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Dependence of quantum correlations of twin beams on the pump finesse of an optical parametric oscillator

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Abstract
The dependence of quantum correlations of twin beams on the pump finesse of an optical parametric oscillator is studied via a semi-classical analysis. It is found that the phase-sum correlation of the output signal and idler beams from an optical parametric oscillator operating above threshold depends on the finesse of the pump field when the spurious pump phase noise generated inside the optical cavity and the excess noise of the input pump field are involved in the Langevin equations. The theoretical calculations can explain the previously experimental results quantitatively.

1. Introduction

As an important device in nonlinear optics, quantum optics and quantum information, the optical parametric oscillator (OPO) has been extensively studied and applied since the 1960s. In particular, it has become one of the most successful tools for the generation of entangled states of light in continuous variable (CV) quantum information systems [1]. Earlier, Reid and Drummond theoretically demonstrated that the Einstein–Podolsky–Rosen (EPR) entangled states can be generated from a nondegenerate OPO (NOPO) operating both above and below its threshold [2–5]. For the first time, CV EPR entanglement was experimentally realized by Ou *et al* with a NOPO below threshold in 1992 [6]. In recent years, the optical CV entangled states with quantum correlations of amplitude and phase quadratures of light fields produced from OPOs or NOPOs below threshold have been used in quantum information systems to realize unconditional quantum teleportation [7], quantum dense coding [8], quantum entanglement swapping [9], quantum key distribution [10, 11] and a variety of quantum communication networks [12–14]. Although the intensity-difference quantum correlation of twin beams from NOPO above threshold was measured experimentally and was effectively applied by several groups since the first experiment achieved by Heidmann *et al* in 1987 [15–20], their phase correlation had not been observed up to 2005 owing to technical difficulty in measuring the phase noise of twin beams with nondegenerate frequencies. In 2005, Laurat *et al* forced the NOPO to oscillate in a strict frequency-degenerate situation by inserting a $\lambda/4$ plate inside the optical cavity with a finesse of ≈102 for the pump laser, and observed a 3 dB phase-sum variance above the shot noise limit (SNL) [21]. Later, 0.8 dB phase correlation below the SNL between twin beams with a different frequency from a NOPO for a pump power of ≈4% above threshold was measured by Villar *et al* by scanning a pair of tunable ring analysis cavities [22]. In the experiment of [22], when the pump power was higher than 1.07 times of the threshold, the phase-sum noise of twin beams was larger than that of the SNL and thus the quantum correlation of the phase quadratures disappeared. Successively, our group detected the phase-sum correlation of the twin beams with two sets of unbalanced Match–Zehnder interferometers [23]. In this experiment, the phase-sum correlation of 1.05 dB lower that the SNL was recorded at a pump power of 230 mW which was almost twice the threshold of 120 mW. In 2006, the phase-sum correlation of 1.35 dB below the SNL between twin beams with a stable frequency difference was obtained with a doubly resonant NOPO without the resonance of the pump field [24].

To explain why the experimentally measured phase-sum correlation of twin beams was always lower than that predicted by theory and why it disappeared easily in some experimental systems, the influence of the excess noise of the pump field was theoretically and experimentally studied recently

[25–28]. In particular, it was discovered by Villar *et al* [28] that the spurious pump noise is generated inside the OPO cavity containing a nonlinear crystal, even for a shot-noise-limited input pump beam and without a parametric oscillation. They analyzed the physical origin of this phenomenon and assumed that the pump phase noise generated inside the cavity due to the effect of the intensity-dependent index of the refraction should be mainly responsible to the lower phase-sum correlation. Thus, they pointed out that the phase shifts accumulated inside the cavity with a lower finesse of pump laser should be smaller, hence the spurious noise should probably also be smaller. Very recently, we experimentally investigated the influence of the excess pump noise on the entanglement of twin beams by adding different excess phase noise on the input pump laser outside the cavity [29]. In this experiment, the noise spectra of the intensity difference and the phase sum of twin beams were measured at three analysis frequencies of 2 MHz, 5 MHz and 10 MHz under three different pump phase noises. The experimental results showed that the measured phase-sum correlations were still worse than those calculated with the theoretical formula in which the excess pump phase noise was involved. We considered this is because the possibly spurious phase noise of the pump laser produced inside the NOPO was not counted in the formula.

It has been proved that in the calculations of the quantum correlations between the output signal and idler from NOPO, the standard full quantum theory almost leads to the same results with those deduced with the semiclassical methods [30–33]. For conveniently comparing with experiments, in this paper, we present a semiclassical analysis of quantum correlations for the intensity difference and the phase sum of twin beams. A set of semiclassical Langevin equations involving the excess pump phase noise and the spurious phase noise produced inside the cavity are given. By solving the Langevin equations, the analytic expressions for the intensity-difference and the phase-sum noise spectra of twin beams are obtained. The expressions are compatible with those in [28, 30], if the excess pump phase noise and the spurious phase noise inside the cavity are not considered. All physical parameters in the expressions are experimentally measurable parameters; thus we can conveniently compare the theoretical calculations and the experimental results. The numerical calculations based on the expressions of the noise spectra show that the phase-sum noise spectrum of twin beams depends on the finesse of the pump laser. Our calculations prove quantitatively the physical analysis on this phenomenon in [28]. The published experimental results in [21–25] can be fit reasonably to the theoretical results if the appropriate parameters characterizing the spurious phase noise and the excess noise of input pump field are chosen.

2. Langevin equations involving the excess pump noise and intracavity spurious phase noise

The semiclassical motion equations for the pump mode α_0, signal mode α_1 and idler mode α_2 inside a triple resonant NOPO can be described by equations (1),

$$\begin{aligned}\tau\dot{\alpha}_1 &= -(\gamma+\mu)\alpha_1 + 2\chi\alpha_0\alpha_2^* + \sqrt{2\gamma}\alpha_1^{\rm in} + \sqrt{2\mu}\beta_1^{\rm in}\\ \tau\dot{\alpha}_2 &= -(\gamma+\mu)\alpha_2 + 2\chi\alpha_0\alpha_1^* + \sqrt{2\gamma}\alpha_2^{\rm in} + \sqrt{2\mu}\beta_2^{\rm in}\\ \tau\dot{\alpha}_0 &= -(\gamma_0+\mu_0)\alpha_1 - 2\chi\alpha_0\alpha_2^* + \sqrt{2\gamma_0}\alpha_1^{\rm in} + \sqrt{2\mu_0}\beta_0^{\rm in}\end{aligned} \tag{1}$$

which can be obtained by adding Gaussian white noise to classical electrodynamics [31]. In equations (1), τ is the round-trip time, which is assumed to be the same for all three fields. χ is the nonlinear coupling parameter. γ_i and μ_i $(i = 0, 1, 2)$ are the one pass losses associated with the coupling mirror of the cavity and with all other losses, respectively. Without losing generality, we assume that the losses of the signal and idler modes are balanced, thus we have $\gamma = \gamma_1 = \gamma_2$ and $\mu = \mu_1 = \mu_2$. $\alpha_i^{\rm in}$ and $\beta_i^{\rm in}$ are the incoming fields, associated with the coupling mirror and with the intracavity loss mechanism, respectively.

Solving equations (1), the stationary state values are obtained:

$$\begin{aligned}\bar{\alpha}_1^2 = \bar{\alpha}_2^2 &= \frac{\gamma_0'\gamma'}{4\chi^2}(\sigma-1)\\ \bar{\alpha}_0^2 &= \frac{\gamma'^2}{4\chi^2}\end{aligned} \tag{2}$$

where the loss parameters $\gamma' = \gamma + \mu$ and $\gamma_0' = \gamma_0 + \mu_0$. In the case above threshold, the pump parameter σ is larger than 1:

$$\sigma = \sqrt{\frac{P}{P_0}} = 2\sqrt{\frac{2\chi^2\gamma_0}{\gamma_0'^2\gamma'^2}}\bar{\alpha}_0^{\rm in} \tag{3}$$

where P, P_0 and $\bar{\alpha}_0^{\rm in}$ stand for the pump power, the threshold power of the NOPO and the mean amplitude of the input field, respectively.

In order to obtain the noise dynamic equations, a semiclassical method is used. We define the fluctuation operators $\delta\alpha_i$ and $\alpha_i = \bar{\alpha}_i + \delta\alpha_i, \bar{\alpha}_i$ is the mean value of α_i. Introducing the real and imaginary parts of the fields, we get the noise operators of the amplitude and phase quadratures:

$$\begin{aligned}p_i &= \delta\alpha_i + \delta\alpha_i^*\\ q_i &= -i(\delta\alpha_i - \delta\alpha_i^*)\end{aligned} \tag{4}$$

It is well known that the amplitude quadratures of the output twin beams are correlated and their phase quadratures are anticorrelated [30]. The amplitude-difference and the phase-sum noise operators of the twin beams are expressed by

$$p = \tfrac{1}{\sqrt{2}}(p_1 - p_2) \qquad q = \tfrac{1}{\sqrt{2}}(q_1 + q_2) \tag{5}$$

From equations (1) and using the input and output relation

$$p^{\rm out}(\omega) = \sqrt{2\gamma}\,p(\omega) - p^{\rm in}(\omega) \tag{6}$$

we obtain the correlation spectrum $p^{\rm out}(\omega)$ of the amplitude difference:

$$p^{\rm out}(\omega) = \frac{1}{2\gamma' + {\rm i}\omega\tau}[\sqrt{2\gamma}\,p^{\rm in}(\omega) + p'^{\rm in}(\omega)] \tag{7}$$

where ω is the analysis frequency; and $p^{\rm in}(\omega)$ are $p'^{\rm in}(\omega)$ are the vacuum noises associated with the cavity mirror and the intracavity loss respectively, both of which can be normalized to 1. We see that any parameter of the pump mode is not involved in the right side of equation (7). That is to say, the amplitude-difference noise of the output twin beams does not depend on the pump intensity and the pump noise. The noise power spectrum of the amplitude difference is given from equation (7):

$$S_p(\omega) = 1 - \frac{TT'}{T'^2 + \omega^2\tau^2} \tag{8}$$

where $T' = T + \delta$, $T = 2\gamma$ is the transmission coefficient of the output mirror and $\delta = 2\mu$ is the intracavity loss of twin beams in the NOPO. Equation (8) is totally the same with the result deduced in [30] which has been extensively applied.

However, for the phase sum we have to consider the influence of the pump noises since it cannot be eliminated. It has been pointed out in [28] that the phase noise of the pump field in a NOPO with a nonlinear crystal will increase. Thus the crystal in an optical cavity can be regarded as a gain medium for the phase noise of the pump field [28]. We introduce a gain factor ε in the Langevin equation for the phase quadrature q_0 to characterize the effect of the spurious phase noise which is continuously gained in the crystal. Substituting equations (4) and (5) into equations (1), we obtain the Langevin equations for the phase motion:

$$\begin{aligned}
\tau\dot{q}_1 &= -\gamma'(q_1 + q_2) + \sqrt{\gamma_0'\gamma'(\sigma - 1)}q_0 + \sqrt{2\gamma}q_1^{\text{in}} + \sqrt{2\mu}q_{\beta 1}^{\text{in}} \\
\tau\dot{q}_2 &= -\gamma'(q_2 + q_1) + \sqrt{\gamma_0'\gamma'(\sigma - 1)}q_0 + \sqrt{2\mu}q_2^{\text{in}} + \sqrt{2\mu}q_{\beta 2}^{\text{in}} \\
\tau\dot{q}_0 &= -\gamma_0' q_0 + \varepsilon q_0 - \sqrt{\gamma_0'\gamma'(\sigma - 1)}(q_1 + q_2) \\
&\quad + \sqrt{2\gamma_0}q_0^{\text{in}} + \sqrt{2\mu_0}q_{\beta 0}^{\text{in}}
\end{aligned} \tag{9}$$

where q_i^{in} and $q_{\beta i}^{\text{in}}$ $(i = 0, 1, 2)$ are the phase quadratures of the incoming fields associated with the cavity mirror and the intracavity loss mechanism respectively, both of which can be normalized to 1. Solving these equations, we get

$$\begin{aligned}
q^{\text{out}} &= \frac{\sqrt{2\gamma}\sqrt{2}\sqrt{\gamma_0'\gamma'(\sigma - 1)}(\sqrt{2\gamma_0}q_0^{\text{in}} + \sqrt{2\mu_0}q_{\beta 0}^{\text{in}}) + (\text{i}\omega\tau + \gamma_0' - \varepsilon)2\sqrt{\gamma\mu}q_\beta^{\text{in}}}{2\gamma'\gamma_0'\sigma - \omega^2\tau^2 - 2\gamma'\varepsilon + \text{i}\omega\tau(\gamma_0' + 2\gamma' - \varepsilon)} \\
&\quad + \frac{[\text{i}\omega\tau(2\gamma - 2\gamma' - \gamma_0' + \varepsilon) + 2\gamma_0'\gamma - 2\gamma\varepsilon - 2\gamma'\gamma_0'\sigma + \omega^2\tau^2 + 2\gamma'\varepsilon]q^{\text{in}}}{2\gamma'\gamma_0'\sigma - \omega^2\tau^2 - 2\gamma'\varepsilon + \text{i}\omega\tau(\gamma_0' + 2\gamma' - \varepsilon)}
\end{aligned} \tag{10}$$

Assuming that the excess noise of the input pump field at a frequency ω is $E(\omega)$, i.e., $\langle|\delta q^{\text{in}}(\omega)|^2\rangle = 1 + E(\omega)$, the noise power spectrum formula of the phase sum is obtained:

$$\begin{aligned}
S_q(\omega) &= 1 - \frac{TT'(T_0'^2 + 4\omega^2\tau^2) + 4T\varepsilon(2T'\varepsilon - TT_0'\sigma^2 - T\varepsilon - \delta\varepsilon)}{(T'T_0'\sigma - 2\omega^2\tau^2 - 2T\varepsilon)^2 + \omega^2\tau^2(T_0' + 2T' - 2\varepsilon)^2} \\
&\quad + \frac{2TT'T_0'T_0(\sigma - 1)}{(T'T_0'\sigma - 2\omega^2\tau^2 - 2T\varepsilon)^2 + \omega^2\tau^2(T_0' + 2T' - 2\varepsilon)^2}E(\omega)
\end{aligned} \tag{11}$$

where $T_0' = T_0 + \delta_0$, T_0 is the transmission coefficient of the input mirror of the NOPO and $\delta_0 = 2\mu_0$ is the intracavity loss of the pump laser in the NOPO. If there is no spurious noise inside the cavity ($\varepsilon = 0$), equation (11) goes to

$$\begin{aligned}
S_q(\omega) &= 1 - \frac{TT'T_0'^2 + 4TT'\omega^2\tau^2}{(T'T_0'\sigma - 2\omega^2\tau^2)^2 + \omega^2\tau^2(T_0' + 2T')^2} \\
&\quad + \frac{2TT'T_0'T_0(\sigma - 1)}{(T'T_0'\sigma - 2\omega^2\tau^2)^2 + \omega^2\tau^2(T_0' + 2T')^2}E(\omega)
\end{aligned} \tag{12}$$

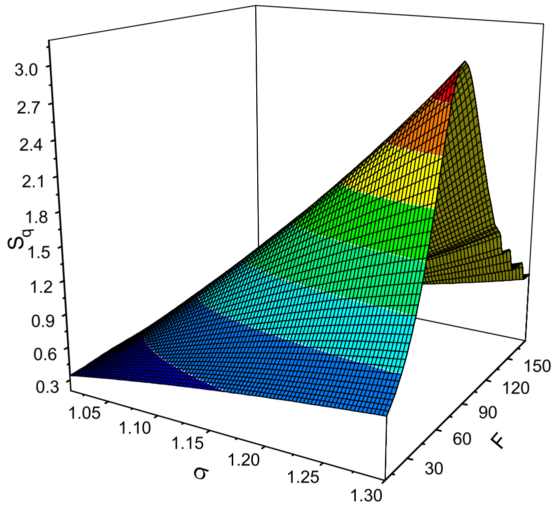

Figure 1. Phase-sum noise versus pump finesse and pump parameters for given $\varepsilon = 0.04$ and $E = 0$.

If the cavity finesse of the pump field is much lower than that of signals ($T_0' \gg T'$), equation (12) can be simplified as:

$$S_q(\omega) = 1 - \frac{TT'}{T'^2\sigma^2 + \omega^2\tau^2} + \frac{2TT'(\sigma - 1)}{T'^2 + \omega^2\tau^2}E(\omega) \tag{13}$$

which is the same with that in [29] where the spurious pump phase noise was not considered. If the pump light is an ideal coherent laser without the excess noise, i.e., $E(\omega) = 0$, equation (13) can be further simplified as:

$$S_q(\omega) = 1 - \frac{TT'}{T'^2\sigma^2 + \omega^2\tau^2} \tag{14}$$

This equation is totally equivalent to equation (25) in [30] which was deduced under the condition without the pump excess phase noise and the intracavity spurious pump phase noise. Thus the equation (11) is a general formula which is compatible with that obtained under the specific requirements.

3. Numerical analysis on the phase-sum correlation of twin beams

In a practical experimental system, the efficiency of the detector is always imperfect. Accounting for the detection efficiency of $\eta < 1$, the noise power spectrum equation (11) of the phase sum becomes

$$\begin{aligned}
S_q(\omega) &= 1 - \eta\frac{TT'(T_0'^2 + 4\omega^2\tau^2) + 4T\varepsilon(2T'\varepsilon - T'T_0'^2\sigma^2 - T\varepsilon - \delta\varepsilon)}{(T'T_0'\sigma - 2\omega^2\tau^2 - 2T'\varepsilon)^2 + \omega^2\tau^2(T_0' + 2T' - 2\varepsilon)^2} \\
&\quad + \eta\frac{2TT'T_0'T_0(\sigma - 1)}{(T'T_0'\sigma - 2\omega^2\tau^2 - 2T'\varepsilon)^2 + \omega^2\tau^2(T_0' + 2T' - 2\varepsilon)^2}.
\end{aligned} \tag{15}$$

From equation (15), we can see that the noise power spectrum of the phase sum depends on a variety of physical parameters. The 3D figures 1–3 show the dependences of the phase-sum noise spectrum (S_q) on the finesse (F) of the pump field as well as the pump parameter (σ) with $\varepsilon = 0.04$ and $E = 0$ (figure 1), the excess pump noise (E) with $\sigma = 1.1$ and $\varepsilon = 0$ (figure 2) and the intracavity spurious noise (ε) with $\sigma = 1.1$ and $E = 0$ (figure 3), respectively. The other parameters in the three

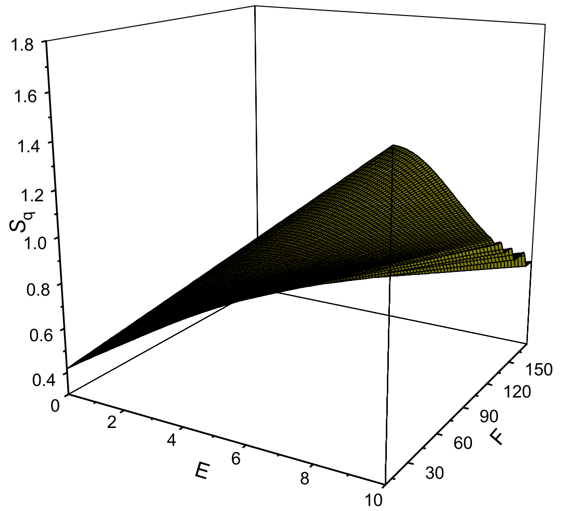

Figure 2. Phase-sum noise versus pump finesse and excess pump phase noise for given $\sigma = 1.1$ and $\varepsilon = 0$.

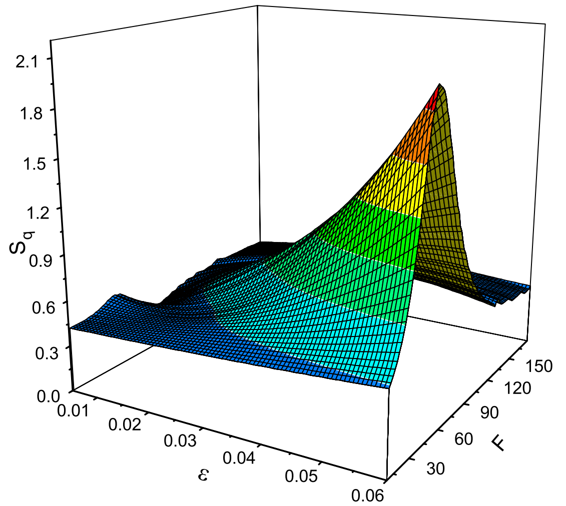

Figure 3. Phase-sum noise versus pump finesse and spurious pump phase noise for given $\sigma = 1.1$ and $E = 0$.

figures are the same: $T = 5\%$, $\delta = \delta_0 = 0.5\%$, $\eta = 90\%$ and $\omega\tau = 0.025$. It is obvious from figure 1 that the phase-sum noise increases along with increasing pump parameter (i.e. increase the pump power when other parameters are constant) and for the higher finesse F the noise increase is significant. For higher pump power and larger finesse, the quantum correlation of the phase-sum disappears, i.e., the phase-sum noises are larger than the normalized SNL ($S_q = 1$). The results can be used to explain the experimental phenomena in [28], in which a critical pump parameter for the phase-sum correlation was measured (see curve ii of figure 4). It is pointed out from figure 2 that for a given finesse the phase-sum noise increases when E increases. However, the influence of the excess pump phase noise on the phase-sum noise of twin beams monotonously degrades as the pump finesse increases if the intracavity spurious pump noise is not considered ($\varepsilon = 0$).

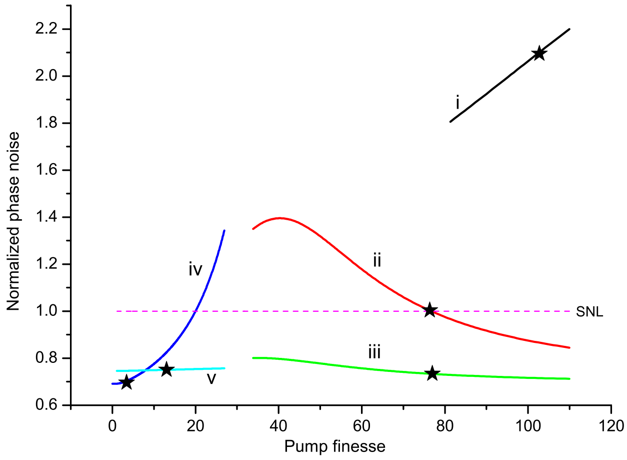

Figure 4. Phase-sum noise versus pump finesse for matching the experimental values. Curves i to v correspond to [21, 28] ($\sigma = 1.28$), [28] ($\sigma = 1.07$), [24, 29]. Stars stand for the experimental values.

The physical reason for the effect is that in the NOPO with low pump finesse, the transmission of the input mirror for the pump field is quite high, so the incoming phase noise together with the pump field is also larger if $E \neq 0$. Figure 3 shows that the phase-sum noise increases when ε increases. For a given finesse, the phase-sum correlation cannot be observed if the intracavity spurious phase noise is higher than a critical value even without the existence of the excess pump noise ($E = 0$). Due to the fact that the phase-sum noise depends on a variety of physical parameters of both pump field and subharmonic fields (see equation (15)), the dependence of the phase sum correlation on the finesse of the pump field is not identical for different NOPOs. The function curves of the phase-sum noise versus the pump finesse will change if other cavity parameters are changed. Generally, there is a maximum on the function curves if $\varepsilon \neq 0$ (see figures 1 and 3). At first the phase-sum noise increases when the pump finesse increases from zero due to the effect of the intracavity spurious noise. However, the intracavity intensity of the pump field is also raised when the pump finesse increases under a given pump power. Thus, the effective nonlinear conversion efficiency in the NOPO will be enhanced, which must result in the increase of the quantum correlation between the signal and idler modes. When the positive effect increasing the intracavity intensity of the pump field is superior to the negative effect gaining the spurious noise, the phase-sum noise will start to decrease if the pump finesse continuously increases. Comparing figures 2 and 3, it is obvious that the influence of the intracavity spurious noise (ε) the dependence of the phase-sum noise on the pump finesse is stronger than the influence of the excess phase noise of the input pump field (E).

4. Comparison of theoretical calculations and previous experiments

After considering the influence of E and ε, the theoretical calculations based on the real system parameters can match with the experimental results if appropriate values of ε and E are selected. In [21], the phase-sum correlation was not observed. We estimate E was probably higher in their system. If taking $\varepsilon = 0.015$ and $E = 5$, the function curve of the phase-sum noise versus the pump finesse according to the experimental parameters of [21] is shown in the curve i of figure 4. The star symbol denotes the experimental result (also for the other curves in figure 4), where the pump finesse is about 102 and the normalized phase-sum noise is about 2.1 corresponding to 3.2 dB above the SNL. The function curves for the experimental system of [28] are drawn as curves ii and iii in figure 4. Since they experimentally proved that the excess noise of the input pump field can be neglected at the analysis frequency of $f = \omega/(2\pi) = 27$ MHz [28], we take $E = 0$ and $\varepsilon = 0.06$. The curves ii and iii correspond to $\sigma = 1.28$ and $\sigma = 1.06$ respectively according to their experimental measurements. Under the low pump power ($\sigma = 1.06$), the phase-sum correlation always exists (all phase-sum noises are smaller than those of the SNL). But under the higher pump power ($\sigma = 1.28$), the phase-sum noises are larger than those of the SNL in the range of the pump finesses from 17 to 76. The theoretical curves are perfectly matched with the experimental results. The star on curve ii corresponds to the critical pump power for the phase-sum correlation in their experiment. In [24], the pump field did not resonate, so we can consider the pump finesse was very low (close to 1). The curve iv is drawn with the parameters of the system of [24] where $\varepsilon = 0.06$ and $E = 5.6$ are taken for matching the experimentally measured phase-sum noise of 0.69 corresponding to 1.6 dB below the SNL. For our experimental system of [29] with the low finesse of $\approx$12, if taking $\varepsilon = 0.005$ and $E = 0.06$, the measured phase-sum noise of 0.75 (1.25 dB below the SNL) will perfectly match with the theoretical curve (see curve v).

Although the values of ε and E in figure 4 are not experimentally measured, these estimated values are reasonable. The excess pump noise depends on the quality of the pump laser, thus it can change in a large range. In figure 2, the values of E are taken from 0 to 10. Generally, the pump noise can be degraded by means of some technical implements, such as adding a mode-cleaner in the path of the pump laser to filter the excess pump noise. For matching the experimental results in [28], we take $E = 0$ since they experimentally proved that the excess noise can be neglected at $f = 27$ MHz. The spurious noise depends on the quality of the non-linear crystal and it would be quite small for a qualified commercial crystal. So the taken values of ε are a few orders smaller than those of E (0.005–0.06). At least these calculations tell us that the previous experimental results on twin-beam generations from NOPOs above threshold achieved by different groups can be explained by means of the semiclassical theory if the intracavity spurious phase noise and the excess phase noise of the input pump field are involved in the Langevin equations.

5. Conclusion

By solving the semiclassical Langevin equations involving the intracavity spurious pump phase noise and the excess noise of the input pump field, we obtained the expressions of the intensity-difference and the phase-sum noise spectra between the output signal and idler modes from a NOPO above threshold. The phase-sum quantum correlation of twin beams not only depends on the cavity parameters of the subharmonic field, but also depends on the finesse and the noises of the pump field. In particular, the phase-sum noise significantly increases when the spurious pump noise produced inside the cavity with a nonlinear crystal becomes higher. The dependence of the phase-sum correlation of twin beams on the system parameters of the NOPO is more complex. Our calculations provide a useful reference for the design of the NOPO serving as a source of optical entangled states. The expressions of the noise spectra presented in this paper are compatible with those obtained previously under the condition without considering the pump noises if taking $\varepsilon = 0$ and $E = 0$. Using the extended expressions, the previously experimental results can be reasonably explained if appropriate parameters characterizing pump noises are applied.

The NOPO above threshold is a helpful device to produce bright tunable entanglement optical beams which could be used to transfer quantum information from one frequency to another and to implement the quantum memory. The entanglement of twin beams with directly detectable intensity can be measured with a pair of analysis cavities [22], or unbalance M–Z interferometers [23, 29] without the need of a local oscillator, thus it might be conveniently applied to realize the quantum key distribution protocols based on entangled states of light [11, 34, 35]. Clearing the excess pump phase noise, minimizing the intracavity spurious phase noise of the pump field and selecting appropriate parameters of optical cavity are the key factors for obtaining twin beams with higher phase-sum correlation. In the design of the NOPO optical system, to add a mode cleaner in the way of the pump laser is necessary to reduce the excess noise. For a given nonlinear crystal with larger ε, we should choose the NOPO with lower pump finesse to decrease the effect of the intracavity phase noise gain. If ε is very small, we may appropriately increase the pump finesse to reduce the threshold pump power and raise the effective nonlinear conversion efficiency inside the NOPO.

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