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Noise-free frequency conversion of quantum states*

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In this paper, the frequency conversion of quantum states based on the intracavity nonlinear interaction is proposed. The fidelity of an input state after frequency conversion is calculated, and it is shown the noise-free frequency conversion of a quantum state can be achieved by injecting a strong signal field. The dependences of conversion efficiency on the pump parameter, extra losses and input state amplitude are also analysed.

Keywords: frequency conversion, quantum state, fidelity

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1. Introduction

Quantum networks based on atoms and photons supply a promising method for a large-scale quantum information process,[1] the atoms acting as quantum nodes are used to process and store quantum states locally,[2,3] and meanwhile the photons serve as quantum channels to link the separated nodes for the exchange of quantum information.[4] It is well known that the transmission wavelengths for photons in telecommunication optical fibers are 1310 nm and 1550 nm,[5,6] and the atoms absorb/emit photons at different wavelengths, e.g. 800 nm for alkaline atoms,[7] thus an optical frequency conversion interface is needed to couple them in a quantum network. In this procedure, the quantum state should be maintained for faithful quantum frequency conversion.[8]

There are many information-preserving unitary transformations allowing a nonlinear frequency conversion via a particle annihilation or creation process, the optimum candidate is frequency up-conversion because of the noise-free and 100% conversion efficiency.[9] Since the conception for noise-free photon frequency up-conversion was proposed by Kumar in 1990,[8] it has been developed both for discrete and continuous variables.[10−13] However, the noise-free frequency conversion of a quantum state is difficult to realize in the down-conversion process, because the process is normally considered as an amplification process with unavoidable quantum noise.[10] Recently, the theoretical and the experimental studies on noise-free parametric amplification of a traveling-wave light have been developed when spontaneous parametric down-conversion is pumped by a weak field and injected with a strong signal field,[14,15] this noise-free down-conversion process is feasible for frequency conversion of a quantum state.

In this paper, an intracavity scheme with frequency down-conversion as well as up-conversion of a quantum state are analysed, the condition for conversion of quantum state is reached. According to the fidelity, the dependences of conversion efficiency on pump parameter, extra intracavity losses and input amplitude are discussed.

2. Theoretical model

The spontaneous photon frequency up and down-converison processes are depicted in Figs. 1(a) and 1(b). First, if one weak light beam and one strong light beam respectively with angular frequencies ω_1 and ω_2 are coupled in a second-order nonlinear crystal to generate a higher frequency $\omega_3 = \omega_1 + \omega_2$, the input mode $\hat{a}_2$ can be treated classically as real amplitude E_2, thus one obtains[8]

$$\hat{a}_1(t) = \hat{a}_1(0)\cos(\chi_1 t) - \hat{a}_3(0)\sin(\chi_1 t), \quad (1)$$

$$\hat{a}_3(t) = \hat{a}_3(0)\cos(\chi_1 t) + \hat{a}_1(0)\sin(\chi_1 t), \quad (2)$$

where $\chi_1 = \chi' E_2$, and χ' is a coupling constant that is proportional to the second-order susceptibility $\chi^{(2)}$

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of the nonlinear material. The quantum states can realize complete frequency conversion at $t = \pi/2\chi_1$

$$\hat{a}_1(t = \pi/2\chi_1) = -\hat{a}_3(0), \qquad (3)$$

$$\hat{a}_3(t = \pi/2\chi_1) = \hat{a}_1(0). \qquad (4)$$

On the other hand, for frequency down-conversion process: $\omega_3 - \omega_2 = \omega_1$, the harmonic mode $\hat{a}_3$ is set as the strong pump[16] and replaced with real amplitude E_3. The input–output relations are[17]

$$\hat{a}_1(t) = \hat{a}_1(0)\cosh(\chi_2 t) - \hat{a}_2^+(0)\sinh(\chi_2 t), \qquad (5)$$

$$\hat{a}_2(t) = \hat{a}_2(0)\cosh(\chi_2 t) - \hat{a}_1^+(0)\sinh(\chi_2 t), \qquad (6)$$

where $\chi_2 = \chi' E_3$. Although the down-conversion probability can be greatly enhanced by the field $\hat{a}_2$, the appearance of the $\hat{a}^+$ terms leads to spontaneous quantum noise.[10] Therefore, it seems that frequency down-conversion is unsuitable for the frequency conversion of quantum state. To solve the problem, Ou[14] proposed a solution by reducing the intensity of pump field $\hat{a}_3$ as shown in Fig. 1(b), thus the process becomes a frequency converter. The solutions are equal to Eqs. (3) and (4). Thus, a complete conversion: $\hat{a}_1(0) \to \hat{a}_3(t)$ and $\hat{a}_3(0) \to -\hat{a}_1(t)$ with unit conversion efficiency are achieved, perhaps without adding any excess noise to the output state.

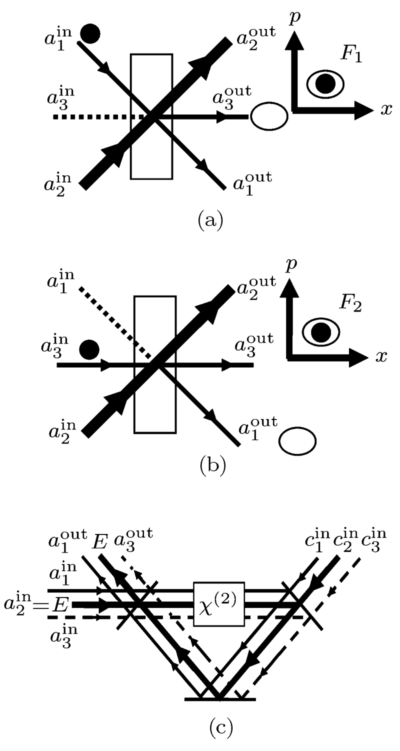

Fig. 1. Intracavity scheme with (a) up-conversion and (b) down-converison in (c) a single-sided cavity.

When the process of frequency conversion of quantum state occurs in a cavity (see Fig. 1(c)), could the quantum state frequency conversion be realized as it is shown with the spontanous parametric process? In this case, the equations of motion for the three modes $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_3$ can be expressed as[18]

$$\begin{aligned}\tau\dot{\hat{a}}_1(t) &= -(\gamma_1+\rho_1)\hat{a}_1(t) + \chi\hat{a}_2^+(t)\hat{a}_3(t)\\ &\quad + \sqrt{2\gamma_1}\hat{a}_1^{\rm in}(t) + \sqrt{2\rho_1}\hat{c}_1^{\rm in}(t), \qquad (7)\\ \tau\dot{\hat{a}}_2(t) &= -(\gamma_2+\rho_2)\hat{a}_2(t) + \chi\hat{a}_1^+(t)\hat{a}_3(t)\\ &\quad + \sqrt{2\gamma_2}\hat{a}_2^{\rm in}(t) + \sqrt{2\rho_2}\hat{c}_2^{\rm in}(t), \qquad (8)\\ \tau\dot{\hat{a}}_3(t) &= -(\gamma_3+\rho_3)\hat{a}_3(t) - \chi\hat{a}_1(t)\hat{a}_2(t)\\ &\quad + \sqrt{2\gamma_3}\hat{a}_3^{\rm in}(t) + \sqrt{2\rho_3}\hat{c}_3^{\rm in}(t), \qquad (9)\end{aligned}$$

where $\hat{a}_i^{\rm in}$ $(i = 1, 2, 3)$ denote the input amplitude operators; $\hat{c}_i^{\rm in}(t)$ are the vacuum noise terms each corresponding to intracavity loss; τ is the roundtrip time in the cavity, which is assumed to be the same for all three fields; $\gamma_i + \rho_i$ $(i = 1, 2, 3)$ are the total loss parameters with γ_i being related to amplitude reflection and transmission coefficients of the coupling mirror, and ρ_i representing the extra intracavity losses.

The steady-state equations of Eqs. (7)–(9) are then obtained as[19]

$$-(\gamma_1+\rho_1)\alpha_1 + \chi\alpha_2^*\alpha_3 + \sqrt{2\gamma_1}\alpha_1^{\rm in} = 0, \qquad (10)$$

$$-(\gamma_2+\rho_2)\alpha_2 + \chi\alpha_1^*\alpha_3 + \sqrt{2\gamma_2}\alpha_2^{\rm in} = 0, \qquad (11)$$

$$-(\gamma_3+\rho_3)\alpha_3 - \chi\alpha_1\alpha_2 + \sqrt{2\gamma_3}\alpha_3^{\rm in} = 0, \qquad (12)$$

where α_1, α_2, and α_3 are the steady state amplitudes of intracavity modes $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$ respectively.

The two cases $\alpha_3^{\rm in} = 0$, $\alpha_1^{\rm in} \ll \alpha_2^{\rm in}$ and $\alpha_1^{\rm in} = 0$, $\alpha_3^{\rm in} \ll \alpha_2^{\rm in}$ are corresponding to the nonsymmetrical pumped up-conversion[20] and the strongly driven down-conversion process[21] respectively. Considering that the input mode $\alpha_2^{\rm in}$ is strong and can be considered to be undepleted, we have $\alpha_2 \approx \alpha_2^{\rm in} = E$. Using the boundary condition[22] $\alpha_i^{\rm out} = \sqrt{2\gamma_i}\alpha_i - \alpha_i^{\rm in}$, the output modes for the up-conversion and the down-conversion respectively are

$$\alpha_3^{\rm out} = \frac{-2\sqrt{\gamma_1\gamma_3}\chi E\alpha_1^{\rm in}}{(\gamma_1+\rho_1)(\gamma_3+\rho_3)+(\chi E)^2}, \qquad (13)$$

$$\alpha_1^{\rm out} = \frac{2\sqrt{\gamma_1\gamma_3}\chi E\alpha_3^{\rm in}}{(\gamma_1+\rho_1)(\gamma_3+\rho_3)+(\chi E)^2}. \qquad (14)$$

The quantum state transfers during intracavity up-conversion $\alpha_1^{\rm in} \to \alpha_3^{\rm out}$ and down-conversion $\alpha_3^{\rm in} \to \alpha_1^{\rm out}$ are strongly dependent on cavity parameters as seen in Eqs. (13) and (14). The conversion efficiency is discussed together with its fluctuations in the following.

The dynamics of quantum fluctuations can be described by linearizing the classical equations of motion around the stationary solution. Setting

$$\begin{aligned}\hat{a}_i(t) &= \alpha_i + \delta\hat{a}_i(t),\\ \hat{a}_i^{\rm in}(t) &= \alpha_i^{\rm in} + \delta\hat{a}_i^{\rm in}(t),\\ \hat{c}_i^{\rm in}(t) &= \delta\hat{c}_i^{\rm in}(t),\end{aligned} \tag{15}$$

substituting Eq. (15) into Eqs. (7)–(9), we obtain the fluctuation dynamics equations

$$\begin{aligned}\tau\delta\dot{a}_1(t) &= -(\gamma_1+\rho_1)\delta a_1(t) + \chi E\delta a_3(t)\\ &\quad + \sqrt{2\gamma_1}\delta a_1^{\rm in}(t) + \sqrt{2\rho_1}c_1^{\rm in}(t), \qquad (16)\\ \tau\delta\dot{a}_3(t) &= -(\gamma_3+\rho_3)\delta a_3(t) - \chi E\delta a_1(t)\\ &\quad + \sqrt{2\gamma_3}\delta a_3^{\rm in}(t) + \sqrt{2\rho_3}c_3^{\rm in}(t). \qquad (17)\end{aligned}$$

We then obtain the fluctuation of output field at the analysis frequency ω

$$\begin{aligned}&\delta a_3^{\rm out}(\omega)\\ &= \frac{1}{({\rm i}\,\omega\tau+\gamma_1+\rho_1)({\rm i}\,\omega\tau+\gamma_3+\rho_3)+(\chi E)^2}\\ &\quad\times\{-2\chi E\sqrt{\gamma_1\gamma_3}\delta a_1^{\rm in}(\omega) - 2\chi E\sqrt{\rho_1\gamma_3}c_1^{\rm in}(\omega)\\ &\quad+[({\rm i}\,\omega\tau+\gamma_1+\rho_1)(-{\rm i}\,\omega\tau+\gamma_3-\rho_3)-(\chi E)^2]\\ &\quad\times\delta a_3^{\rm in}(\omega) + 2\sqrt{\gamma_3\rho_3}({\rm i}\,\omega\tau+\gamma_1+\rho_1)c_3^{\rm in}(\omega)\}, \qquad (18)\\ &\delta a_1^{\rm out}(\omega)\\ &= \frac{1}{({\rm i}\,\omega\tau+\gamma_1+\rho_1)({\rm i}\,\omega\tau+\gamma_3+\rho_3)+(\chi E)^2}\\ &\quad\times\{2\chi E\sqrt{\gamma_1\gamma_3}\delta a_3^{\rm in}(\omega) + 2\chi E\sqrt{\gamma_1\rho_3}c_3^{\rm in}(\omega)\\ &\quad+[(-{\rm i}\,\omega\tau+\gamma_1-\rho_1)({\rm i}\,\omega\tau+\gamma_3+\rho_3)-(\chi E)^2]\\ &\quad\times\delta a_1^{\rm in}(\omega) + 2\sqrt{\gamma_1\rho_1}({\rm i}\,\omega\tau+\gamma_3+\rho_3)c_1^{\rm in}(\omega)\}. \qquad (19)\end{aligned}$$

2.1. Frequency up-conversion process

Using Eq. (18) and the definitions of amplitude and phase quadrature $X = 1/2(a + a^+)$ and $Y = 1/2\,{\rm i}(a - a^+)$, the fluctuation spectra of the quadrature components of $a_3^{\rm out}$ can be written as

$$\delta X_{a_3}^{\rm out} = A\delta X_{a_1}^{\rm in} + B\delta Y_{a_1}^{\rm in} + \sum_{j=1}^{6} C_j\delta L_j, \tag{20}$$

$$\delta Y_{a_3}^{\rm out} = -B\delta X_{a_1}^{\rm in} + A\delta Y_{a_1}^{\rm in} + \sum_{j=1}^{6}(-1)^j C_j\delta L_j. \tag{21}$$

In Eqs. (20) and (21), $X_{a_3}^{\rm out} \to X_{\rm out}$, $X_{a_1}^{\rm in} \to X_{\rm in}$, $Y_{a_3}^{\rm out} \to Y_{\rm out}$, and $Y_{a_1}^{\rm in} \to Y_{\rm in}$, all the other terms $X_{a_3}^{\rm in}$, $Y_{a_3}^{\rm in}$, $X_{c_3}^{\rm in}$, $Y_{c_3}^{\rm in}$, $X_{c_1}^{\rm in}$, and $Y_{c_1}^{\rm in}$ are taken for vacuum noise δL_j. Assuming that the input state $a_1^{\rm in}$ is a phase squeezed state with squeezing degree r, we can obtain

$$\Delta^2 X_{\rm out} = A^2{\rm e}^{2r} + B^2{\rm e}^{-2r} + \sum_{j=1}^{6} C_j^2, \tag{22}$$

$$\Delta^2 Y_{\rm out} = B^2{\rm e}^{2r} + A^2{\rm e}^{-2r} + \sum_{j=1}^{6} C_j^2. \tag{23}$$

2.2. Frequency down-conversion process

From Eq. (19) we obtain the fluctuation spectra of $a_1^{\rm out}$

$$\delta X_{a_1}^{\rm out} = -A\delta X_{a_3}^{\rm in} - B\delta Y_{a_3}^{\rm in} + \sum_{j=1}^{6} D_j\delta L_j, \tag{24}$$

$$\delta Y_{a_1}^{\rm out} = B\delta X_{a_3}^{\rm in} - A\delta Y_{a_3}^{\rm in} + \sum_{j=1}^{6}(-1)^j D_j\delta L_j. \tag{25}$$

In this process, $X_{a_1}^{\rm out} \to X_{\rm out}$, $X_{a_3}^{\rm in} \to X_{\rm in}$, $Y_{a_1}^{\rm out} \to Y_{\rm out}$, and $Y_{a_3}^{\rm in} \to Y_{\rm in}$, assuming that $a_3^{\rm in}$ is a phase squeezed state with squeezed degree r, we can obtain

$$\Delta^2 X_{\rm out} = A^2{\rm e}^{2r} + B^2{\rm e}^{-2r} + \sum_{j=1}^{6} D_j^2, \tag{26}$$

$$\Delta^2 Y_{\rm out} = B^2{\rm e}^{2r} + A^2{\rm e}^{-2r} + \sum_{j=1}^{6} D_j^2, \tag{27}$$

where $\sum\limits_{j=1}^{6} C_j^2 = \sum\limits_{j=1}^{6} D_j^2$.

3. Results

We introduce the fidelity to discuss the conversion efficiency η, which is propotional to the phase space overlap between input state and output state, and written as[23]

$$\begin{aligned}F &= \frac{2}{\sqrt{(1+\Delta^2 X_{\rm out})}\sqrt{(1+\Delta^2 P_{\rm out})}}\\ &\quad\times\exp\bigg[-\frac{(x_{\rm out}-x_{\rm in})2}{2(1+\Delta^2 X_{\rm out})}\\ &\quad-\frac{(p_{\rm out}-p_{\rm in})^2}{2(1+\Delta^2 P_{\rm out})}\bigg],\end{aligned} \tag{28}$$

where $\Delta^2 X_{\rm out}(\Delta^2 P_{\rm out})$ and $x_{\rm out}(p_{\rm out})$ represent the variance and the average of quadrature amplitudes (phases) of the output, and $x_{\rm in}(p_{\rm in})$ are the average values of quadrature amplitudes (phases) of the input state. Substitute Eqs. (13), (22) and (23), and Eqs. (14), (26) and (27) into Eq. (28) respectively, the fidelities of frequency up-conversion and down-conversion can be obtained and we have $F_{\rm up} = F_{\rm down} = F$.

Here, we discuss the frequency conversion of coherent state ($r = 0$). The relations between the fidelity F and the amplitude of input signal $\alpha_{\rm in}$ for different pump parameters are shown in Fig. 2. There exists a complete conversion condition. When extra intracavity losses $\rho_3 = \rho_1 = 0$ and the amplitude of pump field $E = \gamma_1/\chi$, the fidelity $F = 1$. This requirement of pump field is on the order of the threshold of cavity, which is roughly hundreds of milliwatts for a common PPKTP crystal.

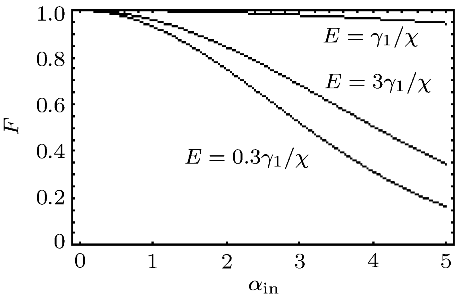

Fig. 2. Curves for fidelity F versus amplitude $\alpha_{\rm in}$ with $\gamma_3 = \gamma_1$ and $\rho_3 = \rho_1 = 0.1\gamma_1$ for three E values.

Once the pump parameters are given, the fidelity decreases monotonically with the increase of the amplitude of input signal $\alpha_{\rm in}$. Recall that the assumption of small input amplitude has been made in almost all theoretical and experimental work.[11,12] The point is to what extent the assumption is valid our results give it a quantitative reference.

The curves for fidelity versus pump parameter for different input amplitudes are given in Fig. 3. The results show that with pump parameter increasing, each fidelity curve first gradually increases and then decreases, and at $\chi E/\gamma_1 = 1$ reaches the highest point. In addition, for the fidelity with the same pump parameter, the lower the initial input amplitude is, the higher the fidelity is.

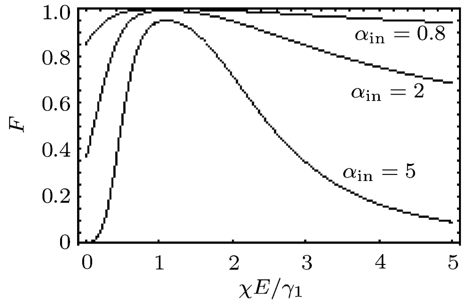

Fig. 3. Curves for fidelity F versus pump parameter $\chi E/\gamma_1$ with $\rho_3 = \rho_1 = 0.1\gamma_1$ and $\gamma_3 = \gamma_1$ for different $\alpha_{\rm in}$ values.

Meanwhile, the fidelity versus intracavity loss is shown in Fig. 4. With the increase of intracavity loss, the fidelity decreases rapidly so a lossless cavity is always hoped for.

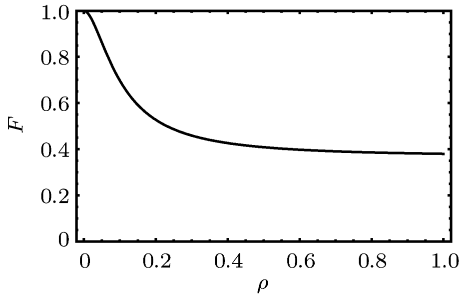

Fig. 4. Fidelity F versus the extra intracavity losses $\rho_3 = \rho_1 = \rho$ with $\gamma_3 = \gamma_1 = 0.1$, $\chi E/\gamma_1 = 1$ and $\alpha_{\rm in} = 2$.

4. Conclusion

The theoretical analyses of intracavity frequency conversion of quantum state are given. The dependences of fidelity on the parameters of pump, cavity losses and amplitude of input state are investigated. This theoretical discussion provides a possible scheme for the frequency conversion of quantum state in a cavity, and may have an application in quantum network.

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