

## Tripartite entanglement from the cavity with second-order harmonic generation

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In this paper, tripartite entanglement among the two pump fields and the second-order harmonic field is established in the process of type-II second-order harmonic generation (SHG) with a triply resonant optical cavity below threshold. A sufficient inseparability criterion for continuous-variable tripartite entanglement proposed by van Loock and Furusawa is used to evaluate the degree of the quadrature-phase-amplitude correlations between the three modes. The dependence of the entanglement on the pump parameter and analysis frequency is also discussed. It is shown that the best entanglement appears at the appropriate pump power and analysis frequency. These three entangled states with different frequencies generated directly from a simple SHG process make the scheme useful for the application in quantum communication and network.

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Quantum entanglement is considered the most important resource for future quantum-information technology. With the development of science and technology, entanglement between more than two systems is going to be the key ingredient for advanced multiparty quantum communication of quantum teleportation networks [1], telecloning [2,3], and controlled dense coding [4,5]. Multipartite entangled light beams with different frequency will be more important than same frequency, since it will be necessary for the storage and communication of quantum information in the nodes of quantum teleportation and networks, e.g., atom clouds [6], quantum dots [7], trapped ions [8], air windows, and fiber windows, with which the different resonance frequencies are needed to carry out such tasks.

Both the generation of multipartite entanglement of continuous variables (CVs) [1,5,9] and discrete variables (DVs) [10–12] were developed theoretically and experimentally. A multipartite entangled state of CVs is an inseparable multimode squeezed state that tends toward a Greenberger-Horne-Zeilinger (GHZ) state in the limit of infinite squeezing, the quantum optical implementation of multipartite entanglement of CVs is thus strongly based on the process of squeezed state generation. First, the tripartite entanglement of CVs was implemented by combining squeezed states emitted by optical parametric amplifiers (OPAs) on beam splitters [1,5,9,13,14]. This can only lead to entangled beams with the same frequencies, because the quantum interference at the beam splitters requires the same frequency and polarization between the two optical paths. Thus the development of multipartite entanglement via only the nonlinear optical process has drawn much attention recently [15–17], and it might supply the way to obtain the multipartite entanglement with different frequencies. The generation of full inseparable three-mode entangled states by interlinked interactions in a  $\chi^{(2)}$  medium was addressed [15], and the direct generation of tripartite entanglement in an optical cavity via parametric down-conversion and sum-frequency generation was theoretically proposed [16]. Very recently, the tripartite pump-signal-idler entanglement in the above-threshold nondegenerate optical parametric oscillator (OPO) was theoretically discussed [18], after which the first observation of three-color optical quantum correlations produced by above-

threshold OPO was demonstrated [19]. Furthermore, experimental results approaching three-color quantum entanglement were also investigated [20].

In the above work, the tripartite entanglements with different frequencies are generated through below- or above-threshold OPO. Apart from the parametric down-conversion in OPO, second-order harmonic generation (SHG) is another interesting and easily accomplished nonlinear process. It also yields a nonclassical light field. It has been demonstrated experimentally that the pump fields reflected from an optical cavity for intracavity SHG are squeezed because of the cascaded nonlinear interaction between subharmonic and harmonic fields inside the cavity [21,22], therefore the entanglement generation with SHG [23–25] has also attracted interest. In Refs. [24,25], it was shown that the entanglement between subharmonic modes from type-II SHG can be established, and the perfect entanglement can be accessed by triple resonant SHG. The different wavelength bipartite entanglement between a fundamental field and its second-order harmonic field was also suggested and experimentally demonstrated [26,27]. Based on these analyses, we first propose a scheme to produce tripartite entangled states with different frequencies via the SHG process. The tripartite quantum entanglement among the two subharmonic modes and the harmonic mode in a single-ended optical cavity of type-II second-order harmonic generation under the condition of below the intracavity OPO threshold is obtained. The dependences of quantum correlations of the amplitude and phase quadratures among the three modes on the pump parameter and the analysis frequencies are also theoretically analyzed. The inseparability of the three modes is confirmed by numerical calculation.

The sketch of SHG is shown in Fig 1. The system consists of a single-sided cavity with three modes  $\hat{a}_0, \hat{a}_1, \hat{a}_2$ . We consider the SHG process in the triply resonating optical cavity with a nonlinear type-II phase matching  $\chi^{(2)}$  crystal. The triple resonance means that the two subharmonic pump modes  $\hat{a}_1, \hat{a}_2$  and the harmonic mode  $\hat{a}_0$  simultaneously resonate in the cavity. Under the ideal case with perfect phase matching and without any detuning, the equations of motion for a single-ended cavity with one mirror used for the input and output coupler can be expressed as [28]

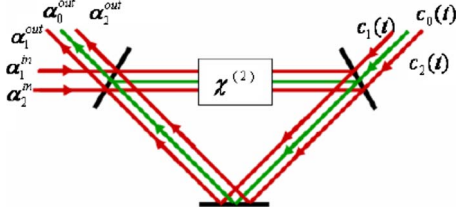


FIG. 1. (Color online) Sketch of SHG.

$$\begin{aligned}
\tau \dot{\hat{a}}_0(t) &= -\gamma_0 \hat{a}_0(t) - \chi \hat{a}_1(t) \hat{a}_2(t) + \sqrt{2\gamma_{b0}} \hat{a}_0^{\text{in}}(t) e^{i\phi_0} \\
&\quad + \sqrt{2\gamma_{c0}} \hat{c}_0(t), \\
\tau \dot{\hat{a}}_1(t) &= -\gamma_1 \hat{a}_1(t) + \chi \hat{a}_2^{\dagger}(t) \hat{a}_0(t) + \sqrt{2\gamma_{b1}} \hat{a}_1^{\text{in}}(t) e^{i\phi_1} \\
&\quad + \sqrt{2\gamma_{c1}} \hat{c}_1(t), \\
\tau \dot{\hat{a}}_2(t) &= -\gamma_2 \hat{a}_2(t) + \chi \hat{a}_1^{\dagger}(t) \hat{a}_0(t) + \sqrt{2\gamma_{b2}} \hat{a}_2^{\text{in}}(t) e^{i\phi_2} \\
&\quad + \sqrt{2\gamma_{c2}} \hat{c}_2(t). \tag{1}
\end{aligned}$$

Here  $\hat{a}_0$ ,  $\hat{a}_1$ , and  $\hat{a}_2$  are the amplitude operators of harmonic and two subharmonic pump fields inside the cavity, respectively.  $\hat{a}_i^{\text{in}}(i=0,1,2)$  denotes the input amplitude operators of the three fields. The roundtrip time  $\tau$  in the cavity is assumed to be the same for all three fields.  $\chi$  is the effective nonlinear coupling parameter, and is proportional to the second-order susceptibility  $\chi^{(2)}$  of the crystal. The total loss parameter for each mode is  $\gamma_i = \gamma_{bi} + \gamma_{ci}$  ( $i=0,1,2$ ), where  $\gamma_{bi}$  is related to the amplitude reflection coefficients  $r_i$  and amplitude transmission coefficients  $t_i$  of the coupling mirror of the optical cavity by the formula

$$r_i = 1 - \gamma_{bi},$$

$$t_i = \sqrt{2\gamma_{bi}},$$

and  $\gamma_{ci}$  represents the extra intracavity loss parameter.  $c_i(t)$  is the vacuum noise term corresponding to intracavity loss.

For simplicity, assuming the two pump modes have the same positive real amplitude  $\beta$  and zero initial phase ( $e^{i\phi_0} = e^{i\phi_1} = e^{i\phi_2} = 1$ ), and the cavity transmission factor and the extra losses for the two pump fields are also assumed to be the same, then

$$\gamma_1 = \gamma_2 = \gamma, \quad \gamma_{b1} = \gamma_{b2} = \gamma_b, \quad \gamma_{c1} = \gamma_{c2} = \gamma_c. \tag{2}$$

The steady-state equation of Eq. (1) is then obtained to be [29]

$$\begin{aligned}
-\gamma_0 \alpha_0 - \chi \alpha_1 \alpha_2 &= 0, \\
-\gamma \alpha_1 + \chi \alpha_2^* \alpha_0 + \sqrt{2\gamma_b} \beta &= 0, \\
-\gamma \alpha_2 + \chi \alpha_1^* \alpha_0 + \sqrt{2\gamma_b} \beta &= 0, \tag{3}
\end{aligned}$$

where  $\alpha_0, \alpha_1, \alpha_2$  are the steady-state amplitudes of the three intracavity modes of  $\hat{a}_0, \hat{a}_1$ , and  $\hat{a}_2$ . Equation (3) shows that both  $\alpha_1$  and  $\alpha_2$  are real numbers. The oscillation threshold  $\beta^{\text{th}}$  and pump parameter  $\sigma$  are expressed by

$$\beta^{\text{th}} = \sqrt{2\gamma^3 \gamma_0 / \chi^2 \gamma_b},$$

$$\sigma = \beta / \beta^{\text{th}}, \tag{4}$$

The steady-state solution of Eq. (3) below the threshold ( $\sigma < 1$ ) is given by

$$\alpha_1 = \alpha_2 = \alpha = \sqrt{\gamma \gamma_0 \sigma'} / \chi, \quad \alpha_0 = -\gamma \sigma'^2 / \chi, \tag{5}$$

where

$$\sigma' = \left( \sigma + \sqrt{\sigma^2 + \frac{1}{27}} \right)^{1/3} - \frac{1}{3} \left( \sigma + \sqrt{\sigma^2 + \frac{1}{27}} \right)^{-1/3}. \tag{6}$$

Here we just consider the case of below threshold.

The dynamics of the quantum fluctuations can be described by linearizing the classical equations of motion around the stationary state. Setting

$$\hat{a}_i(t) = \alpha_i + \delta \hat{a}_i(t) \quad (i=0,1,2),$$

$$\hat{a}_0^{\text{in}}(t) = \delta \hat{a}_0^{\text{in}}(t),$$

$$\hat{a}_i^{\text{in}}(t) = \beta + \delta \hat{a}_i^{\text{in}}(t) \quad (i=1,2). \tag{7}$$

Substituting Eqs. (7) into Eqs. (1), we obtain the fluctuation dynamics equations after Fourier transformation,

$$\begin{aligned}
(i\omega\tau + \gamma_0) \delta \hat{a}_0(\omega) &= -[\chi \alpha_2 \delta \hat{a}_1(\omega) + \chi \alpha_1 \delta \hat{a}_2(\omega)] \\
&\quad + \sqrt{2\gamma_{b0}} \delta \hat{a}_0^{\text{in}}(\omega) + \sqrt{2\gamma_{c0}} \hat{c}_0^{\text{in}}(\omega),
\end{aligned}$$

$$\begin{aligned}
(i\omega\tau + \gamma) \delta \hat{a}_1(\omega) &= \chi \alpha_0 \delta \hat{a}_2^{\dagger}(-\omega) + \chi \alpha_2 \delta \hat{a}_0(\omega) + \sqrt{2\gamma_b} \delta \hat{a}_1^{\text{in}}(\omega) \\
&\quad + \sqrt{2\gamma_c} \hat{c}_1^{\text{in}}(\omega),
\end{aligned}$$

$$\begin{aligned}
(i\omega\tau + \gamma) \delta \hat{a}_2(\omega) &= \chi \alpha_0 \delta \hat{a}_1^{\dagger}(-\omega) + \chi \alpha_1 \delta \hat{a}_0(\omega) + \sqrt{2\gamma_b} \delta \hat{a}_2^{\text{in}}(\omega) \\
&\quad + \sqrt{2\gamma_c} \hat{c}_2^{\text{in}}(\omega), \tag{8}
\end{aligned}$$

where  $\omega$  is the analysis frequency.

Under the condition of below threshold, and using the definitions of the amplitude and phase quadratures,  $\hat{X} = \hat{a} + \hat{a}^{\dagger}$  and  $\hat{Y} = (\hat{a} - \hat{a}^{\dagger})/i$ , the quadrature components of the harmonic field and two pump fields inside the cavity can be written as

$$\delta \hat{X}_0(\omega) = \frac{1}{D} [(i\omega\tau + \gamma + \gamma \sigma'^2) \hat{Q}_{X_0} - \sqrt{\gamma \gamma_0} \sigma' (\hat{Q}_{X_1} + \hat{Q}_{X_2})],$$

$$\begin{aligned}
\delta \hat{X}_1(\omega) &= \frac{\sqrt{\gamma \gamma_0} \sigma'}{D(i\omega\tau + \gamma - \gamma \sigma'^2)} (i\omega\tau + \gamma - \gamma \sigma'^2) \hat{Q}_{X_0} \\
&\quad + \frac{1}{D(i\omega\tau + \gamma - \gamma \sigma'^2)} [(i\omega\tau + \gamma_0)(i\omega\tau + \gamma) \\
&\quad + \gamma \gamma_0 \sigma'^2] \hat{Q}_{X_1} - \frac{1}{D(i\omega\tau + \gamma - \gamma \sigma'^2)} [\gamma \sigma'^2 (i\omega\tau + \gamma_0) \\
&\quad + \gamma \gamma_0 \sigma'^2] \hat{Q}_{X_2},
\end{aligned}$$

$$\begin{aligned}
\delta\hat{X}_2(\omega) &= \frac{\sqrt{\gamma\gamma_0}\sigma'}{D(i\omega\tau + \gamma - \gamma\sigma'^2)}(i\omega\tau + \gamma - \gamma\sigma'^2)\hat{Q}_{X_0} \\
&\quad - \frac{1}{D(i\omega\tau + \gamma - \gamma\sigma'^2)}[\gamma\sigma'^2(i\omega\tau + \gamma_0) + \gamma\gamma_0\sigma'^2]\hat{Q}_{X_1} \\
&\quad + \frac{1}{D(i\omega\tau + \gamma - \gamma\sigma'^2)}[(i\omega\tau + \gamma_0)(i\omega\tau + \gamma) \\
&\quad + \gamma\gamma_0\sigma'^2]\hat{Q}_{X_2}, \\
\delta\hat{Y}_0(\omega) &= \frac{1}{B}[(i\omega\tau + \gamma - \gamma\sigma'^2)\hat{Q}_{Y_0} - \sqrt{\gamma\gamma_0}\sigma'(\hat{Q}_{Y_1} + \hat{Q}_{Y_2})], \\
\delta\hat{Y}_1(\omega) &= \frac{1}{B(i\omega\tau + \gamma + \gamma\sigma'^2)}[\sqrt{\gamma\gamma_0}\sigma'(i\omega\tau + \gamma + \gamma\sigma'^2)]\hat{Q}_{Y_0} \\
&\quad + \frac{1}{B(i\omega\tau + \gamma + \gamma\sigma'^2)}[(i\omega\tau + \gamma_0)(i\omega\tau + \gamma) \\
&\quad + \gamma\gamma_0\sigma'^2]\hat{Q}_{Y_1} + \frac{1}{B(i\omega\tau + \gamma + \gamma\sigma'^2)}[\gamma\sigma'^2(i\omega\tau + \gamma_0) \\
&\quad - \gamma\gamma_0\sigma'^2]\hat{Q}_{Y_2}, \\
\delta\hat{Y}_2(\omega) &= \frac{1}{B(i\omega\tau + \gamma + \gamma\sigma'^2)}[\sqrt{\gamma\gamma_0}\sigma'(i\omega\tau + \gamma + \gamma\sigma'^2)]\hat{Q}_{Y_0} \\
&\quad + \frac{1}{B(i\omega\tau + \gamma + \gamma\sigma'^2)}[\gamma\sigma'^2(i\omega\tau + \gamma_0) - \gamma\gamma_0\sigma'^2]\hat{Q}_{Y_1} \\
&\quad + \frac{1}{B(i\omega\tau + \gamma + \gamma\sigma'^2)}[(i\omega\tau + \gamma_0)(i\omega\tau + \gamma) \\
&\quad + \gamma\gamma_0\sigma'^2]\hat{Q}_{Y_2}, \tag{9}
\end{aligned}$$

where

$$D = (i\omega\tau + \gamma_0)(i\omega\tau + \gamma + \gamma\sigma'^2) + 2\gamma\gamma_0\sigma'^2,$$

$$B = (i\omega\tau + \gamma_0)(i\omega\tau + \gamma - \gamma\sigma'^2) + 2\gamma\gamma_0\sigma'^2,$$

$$\hat{Q}_{X(Y)_0} = \sqrt{2\gamma_{b0}}\delta\hat{X}(\hat{Y})_0^{\text{in}}(\omega) + \sqrt{2\gamma_{c0}}\delta\hat{X}(\hat{Y})_{c0}^{\text{in}}(\omega),$$

$$\hat{Q}_{X(Y)_i} = \sqrt{2\gamma_b}\delta\hat{X}(\hat{Y})_i^{\text{in}}(\omega) + \sqrt{2\gamma_c}\delta\hat{X}(\hat{Y})_{ci}^{\text{in}}(\omega), \quad i = 1, 2.$$

Then the output amplitude and phase quadratures of the cavity can be obtained using the boundary condition [30],

$$\delta\hat{X}_0^{\text{out}}(\omega) = \sqrt{2\gamma_{b0}}\delta\hat{X}_0(\omega) - \delta\hat{X}_0^{\text{in}}(\omega),$$

$$\delta\hat{X}_i^{\text{out}}(\omega) = \sqrt{2\gamma_b}\delta\hat{X}_i(\omega) - \delta\hat{X}_i^{\text{in}}(\omega), \quad i = 1, 2,$$

$$\delta\hat{Y}_0^{\text{out}}(\omega) = \sqrt{2\gamma_{b0}}\delta\hat{Y}_0(\omega) - \delta\hat{Y}_0^{\text{in}}(\omega),$$

$$\delta\hat{Y}_i^{\text{out}}(\omega) = \sqrt{2\gamma_b}\delta\hat{Y}_i(\omega) - \delta\hat{Y}_i^{\text{in}}(\omega), \quad i = 1, 2. \tag{10}$$

The tripartite entanglement among the two subharmonic modes and the harmonic mode are denoted by the correlations of quantum fluctuations of their amplitude and phase quadratures. To quantify the degree of correlations, we introduce the correlation spectra of the total amplitude quadratures of three-mode and relative phase quadratures proposed by van Loock and Furusawa [31],

$$\begin{aligned}
S_{\hat{Y}_1^{\text{out}}(\omega) - \hat{Y}_2^{\text{out}}(\omega)}^{\text{out}} &= \langle [\delta\hat{Y}_1^{\text{out}}(\omega) - \delta\hat{Y}_2^{\text{out}}(\omega)][\delta\hat{Y}_1^{\text{out}}(\omega) \\
&\quad - \delta\hat{Y}_2^{\text{out}}(\omega)]^+ \rangle, \\
S_{\hat{Y}_0^{\text{out}}(\omega) + \hat{Y}_1^{\text{out}}(\omega)}^{\text{out}} &= \langle [\delta\hat{Y}_0^{\text{out}}(\omega) + \delta\hat{Y}_1^{\text{out}}(\omega)][\delta\hat{Y}_0^{\text{out}}(\omega) \\
&\quad + \delta\hat{Y}_1^{\text{out}}(\omega)]^+ \rangle, \\
S_{\hat{Y}_0^{\text{out}}(\omega) + \hat{Y}_2^{\text{out}}(\omega)}^{\text{out}} &= \langle [\delta\hat{Y}_0^{\text{out}}(\omega) + \delta\hat{Y}_2^{\text{out}}(\omega)][\delta\hat{Y}_0^{\text{out}}(\omega) \\
&\quad + \delta\hat{Y}_2^{\text{out}}(\omega)]^+ \rangle, \\
S_{\hat{X}_1^{\text{out}} + \hat{X}_2^{\text{out}} - g_1\hat{X}_0^{\text{out}}}^{\text{out}} &= \langle [\delta\hat{X}_1^{\text{out}}(\omega) + \delta\hat{X}_2^{\text{out}}(\omega) - g_1\delta\hat{X}_0^{\text{out}}(\omega)] \\
&\quad \times [\delta\hat{X}_1^{\text{out}}(\omega) + \delta\hat{X}_2^{\text{out}}(\omega) - g_1\delta\hat{X}_0^{\text{out}}(\omega)]^+ \rangle, \\
S_{\hat{X}_1^{\text{out}} + g_2\hat{X}_2^{\text{out}} - \hat{X}_0^{\text{out}}}^{\text{out}} &= \langle [\delta\hat{X}_1^{\text{out}}(\omega) + g_2\delta\hat{X}_2^{\text{out}}(\omega) - \delta\hat{X}_0^{\text{out}}(\omega)] \\
&\quad \times [\delta\hat{X}_1^{\text{out}}(\omega) + g_2\delta\hat{X}_2^{\text{out}}(\omega) - \delta\hat{X}_0^{\text{out}}(\omega)]^+ \rangle, \\
S_{g_3\hat{X}_1^{\text{out}} + \hat{X}_2^{\text{out}} - \hat{X}_0^{\text{out}}}^{\text{out}} &= \langle [g_3\delta\hat{X}_1^{\text{out}}(\omega) + \delta\hat{X}_2^{\text{out}}(\omega) - \delta\hat{X}_0^{\text{out}}(\omega)] \\
&\quad \times [g_3\delta\hat{X}_1^{\text{out}}(\omega) + \delta\hat{X}_2^{\text{out}}(\omega) - \delta\hat{X}_0^{\text{out}}(\omega)]^+ \rangle. \tag{11}
\end{aligned}$$

To verify the quantum entanglement of the three modes, the sufficient inseparability criteria for CV tripartite entanglement proposed by van Loock and Furusawa [31] are used,

$$S_1^{\text{out}} = \langle \delta^2(\hat{Y}_1^{\text{out}} - \hat{Y}_2^{\text{out}}) \rangle + \langle \delta^2(\hat{X}_1^{\text{out}} + \hat{X}_2^{\text{out}} - g_1\hat{X}_0^{\text{out}}) \rangle \leq 4,$$

$$S_2^{\text{out}} = \langle \delta^2(\hat{Y}_0^{\text{out}} + \hat{Y}_1^{\text{out}}) \rangle + \langle \delta^2(\hat{X}_1^{\text{out}} + g_2\hat{X}_2^{\text{out}} - \hat{X}_0^{\text{out}}) \rangle \leq 4,$$

$$S_3^{\text{out}} = \langle \delta^2(\hat{Y}_0^{\text{out}} + \hat{Y}_2^{\text{out}}) \rangle + \langle \delta^2(g_3\hat{X}_1^{\text{out}} + \hat{X}_2^{\text{out}} - \hat{X}_0^{\text{out}}) \rangle \leq 4, \tag{12}$$

where  $g_1$ ,  $g_2$ , and  $g_3$  are scaling factors. The quantities  $S_1^{\text{out}}$ ,  $S_2^{\text{out}}$ , and  $S_3^{\text{out}}$  can be minimized by choosing the adjustable scaling factors. The correlation spectra of these three quantities with optimized choice of the three parameters  $g_i$  are plotted in Figs. 2 and 3. Figure 2 shows the minimum correlation spectra of  $S_1^{\text{out}}$ ,  $S_2^{\text{out}}$ , and  $S_3^{\text{out}}$  as a function of normalized pump parameter  $\sigma = \beta/\beta^{\text{th}}$ . It can be seen that all of the three correlation values are lower than the quantum limit 4 when pump power is below the threshold, i.e., the tripartite entanglement for three modes can be obtained when  $\sigma \leq 1$ .

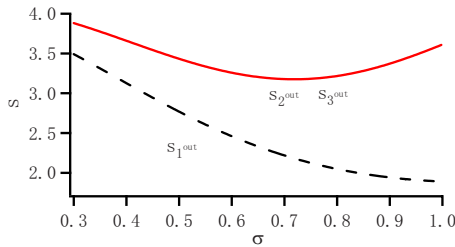


FIG. 2. (Color online) The quantum correlation spectra versus normalized pump parameter ( $\sigma = \beta/\beta^{\text{th}}$ ) with  $\gamma=0.02$ ,  $\gamma_b=0.018$ ,  $\gamma_0=0.1$ ,  $\gamma_{b0}=0.09$ , and  $\Omega=1$ .

The best correlations for  $S_2^{\text{out}}$  and  $S_3^{\text{out}}$  can be accessed when the pump power is near the threshold (roughly at  $\sigma=0.7$ ) but not at the threshold. Figure 3 shows the dependence of correlation spectra of  $S_1^{\text{out}}$ ,  $S_2^{\text{out}}$ , and  $S_3^{\text{out}}$  on the normalized analyzing frequency ( $\Omega = \omega\tau/\gamma$ ) when we take the optimized pump power  $\sigma=0.7$ . It is obvious that all of the correlations (i.e., the tripartite entanglement) always exist in a wide range of analysis frequency.

As is seen in both Figs. 2 and 3, the correlation spectra of  $S_2^{\text{out}}$  and  $S_3^{\text{out}}$  are always equal, i.e.,  $S_2^{\text{out}}=S_3^{\text{out}}$ , and they are always worse than the correlation spectrum of  $S_1^{\text{out}}$ , which means that the correlations between the second-order harmonic field and any one of the two pump fields are symmetric as long as the cavity parameters for the two fields are equal. It can easily understood that both of the pump fields are involved in the generation of second-order harmonic field simultaneously. Meanwhile, because of their simultaneous effect of the pump fields on the SHG, the correlation between them is must better than the correlation between one of them and the generated second-order harmonic field. According to the above discussion, the inseparability criterion is

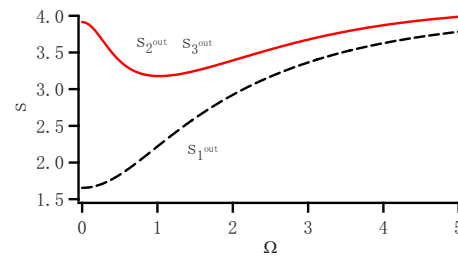


FIG. 3. (Color online) The quantum correlation spectra versus normalized frequency with  $\gamma=0.02$ ,  $\gamma_b=0.018$ ,  $\gamma_0=0.1$ ,  $\gamma_{b0}=0.09$ , and  $\sigma=0.7$ .

satisfied for the three modes of SHG when the pump power is below threshold.

In conclusion, the theoretical discussion proved that the optical cavity with SHG can directly yield tripartite entangled light beams when it operates below threshold. We also calculated the dependences of the resultant correlation spectra on the parameters of the SHG system. Tripartite entanglement with different frequencies for pump fields and second-order harmonic field may be useful in constructing quantum communication networks. The experimental realization of this system is much simpler than the proposals based on cascaded nonlinearities and combined nonlinearities.

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