Generation of continuous-variable tripartite entanglement using cascaded nonlinearities

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It is theoretically shown that tripartite entanglement with different wavelengths can be generated by cascaded nonlinear interaction in an optical parametric oscillator cavity with parametric down conversion and sum-frequency generation. A sufficient inseparability criterion for continuous-variable tripartite entanglement proposed by van Loock and Furusawa was used to evaluate the degree of the quadrature-phase amplitude correlations between the three modes. The dependences of correlation on the cavity parameters and pump intensity are discussed.

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I. INTRODUCTION

Quantum entanglement is the basic resource in quantum communication and computation. The generation of quantum entanglement becomes an essential research of quantum information science and attracts many interests. There are several kinds of experimental realization of entangled states, in which the optical parametric process is one of the most efficient techniques. For example, the bipartite entangled states which have been used in quantum teleportation [1–3] and dense coding [4] are generated by optical parametric amplifier (OPA). Recently other interesting means have been proposed to produce a two-mode entangled state using the triply resonant type-II second-harmonic generation (SHG) system [5,6]. Besides its application in quantum teleportation and dense coding, the entanglement shared by two parties has also been realized as a valuable resource for cryptography [7,8], tomography of state [9,10], etc. With the development of science and technology, entanglement between more than two particles is going to be the key ingredient for advanced multiparty quantum communication of quantum teleportation network [11], telecloning [12,13], and controlled dense coding [14,15].

Continuous-variable (CV) tripartite entanglement was applied in the quantum teleportation experiment in 1998 [1]. van Loock and Braunstein showed theoretically that using single-mode squeezed states and linear optics suffices to produce a truly *N*-partite entangled state [11], and the experimental accessible criterion to verify the full inseparability of entangled state is thus proposed in their work [11] and other published works [16,17]. In addition, a method to produce a three-mode entangled state of bright optical field by distributing a two-mode squeezed state using beam splitters has been demonstrated [14,15].

Quite recently, the generation of multipartite entanglement in a nonlinear optical material with parametric process draws much attention. It is shown theoretically that concurrent interaction in a second-order nonlinear medium placed in an optical resonator can generate multipartite entanglement [18]. The generation of full inseparable three-mode entangled states by interlinked interactions in a $\chi^{(2)}$ medium has been addressed and the preliminary experimental results have been presented too [19]. Based on it, we propose a scheme to produce tripartite entangled states via parametric down conversion (PDC) and sum-frequency generation (SFG) in a $\chi^{(2)}$ medium in an optical parametric oscillator. Compared to the scheme described above in Ref. [19], the resonant optical cavity is used to improve nonlinear coupling efficiency and the output field is a state with correlated quadrature-phase amplitude fluctuations. Furthermore, the three light modes of the entangled state with different wavelengths are acquired. Such phase-matching conditions for both interactions can also hopefully be realized by placing two nonlinear optical crystals or one quasiperiodic optical superlattice crystal into a cavity [18,20]. The dynamical behavior of a self-phase-locked OPO induced by two competing nonlinearities in periodic poled lithiumniobate (PPLN) has been demonstrated, and the potential application of such competing nonlinear process to produce multipartite entanglement has also been analyzed [21].

II. EQUATIONS OF MOTION AND THE SOLUTION OF OUTPUT FIELD

The system consists of a one-sided cavity with three modes a_1, a_2, a_3 and one crystal, which provides the two interactions of parametric down conversion and sum-frequency generation (Fig. 1). The input and output fields of cavity are coupled through the mirror M. Pump a_4 is incident upon the optical medium to create fields a_1 and a_3 by the

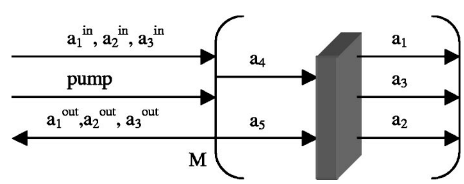

FIG. 1. Sketch of the OPO. a_4, a_5 are pump beams. Pump a_4 creates fields a_1 and a_3 by the process of parametric down conversion, and simultaneously pump a_5 and field a_3 produce sum-frequency field a_2. a_j^{in} $(j=1,2,3)$ are the incoming fields. a_j^{out} are the corresponding outgoing fields.

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process of parametric down conversion, and simultaneously the other pump a_5 interacts with field a_3 to produce sum-frequency field a_2. The frequencies of the pump beams are degenerated, their polarizations are chosen according to the phase-matching conditions. The energy-matching conditions are $\omega_4=\omega_1+\omega_3, \omega_2=\omega_3+\omega_5$.

The interaction Hamiltonian for this system is given by [22]

$$\hat{H}_I = (i\hbar\kappa_1\hat{a}_1^+\hat{a}_3^+e^{-i\omega_4 t} + i\hbar\kappa_2\hat{a}_3\hat{a}_2^+e^{-i\omega_5 t}) + \text{H.c.} \tag{1}$$

$\kappa_{1,2}$ are proportional to the nonlinear susceptibility and pump intensity [19] and are taken to be real commonly [23].

Following the standard procedure in the Heisenberg picture [24], the quantum Langevin equations of motion for the three cavity modes can be expressed as

$$\tau\frac{d\hat{a}_1^+}{dt} = i\omega_1\tau\hat{a}_1^+ + \kappa_1\hat{a}_3e^{i\omega_4 t} - \gamma_1\hat{a}_1^+ + \sqrt{2\gamma_1}\hat{a}_1^{+in},$$

$$\tau\frac{d\hat{a}_2}{dt} = -i\omega_2\tau\hat{a}_2 + \kappa_2\hat{a}_3e^{-i\omega_5 t} - \gamma_2\hat{a}_2 + \sqrt{2\gamma_2}\hat{a}_2^{in},$$

$$\tau\frac{d\hat{a}_3}{dt} = -i\omega_3\tau\hat{a}_3 + \kappa_1\hat{a}_1^+e^{-i\omega_4 t} - \kappa_2\hat{a}_2e^{i\omega_5 t} - \gamma_1\hat{a}_3 + \sqrt{2\gamma_1}\hat{a}_3^{in}, \tag{2}$$

where τ is the cavity round-trip time, $\tau=2L/c$, L is the effective cavity length, and c is the speed of light in vacuum. We assume τ is the same for all three fields. a_j^{in} $(j=1,2,3)$ are the operators corresponding to input fields to the cavity. γ is the damping rate. For simplicity, supposing all of the internal losses of the system are leakage via mirror M with damping constants γ_j which are related to the amplitude reflection coefficients r_j and the amplitude transmission coefficients t_j approximately, $r_j=1-\gamma_j$, $t_j=\sqrt{2\gamma_j}$. Considering the two modes a_1, a_3 are nearly frequency degenerate, we assume the damping rates to be identical $\gamma_1=\gamma_3$.

In order to solve Eq. (2), we transform it into a rotating frame [23] and use the Fourier transformation, then the relationship between the output quantities and the input quantities can be calculated from the boundary conditions $\hat{A}_j^{out}+\hat{A}_j^{in}=\sqrt{2\gamma_j}\hat{A}_j$ at the mirror [22],

$$\underline{\hat{A}}^{out}(\Omega) = [B(i\Omega\tau I - M)^{-1}B - I]\underline{\hat{A}}^{in}(\Omega), \tag{3}$$

with

$$\underline{\hat{A}}^{out}(\Omega) = \begin{pmatrix} \hat{A}_1^{+out}(-\Omega) \\ \hat{A}_2^{out}(\Omega) \\ \hat{A}_3^{out}(\Omega) \end{pmatrix},$$

$$M = \begin{pmatrix} i\Omega\tau+\gamma_1 & 0 & -\kappa_1 \\ 0 & i\Omega\tau+\gamma_2 & -\kappa_2 \\ -\kappa_1 & \kappa_2 & i\Omega\tau+\gamma_1 \end{pmatrix},$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} \sqrt{2\gamma_1} & 0 & 0 \\ 0 & \sqrt{2\gamma_2} & 0 \\ 0 & 0 & \sqrt{2\gamma_1} \end{pmatrix},$$

$$\underline{\hat{A}}^{in}(\Omega) = \begin{pmatrix} \hat{A}_1^{+in}(-\Omega) \\ \hat{A}_2^{in}(\Omega) \\ \hat{A}_3^{in}(\Omega) \end{pmatrix}.$$

III. QUANTUM CORRELATIONS AMONG THE QUADRATURE-PHASE AMPLITUDES OF THE THREE MODES

To study the entanglement characteristics of fields, we need to look at the fluctuations of quadrature amplitude and phase components defined by

$$\hat{X}_j^{out}(\Omega) = \hat{A}_j^{+out}(-\Omega) + \hat{A}_j^{out}(\Omega),$$

$$\hat{Y}_j^{out}(\Omega) = i[\hat{A}_j^{+out}(-\Omega) - \hat{A}_j^{out}(\Omega)]. \tag{4}$$

Then we rewrite Eq. (3) as

$$\hat{X}_1^{out}(\Omega') = G_1(\Omega')\hat{X}_1^{in}(\Omega') - g(\Omega')\hat{X}_2^{in}(\Omega') + h(\Omega')\hat{X}_3^{in}(\Omega'),$$

$$\hat{X}_2^{out}(\Omega') = g(\Omega')\hat{X}_1^{in}(\Omega') + G_2(\Omega')\hat{X}_2^{in}(\Omega') + f(\Omega')\hat{X}_3^{in}(\Omega'),$$

$$\hat{X}_3^{out}(\Omega') = h(\Omega')\hat{X}_1^{in}(\Omega') - f(\Omega')\hat{X}_2^{in}(\Omega') + G_3(\Omega')\hat{X}_3^{in}(\Omega'),$$

$$\hat{Y}_1^{out}(\Omega') = G_1(\Omega')\hat{Y}_1^{in}(\Omega') + g(\Omega')\hat{Y}_2^{in}(\Omega') - h(\Omega')\hat{Y}_3^{in}(\Omega'),$$

$$\hat{Y}_2^{out}(\Omega') = -g(\Omega')\hat{Y}_1^{in}(\Omega') + G_2(\Omega')\hat{Y}_2^{in}(\Omega') + f(\Omega')\hat{Y}_3^{in}(\Omega'),$$

$$\hat{Y}_3^{out}(\Omega') = -h(\Omega')\hat{Y}_1^{in}(\Omega') - f(\Omega')\hat{Y}_2^{in}(\Omega') + G_3(\Omega')\hat{Y}_3^{in}(\Omega') \tag{5}$$

with

$$G_1(\Omega') = [\gamma_1^2\gamma_2(1+\sigma^2+\Omega'^2) + \gamma_1^3\xi^2(1-i\Omega') + i\Omega'\gamma_1^3(1+\sigma^2+\Omega'^2)]/\Delta,$$

$$G_2(\Omega') = [(i\gamma_1^3\Omega' - \gamma_1^2\gamma_2)(-i+\Omega')^2 - i\gamma_1^3\xi^2(-i+\Omega') + (i\gamma_1^3\Omega' - \gamma_1^2\gamma_2)\sigma^2]/\Delta,$$

$$G_3(\Omega') = [-i\gamma_1^3\xi^2(-i+\Omega') + (\gamma_1^2\gamma_2 + i\gamma_1^3\Omega')(1+\sigma^2+\Omega'^2)]/\Delta,$$

$$g(\Omega') = 2\sigma\xi\gamma_1^2\sqrt{\gamma_1\gamma_2}/\Delta,$$

$$h(\Omega') = 2\gamma_1^2\sigma(\gamma_2 + i\gamma_1\Omega')/\Delta,$$

$$f(\Omega') = 2\gamma_1\xi(\gamma_1 + i\gamma_1\Omega')\sqrt{\gamma_1\gamma_2}/\Delta,$$

$$\Delta = \gamma_1^2\xi^2(\gamma_1 + i\gamma_1\Omega') - \gamma_1^2\sigma^2(\gamma_2 + i\gamma_1\Omega') + (\gamma_1 + i\gamma_1\Omega')^2(\gamma_2 + i\gamma_1\Omega').$$

Variables Ω', σ, ξ are defined as $\Omega' = \Omega\tau/\gamma_1$, $\sigma = \kappa_1/\gamma_1$, $\xi = \kappa_2/\gamma_1$. Ω' is the normalized frequency. σ is the pump parameter including both pump power and nonlinear coefficient $\chi^{(2)}$ for down conversion, $\sigma=1$ corresponds to the threshold point. ξ is the pump parameter of the sum-frequency process. If we set $\xi=0$, that means the model is just a double resonating nondegenerate PDC and no SFG. The gains $G_1(\Omega)$ and $G_3(\Omega)$ of the two fields a_1, a_3 depend on the pump strength and can be extremely large as the threshold is approached ($\sigma\to 1$) and $\Omega'\to 0$. Actually, if $\sigma\to 1$ and $\Omega'\to 0$, the two modes of down conversion are perfectly entangled. When $\xi\neq 0$, it is a process with both PDC and SFG, the partial energy conversion from the a_3 field to the a_2 field. And the tripartite entanglement should be obtained.

IV. FULL INSEPARABILITY OF THE OUTPUT LIGHT FIELDS

A sufficient inseparability criterion for CV tripartite entanglement has been proposed by van Loock and Furusawa [16]:

$$\langle \delta^2(\hat{X}_1 - \hat{X}_2)\rangle + \langle \delta^2(\hat{Y}_1 + \hat{Y}_2 + g_3\hat{Y}_3)\rangle < 4,$$

$$\langle \delta^2(\hat{X}_2 - \hat{X}_3)\rangle + \langle \delta^2(g_1\hat{Y}_1 + \hat{Y}_2 + \hat{Y}_3)\rangle < 4,$$

$$\langle \delta^2(\hat{X}_1 - \hat{X}_3)\rangle + \langle \delta^2(\hat{Y}_1 + g_2\hat{Y}_2 + \hat{Y}_3)\rangle < 4, \tag{6}$$

where g_1, g_2, g_3 are scaling factors. Choosing adjustable scaling factors can minimize the quantities of the left side. The satisfaction of any pair of the inequalities is sufficient for full inseparability of three-party entanglement. The smaller the values of the left-hand side of the inequalities are, the larger the correlation degree that we will obtain.

So to quantify the degree of the correlation, we introduce the correlation spectra of the total phase quadratures of three-mode and relative amplitude quadratures:

$$\hat{S}^{out}_{X_1-X_2} = \langle[\hat{X}^{out}_1(\Omega') - \hat{X}^{out}_2(\Omega')][\hat{X}^{out}_1(\Omega') - \hat{X}^{out}_2(\Omega')]^+\rangle = |G_1(\Omega') - g(\Omega')|^2 + |g(\Omega') + G_2(\Omega')|^2 + |h(\Omega') - f(\Omega')|^2,$$

$$\hat{S}^{out}_{X_1-X_3} = \langle[\hat{X}^{out}_1(\Omega') - \hat{X}^{out}_3(\Omega')][\hat{X}^{out}_1(\Omega') - \hat{X}^{out}_3(\Omega')]^+\rangle = |G_1(\Omega') - h(\Omega')|^2 + |f(\Omega') - g(\Omega')|^2 + |h(\Omega') - G_3(\Omega')|^2,$$

$$\hat{S}^{out}_{X_2-X_3} = \langle[\hat{X}^{out}_2(\Omega') - \hat{X}^{out}_3(\Omega')][\hat{X}^{out}_2(\Omega') - \hat{X}^{out}_3(\Omega')]^+\rangle = |g(\Omega') - h(\Omega')|^2 + |G_2(\Omega') + f(\Omega')|^2 + |f(\Omega') - G_3(\Omega')|^2,$$

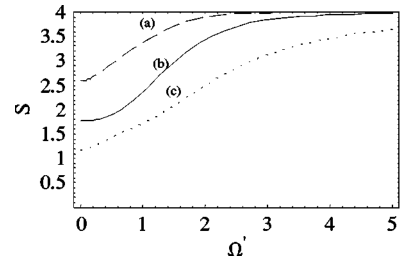

FIG. 2. The quantum correlation spectra of $\hat{S}^{out}_{X_2-X_3} + \hat{S}^{out}_{g_1Y_1+Y_2+Y_3}$ [curve (a)], $\hat{S}^{out}_{X_1-X_2} + \hat{S}^{out}_{Y_1+Y_2+g_3Y_3}$ [curve (b)], and $\hat{S}^{out}_{X_1-X_3} + \hat{S}^{out}_{Y_1+g_2Y_2+Y_3}$ [curve (c)] versus normalized frequency $\Omega'(\Omega'=\omega\tau/\gamma_1)$ for $\gamma_1 = 0.02$, $\gamma_2=0.03$, $\sigma=0.6$, $\xi=1$. g has been chosen to minimize the variances.

$$\hat{S}^{out}_{g_1Y_1+Y_2+Y_3} = \langle[g_1\hat{Y}^{out}_1(\Omega') + \hat{Y}^{out}_2(\Omega') + \hat{Y}^{out}_3(\Omega')][g_1\hat{Y}^{out}_1(\Omega') + \hat{Y}^{out}_2(\Omega') + Y^{out}_3(\Omega')]^+\rangle = |g_1G_1(\Omega') - g(\Omega') - h(\Omega')|^2 + |g_1g(\Omega') + G_2(\Omega') - f(\Omega')|^2 + |-g_1h(\Omega') + f(\Omega') + G_3(\Omega')|^2,$$

$$\hat{S}^{out}_{Y_1+g_2Y_2+Y_3} = \langle[\hat{Y}^{out}_1(\Omega') + g_2\hat{Y}^{out}_2(\Omega') + \hat{Y}^{out}_3(\Omega')][\hat{Y}^{out}_1(\Omega') + g_2\hat{Y}^{out}_2(\Omega') + Y^{out}_3(\Omega')]^+\rangle = |G_1(\Omega') - g_2g(\Omega') - h(\Omega')|^2 + |g(\Omega') + g_2G_2(\Omega') - f(\Omega')|^2 + |-h(\Omega') + g_2f(\Omega') + G_3(\Omega')|^2,$$

$$\hat{S}^{out}_{Y_1+Y_2+g_3Y_3} = \langle[\hat{Y}^{out}_1(\Omega') + \hat{Y}^{out}_2(\Omega') + g_3\hat{Y}^{out}_3(\Omega')][\hat{Y}^{out}_1(\Omega') + \hat{Y}^{out}_2(\Omega') + g_3Y^{out}_3(\Omega')]^+\rangle = |G_1(\Omega') - g(\Omega') - g_3h(\Omega')|^2 + |g(\Omega') + G_2(\Omega') - g_3f(\Omega')|^2 + |-h(\Omega') + f(\Omega') + g_3G_3(\Omega')|^2. \tag{7}$$

The spectra for the three correlation functions of the three modes described in Eq. (7) vs normalized analyzing frequency are plotted in Fig. 2 at $\sigma=0.6$ and $\xi=1$. The curves (a), (b), and (c) are for the correlation degrees of $\hat{S}^{out}_{X_2-X_3} + \hat{S}^{out}_{g_1Y_1+Y_2+Y_3}$, $\hat{S}^{out}_{X_1-X_2} + \hat{S}^{out}_{Y_1+Y_2+g_3Y_3}$, $\hat{S}^{out}_{X_1-X_3} + \hat{S}^{out}_{Y_1+g_2Y_2+Y_3}$, respectively. It can be seen that all of the three correlation values are below the limit 4, which shows that the correlations for three modes always exist in a wide frequency range, and the large correlations can be achieved at low analyzing frequency. Figure 3 shows the dependences of the three correlation spectra $\hat{S}^{out}_{X_2-X_3} + \hat{S}^{out}_{g_1Y_1+Y_2+Y_3}$, $\hat{S}^{out}_{X_1-X_2} + \hat{S}^{out}_{Y_1+Y_2+g_3Y_3}$, $\hat{S}^{out}_{X_1-X_3} + \hat{S}^{out}_{Y_1+g_2Y_2+Y_3}$ on pump parameter σ for $\xi=1$ and $\Omega'=0.8$. The inseparability criterion is satisfied for the pump power at threshold and below threshold. The best correlations for all three modes can be obtained roughly at $\sigma\approx 0.7$. The three correlation spectra $\hat{S}^{out}_{X_2-X_3} + \hat{S}^{out}_{g_1Y_1+Y_2+Y_3}$, $\hat{S}^{out}_{X_1-X_2}$

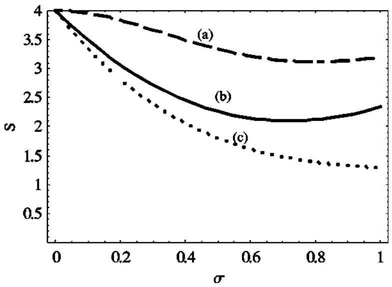

FIG. 3. The quantum correlation spectra of $\hat{S}^{out}_{X_2-X_3}+\hat{S}^{out}_{g_1Y_1+Y_2+Y_3}$ [curve (a)], $\hat{S}^{out}_{X_1-X_2}+\hat{S}^{out}_{Y_1+Y_2+g_3Y_3}$ [curve (b)], and $\hat{S}^{out}_{X_1-X_3}+\hat{S}^{out}_{Y_1+g_2Y_2+Y_3}$ [curve (c)] as functions of pump parameter $\sigma(\sigma=\kappa_1/\gamma_1)$, $\gamma_1=0.02$, $\gamma_2=0.03$, $\xi=1$, and $\Omega'=0.8$. $\sigma=1$ corresponds to the oscillation threshold.

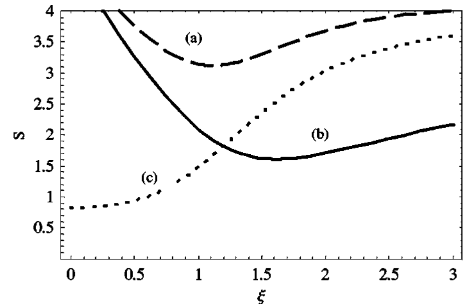

FIG. 4. The quantum correlation spectra as functions of pump parameter $\xi(\xi=\kappa_2/\gamma_1)$ for $\gamma_1=0.02$, $\gamma_2=0.03$, $\sigma=0.7$, and $\Omega'=0.8$. (a) $\hat{S}^{out}_{X_2-X_3}+\hat{S}^{out}_{g_1Y_1+Y_2+Y_3}$, (b) $\hat{S}^{out}_{X_1-X_2}+\hat{S}^{out}_{Y_1+Y_2+g_3Y_3}$, (c) $\hat{S}^{out}_{X_1-X_3}+\hat{S}^{out}_{Y_1+g_2Y_2+Y_3}$.

$+\hat{S}^{out}_{Y_1+Y_2+g_3Y_3}$, $\hat{S}^{out}_{X_1-X_3}+\hat{S}^{out}_{Y_1+g_2Y_2+Y_3}$ versus pump parameter ξ for $\sigma=0.7$ and $\Omega'=0.8$ are displayed in Fig. 4. The correlation degree between the two modes produced by parametric down conversion is largest for $\xi=0$, meanwhile the correlations between the other two modes do not exist. The correlation between the two modes produced by parametric down conversion is smaller and the correlations between the other two modes are larger with the increasing of pump parameter ξ. The largest entanglement is acquired for ξ around 1. And from both Figs. 2 and 3, we can see that the degree of correlation between the two modes produced by parametric down conversion is larger than correlations between the other two modes.

V. CONCLUSIONS

A scheme to generate tripartite entangled states based on cascaded nonlinear processes is proposed. The inseparability of the three output fields is verified theoretically, and the correlations of quadrature phases between three modes are discussed. The experimental realization of this scheme is desirable with a compact OPO in which two $\chi^{(2)}$ nonlinear mediums or a PPLN crystal is used. On the other hand, the entangled three output fields can be in different frequency. All of these properties make the scheme very significant for the application in quantum communication.

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