Bright quadripartite continuous variable entanglement from coupled intracavity nonlinearities

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The continuous variable quadripartite entanglement properties of the output fields by the coupled intracavity parametric downconversions processes are analyzed theoretically. In the above-threshold region, it shows that the four output lights are multicolored entangled beams in separable locations with four-mode amplitude quadratures correlation and relative phase quadratures correlation. © 2010 Optical Society of America

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1. INTRODUCTION

Continuous variable (CV) multipartite entanglement is crucial and useful for quantum information, especially for quantum teleportation networks [1], telecloning [2,3], and controlled dense coding [4,5]. Lots of theoretical and experimental work has been concentrated on the preparation of multipartite entanglement sources in recent years. Until now, the schemes to generate three-mode or multimode entanglement have been reported in many papers in which this type of entanglement can be prepared by combining squeezed beams with linear optics [5–9] or directly from cascaded nonlinear processes [10–16]. In this paper, we present CV quadripartite entanglement by coupled parametric downconversions (CPDC) in a cavity, which is called coupled parametric oscillators in [17,18].

The device of coupled parametric oscillators consists of an optical cavity and two parallel nonlinear waveguides with a $\chi^{(2)}$ component or a nonlinear crystal where the coupling is realized by evanescent overlaps of the intracavity modes. This type of nonlinear optical coupler has been investigated both theoretically and experimentally [19,20]. In 2003, J. Herec studied the bipartite entanglement from coupled spontaneous parametric downconversions in the traveling wave configuration [21]. Then M. Bache introduced quantum optical dimer, where coupled second-harmonic generation (SHG) took place in a Fabry–Perot cavity [22]. They predicted intensity correlations between the modes outside the cavity. Later, the M. K. Olsen group analyzed entanglement and the Einstein–Podolsky–Rosen paradox between the output modes from coupled intracavity downconverters both in the below-threshold region [17] and the above-threshold region [18]. The system is all-integrated, which make it a compact source of entangled light. In addition, spatial separation of the entangled modes could avoid the division by beam splitters before measurement as compared to collinear entangled beams.

To overcome the exponential decay of the communication rate with the distance, the protocol of quantum repeater has been proposed in quantum telecommunication [23]. The idea of a quantum repeater is to insert quantum memory elements into the transmission channel every attenuation length. Considering that atoms are ideal candidates for storage and light is a natural carrier of quantum information, light of different frequencies will be necessary to connect the photonic- and matter-based physical systems [24]. In the present work, we show that the CPDC device can produce four entangled modes centered at 1560 nm, 1560 nm, 780 nm, and 780 nm, respectively. As 1560 nm is a transmission window of optical fiber and 780 nm is the absorption line of rubidium atom, the "two-color" quadripartite entanglement source can be applied for a quantum communication network.

2. EQUATIONS OF MOTION

The model has been described in [22]. Two $\chi^{(2)}$ nonlinear waveguides named A and B are put in a one-sided cavity (Fig. 1), which provides two interactions of type I degenerate parametric downconversions. Pump a_2 and b_2 at frequency ϖ_2 are incident upon the nonlinear medium at spatially separated locations. The two inputs may be from two lasers or be created from one laser using beam splitters. By the processes of degenerate parametric downconversion, a_1 and b_1 of frequency ϖ_1 are created in waveguides A and B, respectively, where $\varpi_2 \simeq 2\varpi_1$. We assume the optical modes inside the nonlinear media are perfectly phase matched and the two nonlinearities are equal.

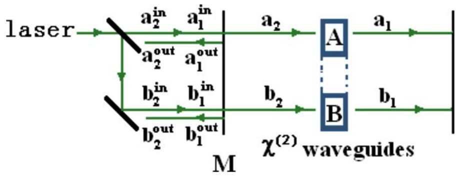

Fig. 1. (Color online) Sketch of coupled optical oscillators. Two nonlinear waveguides A and B are put inside a cavity. Pump a_2 and b_2 from one laser are incident into the cavity through mirror M. Fields a_1 and b_1 are created, respectively, by the processes of parametric downconversion. $a_1^{in},a_2^{in},b_1^{in},b_2^{in}$ are the incoming fields, $a_1^{out},a_2^{out},b_1^{out},b_2^{out}$ are the corresponding outgoing fields.

The total Hamiltonian for this system is given by

$$\hat{H}^{tot}=\hat{H}^{sys}+\hat{H}^{couple}+\hat{H}^{bath}, \tag{1}$$

where $\hat{H}^{sys}$ is the system Hamiltonian including nonlinear interactions, $\hat{H}^{couple}$ represents the coupling between the waveguides and is modeled as a linear process for description of evanescent waves, and $\hat{H}^{bath}$ is the Hamiltonian of the heat bath. The systematic Hamiltonian is given by

$$\hat{H}^{sys}=[(-\hbar\delta_1\hat{a}_1^{+}\hat{a}_1-\hbar\delta_2\hat{a}_2^{+}\hat{a}_2)+(-\hbar\delta_1\hat{b}_1^{+}\hat{b}_1-\hbar\delta_2\hat{b}_2^{+}\hat{b}_2)] + \left[\frac{i\hbar\kappa}{2}(\hat{a}_1^{+2}\hat{a}_2-\hat{a}_1^{2}\hat{a}_2^{+})+\frac{i\hbar\kappa}{2}(\hat{b}_1^{+2}\hat{b}_2-\hat{b}_1^{2}\hat{b}_2^{+})\right]. \tag{2}$$

The first square bracket in the right side is the free Hamiltonian of the cavity modes, and the second part presents the nonlinear interaction between the pump modes a_2, b_2 and the signal modes a_1, b_1 of cavity. κ is the effective nonlinear coefficient and is proportional to the nonlinear susceptibility $\chi^{(2)}$, and $\delta_1=\varpi_1-\varpi_1^{cav}$ and $\delta_2=\varpi_2-\varpi_2^{cav}$ represent the cavity detunings from resonances.

The coupling Hamiltonian is given as

$$\hat{H}^{couple}=\hbar J_1(\hat{a}_1\hat{b}_1^{+}+\hat{b}_1\hat{a}_1^{+})+\hbar J_2(\hat{a}_2\hat{b}_2^{+}+\hat{b}_2\hat{a}_2^{+}). \tag{3}$$

J_1 and J_2 are the linear coupling parameters at frequencies ϖ_1 and ϖ_2. They are sensitive to the specific setup; for example, the distance between guides, the modes in the guides, cavity length, and so on. J_1 is generally higher than J_2 since a_1, b_1 modes decay slower than a_2, b_2 modes [22].

The Hamiltonian of the heat bath is the function of bath operators Γ_{aj}, Γ_{bj} and cavity modes a_j, b_j $(j=1,2)$,

$$\hat{H}^{bath}=i\hbar\int_{-\infty}^{+\infty}d\varpi\kappa(\varpi)[\hat{\Gamma}_{a1}^{+}\hat{a}_1+\hat{\Gamma}_{a2}^{+}\hat{a}_2+\hat{\Gamma}_{b1}^{+}\hat{b}_1+\hat{\Gamma}_{b2}^{+}\hat{b}_2]+h.c. \tag{4}$$

$\kappa(\varpi)$ is the coupling constant and is independent of frequency

$$\kappa(\varpi)=\sqrt{\frac{\gamma}{\pi}}, \tag{5}$$

where γ denotes the damping rate. For simplicity, suppose that all of the internal losses of the system are due to leakage via mirror M with damping constants γ_{a1}, γ_{a2}, γ_{b1}, and γ_{b2}, and assume that $\gamma_{a1}=\gamma_{b1}=\gamma_1$, $\gamma_{a2}=\gamma_{b2}=\gamma_2$. When the losses are very small, γ_j are related to the amplitude reflection coefficients r_j and the amplitude transmission coefficients t_j approximately, $r_j=1-\gamma_j$, $t_j=\sqrt{2\gamma_j}$.

In the Heisenberg picture, we obtain the quantum Langevin equations of motion for the four cavity modes,

$$\tau\frac{d\hat{a}_1}{dt}=(-\gamma_1+i\Delta_1)\hat{a}_1+\kappa\hat{a}_1^{+}\hat{a}_2-iJ_1\hat{b}_1+\sqrt{2\gamma_1}\hat{a}_1^{in},$$

$$\tau\frac{d\hat{a}_2}{dt}=(-\gamma_2+i\Delta_2)\hat{a}_2-\frac{\kappa}{2}\hat{a}_1^{2}-iJ_2\hat{b}_2+\sqrt{2\gamma_2}\hat{a}_2^{in},$$

$$\tau\frac{d\hat{b}_1}{dt}=(-\gamma_1+i\Delta_1)\hat{b}_1+\kappa\hat{b}_1^{+}\hat{b}_2-iJ_1\hat{a}_1+\sqrt{2\gamma_1}\hat{b}_1^{in},$$

$$\tau\frac{d\hat{b}_2}{dt}=(-\gamma_2+i\Delta_2)\hat{b}_2-\frac{\kappa}{2}\hat{b}_1^{2}-iJ_2\hat{a}_2+\sqrt{2\gamma_2}\hat{b}_2^{in}. \tag{6}$$

Here τ is the cavity round trip time and is assumed to be the same for all four fields. The detunings are described by the dimensionless variables $\Delta_1=\delta_1\tau$ and $\Delta_2=\delta_2\tau$, correspondingly. $\hat{a}_1^{in}$, $\hat{a}_2^{in}$, $\hat{b}_1^{in}$, and $\hat{b}_2^{in}$ are the operators corresponding to the input fields.

3. THE SOLUTIONS OF STEADY STATE AND QUADRATURE FLUCTUATIONS

In order to analyze the quantum entanglement, we calculate the steady-state solutions of Eqs. (6) first. By setting the left side of the equations to be zero and replacing all operators with their expectation values, the steady-state equations are

$$(-\gamma_1+i\Delta_1)\alpha_1+\kappa\alpha_1^{*}\alpha_2-iJ_1\beta_1=0,$$

$$(-\gamma_2+i\Delta_2)\alpha_2-\frac{\kappa}{2}\alpha_1^{2}-iJ_2\beta_2+\sqrt{2\gamma_2}\alpha_2^{in}=0,$$

$$(-\gamma_1+i\Delta_1)\beta_1+\kappa\beta_1^{*}\beta_2-iJ_1\alpha_1=0,$$

$$(-\gamma_2+i\Delta_2)\beta_2-\frac{\kappa}{2}\beta_1^{2}-iJ_2\alpha_2+\sqrt{2\gamma_2}\beta_2^{in}=0. \tag{7}$$

Here α_1, α_2, β_1, and β_2 are the steady-state amplitudes of the four intracavity modes a_1, a_2, b_1, and b_2. α_2^{in} and β_2^{in} denote the amplitudes of two input pump fields outside the coupler. In the symmetric case ($\alpha_2=\beta_2$ and $\sqrt{2\gamma_2}\alpha_2^{in}=\sqrt{2\gamma_2}\beta_2^{in}=\varepsilon$), the steady-state solutions below the threshold are

$$\alpha_1 = \beta_1 = 0,$$

$$\alpha_2 = \beta_2 = \frac{\varepsilon}{\gamma_2 + i(J_2 - \Delta_2)}. \quad (8)$$

The steady-state solutions above the threshold are

$$\alpha_1 = \beta_1 = A_1 e^{i\theta_1},$$

$$A_1 = \sqrt{\frac{2}{\kappa^2}[\sqrt{\varepsilon^2\kappa^2 - (\gamma_1\mu_2 + \gamma_2\mu_1)^2} + (\mu_1\mu_2 - \gamma_1\gamma_2)]},$$

$$\theta_1 = \arccos\sqrt{\frac{1}{2} + \frac{\sqrt{\varepsilon^2\kappa^2 - (\gamma_1\mu_2 + \gamma_2\mu_1)^2}}{2\varepsilon\kappa}},$$

$$\alpha_2 = \beta_2 = A_2 e^{i\theta_2},$$

$$A_2 = \frac{\sqrt{\gamma_1^2 + \mu_1^2}}{\kappa},$$

$$\theta_2 = \arctan\frac{\mu_1}{\gamma_1} + 2\theta_1,$$

$$\mu_1 = J_1 - \Delta_1,$$

$$\mu_2 = J_2 - \Delta_2. \quad (9)$$

The oscillation threshold is expressed by

$$\varepsilon_{th} = \frac{\sqrt{[\gamma_1^2 + (J_1 - \Delta_1)^2][\gamma_2^2 + (J_2 - \Delta_2)^2]}}{\kappa}. \quad (10)$$

For the CV, we need to look at the fluctuations of the quadrature amplitude and the phase components. Following the semiclassical method, the intracavity fields can be written as

$$\hat{a}_j = \alpha_j + \frac{1}{2}(\delta\hat{X}_{aj} + i\delta\hat{Y}_{aj}), \quad (11)$$

$$\hat{b}_j = \beta_j + \frac{1}{2}(\delta\hat{X}_{bj} + i\delta\hat{Y}_{bj}). \quad (12)$$

Here $\delta\hat{X}_{aj}$, $\delta\hat{X}_{bj}$ $(j=1,2)$ define the fluctuations of the amplitude quadrature and $\delta\hat{Y}_{aj}$, $\delta\hat{Y}_{bj}$ $(j=1,2)$ describe the fluctuations of the phase quadrature. Under the condition of the above threshold, combining steady-state solution expressions (9) and Eqs. (6) we have

$$\tau\begin{pmatrix} \frac{d\delta\hat{X}_{a1}}{dt} \\ \frac{d\delta\hat{X}_{a2}}{dt} \\ \frac{d\delta\hat{X}_{b1}}{dt} \\ \frac{d\delta\hat{X}_{b2}}{dt} \\ \frac{d\delta\hat{Y}_{a1}}{dt} \\ \frac{d\delta\hat{Y}_{a2}}{dt} \\ \frac{d\delta\hat{Y}_{b1}}{dt} \\ \frac{d\delta\hat{Y}_{b2}}{dt} \end{pmatrix} = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix}\begin{pmatrix} \delta\hat{X}_{a1} \\ \delta\hat{X}_{a2} \\ \delta\hat{X}_{b1} \\ \delta\hat{X}_{b2} \\ \delta\hat{Y}_{a1} \\ \delta\hat{Y}_{a2} \\ \delta\hat{Y}_{b1} \\ \delta\hat{Y}_{b2} \end{pmatrix} + \begin{pmatrix} N & 0 \\ 0 & N \end{pmatrix}\begin{pmatrix} \delta\hat{X}^{in}_{a1} \\ \delta\hat{X}^{in}_{a2} \\ \delta\hat{X}^{in}_{b1} \\ \delta\hat{X}^{in}_{b2} \\ \delta\hat{Y}^{in}_{a1} \\ \delta\hat{Y}^{in}_{a2} \\ \delta\hat{Y}^{in}_{b1} \\ \delta\hat{Y}^{in}_{b2} \end{pmatrix}, \quad (13)$$

with

$$M_1 = \begin{pmatrix} -\gamma_1 + f & h & 0 & 0 \\ -h & -\gamma_2 & 0 & 0 \\ 0 & 0 & -\gamma_1 + f & h \\ 0 & 0 & -h & -\gamma_2 \end{pmatrix},$$

$$M_2 = \begin{pmatrix} -\Delta_1 + p & q & J_1 & 0 \\ q & -\Delta_2 & 0 & J_2 \\ J_1 & 0 & -\Delta_1 + p & q \\ 0 & J_2 & q & -\Delta_2 \end{pmatrix},$$

$$M_3 = \begin{pmatrix} \Delta_1 + p & -q & -J_1 & 0 \\ -q & \Delta_2 & 0 & -J_2 \\ -J_1 & 0 & \Delta_1 + p & -q \\ 0 & -J_2 & -q & \Delta_2 \end{pmatrix},$$

$$M_4 = \begin{pmatrix} -\gamma_1 - f & h & 0 & 0 \\ -h & -\gamma_2 & 0 & 0 \\ 0 & 0 & -\gamma_1 - f & h \\ 0 & 0 & -h & -\gamma_2 \end{pmatrix},$$

$$N = \begin{pmatrix} \sqrt{2\gamma_1} & 0 & 0 & 0 \\ 0 & \sqrt{2\gamma_2} & 0 & 0 \\ 0 & 0 & \sqrt{2\gamma_1} & 0 \\ 0 & 0 & 0 & \sqrt{2\gamma_2} \end{pmatrix},$$

$$f = \kappa A_2 \cos\theta_2,$$

$$h = \kappa A_1 \cos\theta_1,$$

$$p = \kappa A_2 \sin \theta_2,$$

$$q = \kappa A_1 \sin \theta_1.$$

$\delta\hat{X}^{in}_{aj(bj)}$ and $\delta\hat{Y}^{in}_{aj(bj)}$ $(j=1,2)$ are the non-correlated fluctuations entering the cavity through the coupling mirror and have variances $\langle|\delta\hat{X}^{in}_{aj(bj)}|^2\rangle=\langle|\delta\hat{Y}^{in}_{aj(bj)}|^2\rangle=1$.

4. ENTANGLEMENT OF THE OUTPUT LIGHT FIELDS

The output fluctuations in the frequency space can be calculated after the Fourier transformation. The relationship between the output quantities and the input quantities are [25]

$$(out) = \left\{ \begin{pmatrix} N & 0 \\ 0 & N \end{pmatrix} \left[i\varpi\tau I - \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} \right] \begin{pmatrix} N & 0 \\ 0 & N \end{pmatrix} - I \right\} (in),$$

$$(out) = \begin{pmatrix} \delta\hat{X}^{out}_{a1}(\varpi) \\ \delta\hat{X}^{out}_{a2}(\varpi) \\ \delta\hat{X}^{out}_{b1}(\varpi) \\ \delta\hat{X}^{out}_{b2}(\varpi) \\ \delta\hat{Y}^{out}_{a1}(\varpi) \\ \delta\hat{Y}^{out}_{a2}(\varpi) \\ \delta\hat{Y}^{out}_{b1}(\varpi) \\ \delta\hat{Y}^{out}_{b2}(\varpi) \end{pmatrix}, \quad (in) = \begin{pmatrix} \delta\hat{X}^{in}_{a1}(\varpi) \\ \delta\hat{X}^{in}_{a2}(\varpi) \\ \delta\hat{X}^{in}_{b1}(\varpi) \\ \delta\hat{X}^{in}_{b2}(\varpi) \\ \delta\hat{Y}^{in}_{a1}(\varpi) \\ \delta\hat{Y}^{in}_{a2}(\varpi) \\ \delta\hat{Y}^{in}_{b1}(\varpi) \\ \delta\hat{Y}^{in}_{b2}(\varpi) \end{pmatrix}. \quad (14)$$

I is the unit matrix. Based on the full inseparability criteria of multipartite CV entanglement proposed by P. van Loock and A. Furusawa [26], we have the following six inequalities:

$$\langle \delta^2(\hat{Y}_{a1} - \hat{Y}_{a2})\rangle + \langle \delta^2(\hat{X}_{a1} + \hat{X}_{a2} + g_{b1}\hat{X}_{b1} + g_{b2}\hat{X}_{b2})\rangle > 4, \quad (15)$$

$$\langle \delta^2(\hat{Y}_{a1} + \hat{Y}_{b1})\rangle + \langle \delta^2(\hat{X}_{a1} + g_{a2}\hat{X}_{a2} - \hat{X}_{b1} + g_{b2}\hat{X}_{b2})\rangle > 4, \quad (16)$$

$$\langle \delta^2(\hat{Y}_{a1} + \hat{Y}_{b2})\rangle + \langle \delta^2(\hat{X}_{a1} + g_{a2}\hat{X}_{a2} + g_{b1}\hat{X}_{b1} - \hat{X}_{b2})\rangle > 4, \quad (17)$$

$$\langle \delta^2(\hat{Y}_{a2} + \hat{Y}_{b1})\rangle + \langle \delta^2(g_{a1}\hat{X}_{a1} + \hat{X}_{a2} - \hat{X}_{b1} + g_{b2}\hat{X}_{b2})\rangle > 4, \quad (18)$$

$$\langle \delta^2(\hat{Y}_{a2} + \hat{Y}_{b2})\rangle + \langle \delta^2(g_{a1}\hat{X}_{a1} + \hat{X}_{a2} + g_{b1}\hat{X}_{b1} - \hat{X}_{b2})\rangle > 4, \quad (19)$$

$$\langle \delta^2(\hat{Y}_{b1} - \hat{Y}_{b2})\rangle + \langle \delta^2(g_{a1}\hat{X}_{a1} + g_{a2}\hat{X}_{a2} + \hat{X}_{b1} + \hat{X}_{b2})\rangle > 4, \quad (20)$$

where g_{a1}, g_{a2}, g_{b1}, and g_{b2} are scaling factors. There are seven kinds of separable or partially separable states for four-party entanglement and each satisfies several inequalities expressed above, which can be written in the form of a statistical mixture of reduced density operators with weights $\eta_m \geq 0$:

$$\hat{\rho} = \sum_m \eta_m \hat{\rho}_{m,a1a2b1} \otimes \hat{\rho}_{m,b2} \Rightarrow (17),(19),(20),$$

$$\hat{\rho} = \sum_m \eta_m \hat{\rho}_{m,a1a2b2} \otimes \hat{\rho}_{m,b1} \Rightarrow (16),(18),(20),$$

$$\hat{\rho} = \sum_m \eta_m \hat{\rho}_{m,a1b1b2} \otimes \hat{\rho}_{m,a2} \Rightarrow (15),(18),(19),$$

$$\hat{\rho} = \sum_m \eta_m \hat{\rho}_{m,a2b1b2} \otimes \hat{\rho}_{m,a1} \Rightarrow (15),(16),(17),$$

$$\hat{\rho} = \sum_m \eta_m \hat{\rho}_{m,a1a2} \otimes \hat{\rho}_{m,b1b2} \Rightarrow (16),(17),(18),(19),$$

$$\hat{\rho} = \sum_m \eta_m \hat{\rho}_{m,a1b1} \otimes \hat{\rho}_{m,a2b2} \Rightarrow (15),(17),(18),(20),$$

$$\hat{\rho} = \sum_m \eta_m \hat{\rho}_{m,a1b2} \otimes \hat{\rho}_{m,a2b1} \Rightarrow (15),(16),(19),(20). \quad (21)$$

Considering violations of the inequalities of Eqs. (15) and (16), all separable forms are excluded except the form $\hat{\rho}=\Sigma_m \eta_m \hat{\rho}_{m,a1a2b1} \otimes \hat{\rho}_{m,b2}$. In order to rule out this separable state, one of the inequalities in Eqs. (17) and (19) or (20) should be violated in addition. Hence we may choose

$$S_{a1a2} = \langle \delta^2(\hat{Y}_{a1} - \hat{Y}_{a2})\rangle + \langle \delta^2(\hat{X}_{a1} + \hat{X}_{a2} + g_{b1}\hat{X}_{b1} + g_{b2}\hat{X}_{b2})\rangle < 4, \quad (22)$$

$$S_{a1b1} = \langle \delta^2(\hat{Y}_{a1} + \hat{Y}_{b1})\rangle + \langle \delta^2(\hat{X}_{a1} + g_{a2}\hat{X}_{a2} - \hat{X}_{b1} + g_{b2}\hat{X}_{b2})\rangle < 4, \quad (23)$$

$$S_{b1b2} = \langle \delta^2(\hat{Y}_{b1} - \hat{Y}_{b2})\rangle + \langle \delta^2(g_{a1}\hat{X}_{a1} + g_{a2}\hat{X}_{a2} + \hat{X}_{b1} + \hat{X}_{b2})\rangle < 4. \quad (24)$$

The satisfaction of the three inequalities is sufficient for concluding CV GHZ-type four-party entanglement. With detuning $\Delta_1=J_1$ and $\Delta_2=J_2$, the steady-state solutions of Eq. (9)are simplified to

$$\alpha_1 = \beta_1 = A_1 = \frac{1}{\kappa}\sqrt{2\varepsilon\kappa - 2\gamma_1\gamma_2},$$

$$\alpha_2 = \beta_2 = A_2 = \frac{\gamma_1}{\kappa},$$

$$\varepsilon_{th} = \frac{\gamma_1\gamma_2}{\kappa}. \quad (25)$$

These are the same solutions as for the degenerate optical parametric oscillator (OPO) above threshold.

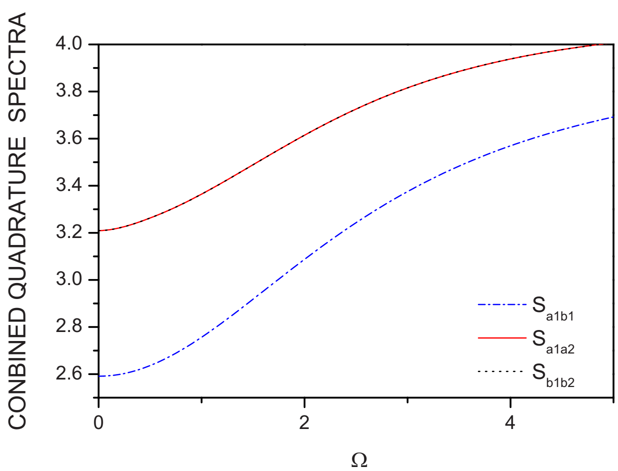

Fig. 2. (Color online) Quantum correlation spectra versus normalized frequency $\Omega=\varpi\tau/\gamma_1$ for $\sigma=\varepsilon/\varepsilon_{th}=1.2$.

The correlation spectra characters are shown numerically in Figs. 2–4. The minimum correlation fluctuation spectra S_{a1a2}, S_{a1b1}, S_{b1b2} versus the analysis frequency $\Omega=\varpi\tau/\gamma_1$ are plotted in Fig. 2, with pump parameter $\sigma=\varepsilon/\varepsilon_{th}=1.2$. The other parameters are set as $\gamma_1=0.01$, $\gamma_2=0.05$, $\Delta_1=J_1=10\gamma_1$, and $\Delta_2=J_2=\frac{1}{5}\gamma_2$. Obviously, all values of S_{a1a2}, S_{a1b1}, and S_{b1b2} are below the quantum limit of 4 in a wide frequency range, and the maximum correlation is obtained at zero frequency. With the same parameter values, we plot the fluctuation spectra as a function of the normalized pumping power σ at $\Omega=0$ in Fig. 3. It shows that the three violations are satisfied when $\sigma>1.1$, and the best correlations for all of four modes are achieved at about $\sigma=1.2$. In both Figs. 2 and 3, it can be seen that the symmetry of the waveguides A and B makes $S_{a1a2}=S_{b1b2}$, and they are always worse than the correlation spectrum S_{a1b1}. The correlation spectrum S_{a1b1} shows the bipartite entanglement between a_1 and b_1 modes as in [17,18]. Meanwhile, the harmonic entanglement [27,28] of a_1, a_2 and b_1, b_2 also can be illustrated by S_{a1a2} and S_{b1b2}. In the end, we investigate the effect of the linear coupling parameters in the system. It is found that J_2 has a small influence on the correlations. Figure 4 shows the dependences of three correlation spectra on the coupling parameter J_1 with $\Omega=0$, $\sigma=1.2$, $\gamma_1=0.01$, $\gamma_2=0.05$ and $\Delta_2=J_2=\frac{1}{5}\gamma_2$. It is noted that S_{a1a2} and S_{b1b2} decrease when the J_1 increases, and meanwhile the minimum value of S_{a1b1} is obtained when $J_1=0.015$. The minimum value of $S_{a1a2}+S_{b1b2}+S_{a1b1}$ is achieved if we choose $J_1=0.1$. According to the above discussion, the inseparability criterion is satisfied for the four modes of CPDC when the pump power is above threshold.

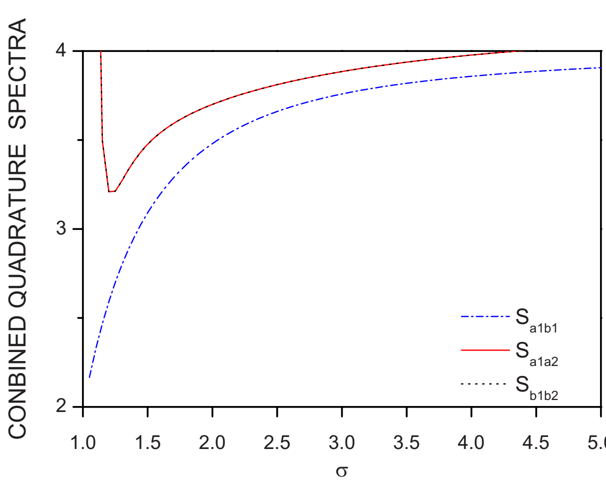

Fig. 3. (Color online) Quantum correlation spectra as functions of pump parameter $\sigma=\varepsilon/\varepsilon_{th}$ at $\Omega=0$.

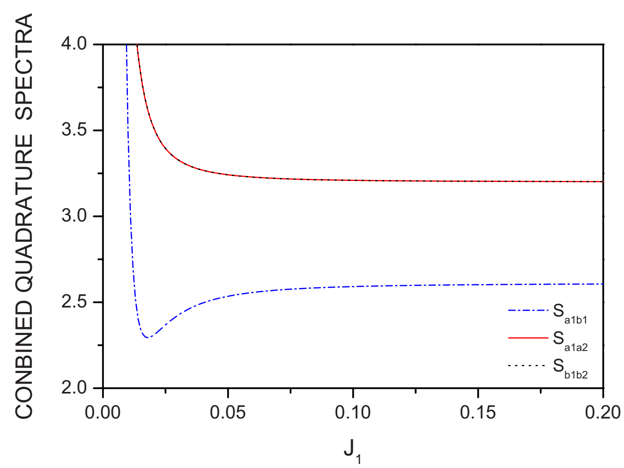

Fig. 4. (Color online) Quantum correlation spectra as functions of coupling parameter J_1 at $\Omega=0$ and $\sigma=1.2$.

5. CONCLUSION

We have calculated the correlation spectra of four output lights from a coupled optical system. The results show that a CV GHZ-type multicolored inseparable state is produced in a large range when the system operates above threshold. Unlike other schemes, the two sets of output fields can be degenerate in both frequency and polarization, and they can exit the cavity at spatially separated locations. The all-integrated configuration makes the system a stable tunable source for bright multimode entangled beams applied in the quantum communication network.

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