section, we show that teleportation of unknown one-particle state can be realized by using four-photon entangled state directly from the degenerate two-crystal geometry

SPDC [8] from a sender to either one of three receivers probabilistically.

We assume that the sender (Alice) wants to teleport an arbitrary and unknown one-particle state to either one of three receivers Bob, Charlie, and Diana. The arbitrary and unknown one-particle state that will be teleported can be given by

$$|\phi\rangle_5 = a|0\rangle + b|1\rangle. \tag{11}$$

Here, a and b are complex and satisfy $|a|^2 + |b|^2 = 1$; 0 is horizontal polarization direction and 1 is vertical polarization direction. To perform the teleportation, We utilize the four-photon entangled state directly from the degenerate collinear SPDC of a two-crystal geometry as the quantum channel. This state can be written as [8]

$$\begin{aligned}|\Psi^{(4)}\rangle = {} & \sqrt{3}/3(|1111\rangle_{1234} + |0000\rangle_{1234}) \\ & + \sqrt{2}/6(|1100\rangle_{1234} + |0011\rangle_{1234} \\ & + |1010\rangle_{1234} + |0101\rangle_{1234} \\ & + |0110\rangle_{1234} + |1001\rangle_{1234}).\end{aligned} \tag{12}$$

Particle 1 is kept by Alice, while particles 2, 3, and 4 are sent to Bob, Charlie, and Diana, respectively. Now the quantum state of the whole system can be given by

$$|\Theta\rangle_s = |\Psi^{(4)}\rangle \otimes |\phi\rangle_5. \tag{13}$$

State $|\Theta\rangle_s$ can be rewritten as

$$\begin{aligned}|\Theta\rangle_s = {} & |\Phi^{\pm}\rangle_{15}\left[\frac{\sqrt{3}a}{3}|000\rangle_{234} + \frac{\sqrt{2}a}{6}(|011\rangle + |101\rangle \right. \\ & +|110\rangle)_{234} \pm \frac{\sqrt{3}b}{3}|111\rangle_{234} \pm \frac{\sqrt{2}b}{6}(|100\rangle \\ & \left. +|010\rangle + |001\rangle)_{234}\right] + |\Psi^{\pm}\rangle_{15}\left[\pm\frac{\sqrt{3}a}{3}|111\rangle_{234} \right. \\ & \pm\frac{\sqrt{2}a}{6}(|100\rangle + |010\rangle + |001\rangle)_{234} \\ & +\frac{\sqrt{3}b}{3}|000\rangle_{234} + \frac{\sqrt{2}b}{6}(|011\rangle + |101\rangle \\ & \left. +|110\rangle)_{234}\right],\end{aligned} \tag{14}$$

where $|\Phi^{\pm}\rangle$ and $|\Psi^{\pm}\rangle$are the four Bell states defined by

$$|\Phi^{\pm}\rangle = \tfrac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad |\Psi^{\pm}\rangle = \tfrac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle). \tag{15}$$

After Alice verifies that all the three Bob, Charlie, and Diana receive their qubits, then she performs a Bell-basis measurement on her qubits 1 and 5 and declares her measurement result publicly. From Eq. (14), the state of qubits 2, 3, and 4 is projected to a pure entangled state of three particles after Bell basis measurement of Alice, and the quantum information is now transferred to the pure entangled state which is shared among Bob, Charlie, and Diana. If Alice gets the result of $|\Phi^{+}\rangle_{15}$, then particles 2, 3, and 4 are projected to the state

$$\begin{aligned}|\Theta\rangle_{234} = {} & \tfrac{\sqrt{3}a}{3}|000\rangle_{234} + \tfrac{\sqrt{2}a}{6}(|011\rangle + |101\rangle + |110\rangle)_{234} \\ & + \tfrac{\sqrt{3}b}{3}|111\rangle_{234} + \tfrac{\sqrt{2}b}{6}(|100\rangle + |010\rangle + |001\rangle)_{234}.\end{aligned} \tag{16}$$

If Bob and Charlie want to cooperate with Charlie, they both perform the projection measurements on their qubits 2 and 3 in the basis $\{|1\rangle, |0\rangle\}$ and inform their outcomes to Diana. We assume Bob's outcome is $|0\rangle_2$ and Charlie's outcome is $|1\rangle_3$, then particle 4 will be projected to the state

$$|\Theta\rangle_4 = \tfrac{\sqrt{2}}{6}(a|1\rangle_4 + b|0\rangle_4). \tag{17}$$

At this stage, Charlie can reconstruct the unknown state by appropriately rotating his qubit

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}|\Theta\rangle_4 = a|0\rangle_4 + b|1\rangle_4. \tag{18}$$

The probabilities of all possible measurement results of Alice, Bob, and Charlie, and the corresponding unitary operators of Diana are given in Table 1 (where σ_x, σ_y, and σ_z are the Pauli operators; I is a unit operator). It is easy to calculate from Table 1 that the total probability of success is 4/9. Also, the probability is independent on the coefficients of the state $|\phi\rangle_5$. This is the same with the scheme based on the GHZ state [9,20] and different from the scheme based on the W state [12].

Table 1
The probabilities of all possible measurement results of Alice, Bob and Charlie and the corresponding unitary operators of Diana

	$\|\Phi^+\rangle_{Alice}$	$\|\Phi^-\rangle_{Alice}$	$\|\Psi^+\rangle_{Alice}$	$\|\Psi^-\rangle_{Alice}$
$\|0\rangle_{Bob}\|0\rangle_{Charlie}$	×	×	×	×
$\|0\rangle_{Bob}\|1\rangle_{Charlie}$	$\frac{1}{18}, \sigma_x$	$\frac{1}{18}, \sigma_y$	$\frac{1}{18}, I$	$\frac{1}{18}, \sigma_z$
$\|1\rangle_{Bob}\|0\rangle_{Charlie}$	$\frac{1}{18}, \sigma_x$	$\frac{1}{18}, \sigma_y$	$\frac{1}{18}, I$	$\frac{1}{18}, \sigma_z$
$\|1\rangle_{Bob}\ \|1\rangle_{Charlie}$	×	×	×	×

(×) Means the teleportation is failed.

4. Conclusion

In conclusion, we have studied the multi-photon entangled states from a two-crystal geometry non-degenerate collinear SPDC and show that two-color W-type entangled states that are very useful for quantum teleportation and quantum cloning can be generated conveniently from this setup. We also pointed out that quantum teleportation of unknown one-particle state from a sender to either one of three receivers can be realized probabilistically by using the four-photon entangled state directly from a degenerate two-crystal geometry parametric down-conversion.

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