Quantum frequency up-conversion with a cavity*

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The quantum state transfer from subharmonic frequency to harmonic frequency based on asymmetrically pumped second harmonic generation in a cavity is investigated theoretically. The performance of noise-free frequency up-conversion is evaluated by the signal transfer coefficient and the conversion efficiency, in which both the quadrature fluctuation and the average photon number are taken into consideration. It is shown that the quantum property can be preserved during frequency up-conversion via operating the cavity far below the threshold. The dependences of the transfer coefficient and the conversion efficiency on pump parameter, analysing frequency, and cavity extra loss are also discussed.

Keywords: frequency up-conversion, signal transfer coefficient, conversion efficiency

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1. Introduction

Parametric frequency conversion is a full-fledged technique to generate tunable coherent radiation and squeezed light,[1,2] it has applications in interferometry, accurate measurement, and spectroscopy. Now it has become a major ingredient in quantum network for large-scale quantum information process.[3] In a quantum network, the atoms are used as quantum nodes to process and store quantum states locally[4] and the photons acted as quantum channels to link the separated nodes for the exchange of quantum information.[5] It is well known that the quantum information is sensitive to loss and photon whose wavelength near communication band of optical fibre (between 1310 nm and 1550 nm) has low loss.[6] However, the atoms absorb/emit photons at a different wavelength, e.g. 800 nm for alkaline atoms.[7] Thus a quantum interface is needed to couple the photons of communication band with atoms.[8] Fundamentally, these endeavours include quantum frequency conversion between an optical system and atom medium, such a quantum connectivity can be achieved by optical interaction between photons and atoms[9−12] in cavity quantum electrodynamics (QED),[7,13] electromagnetically induced transparency (EIT),[14] Raman[15] and four-wave mixing processes.[16]

Theoretically, the information-preserving unitary transformation between two different frequencies can be realized via particle annihilation or creation process, the frequency up-conversion was thought to be optimum candidate because of noise-free and 100% conversion efficiency.[17] Since the conception for noise-free photon frequency up-conversion was proposed[8] and experimentally realized,[18] it has been extensively developed for both discrete and continuous variables.[17,19,20] However, in all of the aforementioned experiments, the up-conversion efficiency is lower than 50%. Motivated by the results that the frequency conversion in a resonant cavity has been demonstrated as an efficient way to enhance the effective conversion for nonclassical state generation,[21,22] in our previous work[23] we proposed an intracavity model of frequency conversion, in which the fidelity was utilized to measure the quadrature component variation in phase space. In the present paper, we present the frequency up-conversion of quantum state in a second-harmonic-generation (SHG) cavity which is asymmetrically pumped, both the conversion efficiency and the signal transfer coefficient instead of the fidelity are used to characterize and quantify the quantum property of frequency up-conversion process accurately. The condition for noise-free conversion is analysed, and the dependences of signal transfer coef-

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ficient and conversion efficiency on pump parameter, signal mode amplitude, and intracavity loss are also discussed.

2. Theoretical model

The scheme of frequency up-conversion of quantum state utilizing the intracavity asymmetrically pumped SHG process is shown in Fig. 1. The system consists of three modes, which are two subharmonic modes $\hat{a}_1$, $\hat{a}_2$, and harmonic mode $\hat{a}_3$ with frequencies ω_1, ω_2, and ω_3, respectively, where $\omega_3 = \omega_1 + \omega_2$ for energy conservation. They are coupled by a χ^2 nonlinear crystal inside an optical cavity. Following the usual terminology, we shall call the fields represented by these operators signal ($\hat{a}_1$), pump ($\hat{a}_2$), and harmonic ($\hat{a}_3$), respectively. If a lower frequency quantum state denoted by $\hat{a}_1^{\rm in}$ (input signal mode) was converted into a higher frequency output harmonic mode $\hat{a}_3^{\rm out}$ with 100% conversion efficiency in the absence of noise, then the requirements are

$$\delta^2 X(Y)_3^{\rm out} = \delta^2 X(Y)_1^{\rm in}, \tag{1}$$

$$\left\langle \hat{n}_3^{\rm out} \right\rangle = \left\langle \hat{n}_1^{\rm in} \right\rangle, \tag{2}$$

where $\delta^2 X(Y)$ is quadrature amplitude (phase) component fluctuation, and $\langle \hat{n} \rangle$ is the average photon number of corresponding modes.

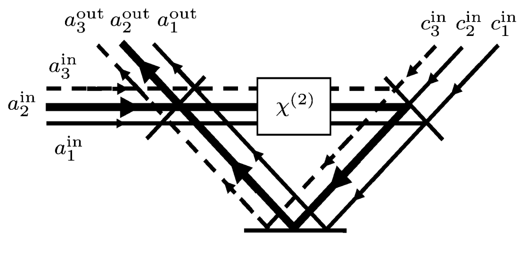

Fig. 1. Sketch of intracavity SHG process.

To consider both the quadrature component fluctuation and the average photon number of the quantum state to be converted from a lower frequency ω_1 into higher frequency ω_3, we use the signal transfer coefficient $T_{X(Y)}$ and conversion efficiency η to evaluate the performance of the frequency conversion, which are commonly defined as[16,24]

$$T_{X(Y)} = \frac{{\rm SNR}[X(Y)_3^{\rm out}]}{{\rm SNR}[X(Y)_1^{\rm in}]}, \tag{3}$$

$$\eta = \frac{\langle \hat{n}_3^{\rm out} \rangle}{\langle \hat{n}_1^{\rm in} \rangle} = \frac{\langle (\hat{a}_3^{\rm out})^{+} \hat{a}_3^{\rm out} \rangle}{\langle (\hat{a}_1^{\rm in})^{+} \hat{a}_1^{\rm in} \rangle}, \tag{4}$$

where ${\rm SNR}[X(Y)^{\rm in(out)}]$ is signal-to-noise ratio of the quadrature amplitude (phase) component of the input (output) mode. For an ideal frequency conversion of a quantum state, the signal-to-noise ratio on the output harmonic mode $\hat{a}_3^{\rm out}$ is identical to that of the input signal mode $\hat{a}_1^{\rm in}$, then we have $T_{X(Y)} = 1$ and the average photon number of two modes should be equal, i.e. $\eta = 1$. This is equivalent to maintaining the same quadrature component distribution in phase space before and after frequency conversion.

We consider the SHG process in the triply resonating optical cavity, which means the two subharmonic modes signal ($\hat{a}_1$ and $\hat{a}_2$) and the harmonic mode ($\hat{a}_3$) simultaneously resonate in the cavity. Under the ideal case with perfect phase matching, zero detuning, and small loss, the evolution equations for this system with one mirror used for the input and the output couplers can be given by[25]

$$\begin{aligned} \tau \dot{\hat{a}}_1(t) &= -(\gamma_1 + \rho_1)\hat{a}_1(t) + \chi \hat{a}_2^{+}(t)\hat{a}_3(t) \\ &\quad + \sqrt{2\gamma_1}\hat{a}_1^{\rm in}(t){\rm e}^{{\rm i}\,\theta_{10}} + \sqrt{2\rho_1}\hat{c}_1^{\rm in}(t), \end{aligned} \tag{5}$$

$$\begin{aligned} \tau \dot{\hat{a}}_2(t) &= -(\gamma_2 + \rho_2)\hat{a}_2(t) + \chi \hat{a}_1^{+}(t)\hat{a}_3(t) \\ &\quad + \sqrt{2\gamma_2}\hat{a}_2^{\rm in}(t){\rm e}^{{\rm i}\,\theta_{20}} + \sqrt{2\rho_2}\hat{c}_2^{\rm in}(t), \end{aligned} \tag{6}$$

$$\begin{aligned} \tau \dot{\hat{a}}_3(t) &= -(\gamma_3 + \rho_3)\hat{a}_3(t) - \chi \hat{a}_1(t)\hat{a}_2(t) \\ &\quad + \sqrt{2\gamma_3}\hat{a}_3^{\rm in}(t){\rm e}^{{\rm i}\,\theta_{30}} + \sqrt{2\rho_3}\hat{c}_3^{\rm in}(t), \end{aligned} \tag{7}$$

where $\hat{a}_i^{\rm in}$ ($i = 1, 2, 3$) denote the input amplitude operators, $\hat{c}_i^{\rm in}(t)$ is the vacuum noise term corresponding to intracavity losses, and χ is the effective nonlinear coupling parameter and is proportional to the second-order susceptibility $\chi^{(2)}$ of crystal. The roundtrip time τ in the cavity is assumed to be the same for all three fields. The total loss parameter is $\gamma_i + \rho_i$ ($i = 1, 2, 3$), where γ_i is related to amplitude reflection and transmission coefficient of the coupling mirror, and ρ_i ($i = 1, 2, 3$) represents the extra intracavity loss parameters.

Assuming that the two input subharmonic modes $\hat{a}_1^{\rm in}$ and $\hat{a}_2^{\rm in}$ have real amplitudes β_1 and β_2, the input harmonic mode $\hat{a}_3^{\rm in}$ is the mode in vacuum, the initial phases are taken to be $\theta_{10} = \pi/4$ and $\theta_{20} = \theta_{30} = 0$, and the cavity transmission factor and the extra loss for the two subharmonic modes are the same, then we have

$$\gamma_1 = \gamma_2 = \gamma, \quad \rho_1 = \rho_2 = \rho.$$

The steady-state equations of Eqs. (5)–(7) are then obtained as[26]

$$\begin{aligned} &-(\gamma + \rho)\bar{\alpha}_1 {\rm e}^{{\rm i}\,\theta_1} + \chi \bar{\alpha}_2^{*} \bar{\alpha}_3 {\rm e}^{{\rm i}(\theta_3 - \theta_2)} \\ &\quad + \sqrt{2\gamma}\beta_1 {\rm e}^{{\rm i}\,\pi/4} = 0, \end{aligned} \tag{8}$$

$$-(\gamma+\rho)\bar{\alpha}_2 e^{\mathrm{i}\,\theta_2}+\chi\bar{\alpha}_1^*\bar{\alpha}_3 e^{\mathrm{i}(\theta_3-\theta_1)}+\sqrt{2\gamma}\beta_2=0, \quad (9)$$
$$-(\gamma_3+\rho_3)\bar{\alpha}_3 e^{\mathrm{i}\,\theta_3}-\chi\bar{\alpha}_1\bar{\alpha}_2 e^{\mathrm{i}(\theta_1+\theta_2)}=0, \quad (10)$$

where $\bar{\alpha}_1$, $\bar{\alpha}_2$, and $\bar{\alpha}_3$ are the steady-state solutions of intracavity modes $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$ respectively. The SHG oscillation threshold $\varepsilon_{\mathrm{th}}$ and steady-state solutions are given by

$$\varepsilon_{\mathrm{th}}=\sqrt{\frac{2(\gamma+\rho)^3(\gamma_3+\rho_3)}{\chi^2\gamma}}, \quad (11)$$
$$\theta_1=\theta_3-\theta_2=\pi/4, \quad (12)$$
$$\bar{\alpha}_1\Big(\gamma+\rho+\frac{2\gamma\chi^2\beta_2^2}{(\gamma_3+\rho_3)[\gamma+\rho+\bar{\alpha}_1^2\chi^2/(\gamma_3+\rho_3)]^2}\Big)=\sqrt{2\gamma}\beta_1. \quad (13)$$

Equation (13) is a five-order equation about $\bar{\alpha}_1$, only the numerical solutions can be obtained when the other physical quantities are given. Substitute the numerical solutions $\bar{\alpha}_1$ into Eqs. (8)–(10), $\bar{\alpha}_2$ and $\bar{\alpha}_3$ can be given as

$$\bar{\alpha}_2=\frac{\sqrt{2\gamma}\beta_2}{\gamma+\rho+\bar{\alpha}_1^2\chi^2/(\gamma_3+\rho_3)}, \quad (14)$$
$$\bar{\alpha}_3=\frac{-\sqrt{2\gamma}\chi\beta_2\bar{\alpha}_1}{(\gamma_3+\rho_3)[\gamma+\rho+\bar{\alpha}_1^2\chi^2/(\gamma_3+\rho_3)]}. \quad (15)$$

Using the boundary condition[27] $\bar{\alpha}_i^{\mathrm{out}}=\sqrt{2\gamma_i}\bar{\alpha}_i-\bar{\alpha}_i^{\mathrm{in}}$, the conversion efficiency η is given by

$$\eta=\frac{\left|\sqrt{2\gamma_3}\bar{\alpha}_3\right|^2}{|\beta_1|^2}. \quad (16)$$

The dynamics of quantum fluctuations can be described by linearizing the equations of motion around the stationary solution through setting

$$\hat{a}_i(t)=\bar{\alpha}_i+\delta\hat{a}_i(t),$$
$$\hat{a}_i^{\mathrm{in}}(t)=\bar{\alpha}_i^{\mathrm{in}}+\delta\hat{a}_i^{\mathrm{in}}(t),$$
$$\hat{c}_i^{\mathrm{in}}(t)=\delta\hat{c}_i^{\mathrm{in}}(t). \quad (17)$$

Substituting Eq. (17) into Eqs. (5)–(7), we obtain the equations as

$$\tau\delta\dot{a}_1(t)=-(\gamma+\rho)\delta a_1(t)+\chi\bar{\alpha}_2\delta a_3(t)+\chi\bar{\alpha}_3\delta a_2^{+}(t)+\sqrt{2\gamma}\delta a_1^{\mathrm{in}}(t)+\sqrt{2\rho}c_1^{\mathrm{in}}(t), \quad (18)$$
$$\tau\delta\dot{a}_2(t)=-(\gamma+\rho)\delta a_2(t)+\chi\bar{\alpha}_1\delta a_3(t)+\chi\bar{\alpha}_3\delta a_1^{+}(t)+\sqrt{2\gamma}\delta a_2^{\mathrm{in}}(t)+\sqrt{2\rho}c_2^{\mathrm{in}}(t), \quad (19)$$
$$\tau\delta\dot{a}_3(t)=-(\gamma_3+\rho_3)\delta a_3(t)-\chi\bar{\alpha}_1\delta a_2(t)-\chi\bar{\alpha}_2\delta a_1(t)+\sqrt{2\gamma_3}\delta a_3^{\mathrm{in}}(t)+\sqrt{2\rho_3}c_3^{\mathrm{in}}(t). \quad (20)$$

Using the definitions of the amplitude and phase quadratures $X=a+a^{+}$ and $Y=(a-a^{+})/\mathrm{i}$, we obtain the fluctuation of output harmonic mode after Fourier transform

$$\delta X_3^{\mathrm{out}}(\omega)=\frac{1}{R_1}[A_1\delta X_1^{\mathrm{in}}(\omega)+B_1\delta X_2^{\mathrm{in}}(\omega)+C_1\delta X_3^{\mathrm{in}}(\omega)+D_1\delta X_{c1}^{\mathrm{in}}(\omega)+G_1\delta X_{c2}^{\mathrm{in}}(\omega)+H_1\delta X_{c3}^{\mathrm{in}}(\omega)], \quad (21)$$

where

$$\begin{aligned}
R_1 &= 2\bar{\alpha}_1\bar{\alpha}_2\bar{\alpha}_3\chi^3+\bar{\alpha}_1^2\chi^2(\gamma+\rho+\mathrm{i}\,\omega\tau)+\bar{\alpha}_2^2\chi^2(\gamma+\rho+\mathrm{i}\,\omega\tau)-\bar{\alpha}_3^2\chi^2(\gamma_3+\rho_3+\mathrm{i}\,\omega\tau)\\
&\quad+(\gamma+\rho+\mathrm{i}\,\omega\tau)^2(\gamma_3+\rho_3+\mathrm{i}\,\omega\tau),\\
A_1 &= -2\sqrt{\gamma\gamma_3}\chi(\bar{\alpha}_2\gamma+\bar{\alpha}_2\rho+\bar{\alpha}_1\bar{\alpha}_3\chi+\mathrm{i}\,\omega\tau\bar{\alpha}_2),\\
B_1 &= -2\sqrt{\gamma\gamma_3}\chi(\bar{\alpha}_1\gamma+\bar{\alpha}_1\rho+\bar{\alpha}_2\bar{\alpha}_3\chi+\mathrm{i}\,\omega\tau\bar{\alpha}_1),\\
C_1 &= 2\gamma_3[\gamma^2+2\gamma\rho+\rho^2+\bar{\alpha}_3^2\chi^2+\omega^2\tau^2+\mathrm{i}\,\omega\tau(2\gamma+2\rho)]-R_1,\\
D_1 &= -2\sqrt{\gamma_3\rho}\chi(\bar{\alpha}_2\gamma+\bar{\alpha}_2\rho+\bar{\alpha}_1\bar{\alpha}_3\chi+\mathrm{i}\,\omega\tau\bar{\alpha}_2),\\
G_1 &= -2\sqrt{\gamma_3\rho}\chi(\bar{\alpha}_2\gamma+\bar{\alpha}_1\rho+\bar{\alpha}_2\bar{\alpha}_3\chi+\mathrm{i}\,\omega\tau\bar{\alpha}_1),\\
H_1 &= 2\sqrt{\gamma_3\rho_3}[\gamma^2+2\gamma\rho+\rho^2+\bar{\alpha}_3^2\chi^2+\omega^2\tau^2+\mathrm{i}\,\omega\tau(2\gamma+2\rho)],
\end{aligned}$$

$$\delta Y_3^{\mathrm{out}}(\omega)=\frac{1}{R_2}[A_2\delta Y_1^{\mathrm{in}}(\omega)+B_2\delta Y_2^{\mathrm{in}}(\omega)+C_2\delta Y_3^{\mathrm{in}}(\omega)+D_2\delta Y_{c1}^{\mathrm{in}}(\omega)+G_2\delta Y_{c2}^{\mathrm{in}}(\omega)+H_2\delta Y_{c3}^{\mathrm{in}}(\omega)], \quad (22)$$

with

$$\begin{aligned}
R_2 &= -2\bar{\alpha}_1\bar{\alpha}_2\bar{\alpha}_3\chi^3+\bar{\alpha}_1^2\chi^2(\gamma+\rho+\mathrm{i}\,\omega\tau)+\bar{\alpha}_2^2\chi^2(\gamma+\rho+\mathrm{i}\,\omega\tau)-\bar{\alpha}_3^2\chi^2(\gamma_3+\rho_3+\mathrm{i}\,\omega\tau)\\
&\quad+(\gamma+\rho+\mathrm{i}\,\omega\tau)^2(\gamma_3+\rho_3+\mathrm{i}\,\omega\tau),\\
A_2 &= -2\sqrt{\gamma\gamma_3}\chi(\bar{\alpha}_2\gamma+\bar{\alpha}_2\rho-\bar{\alpha}_1\bar{\alpha}_3\chi+\mathrm{i}\,\omega\tau\bar{\alpha}_2),\\
B_2 &= -2\sqrt{\gamma\gamma_3}\chi(\bar{\alpha}_1\gamma+\bar{\alpha}_1\rho-\bar{\alpha}_2\bar{\alpha}_3\chi+\mathrm{i}\,\omega\tau\bar{\alpha}_1),\\
C_2 &= 2\gamma_3[\gamma^2+2\gamma\rho+\rho^2-\bar{\alpha}_3^2\chi^2-\omega^2\tau^2+\mathrm{i}\,\omega\tau(2\gamma+2\rho)]-R_2,\\
D_2 &= -2\sqrt{\gamma_3\rho}\chi(\bar{\alpha}_2\gamma+\bar{\alpha}_2\rho-\bar{\alpha}_1\bar{\alpha}_3\chi+\mathrm{i}\,\omega\tau\bar{\alpha}_2),\\
G_2 &= -2\sqrt{\gamma_3\rho}\chi(\bar{\alpha}_1\gamma+\bar{\alpha}_1\rho-\bar{\alpha}_2\bar{\alpha}_3\chi+\mathrm{i}\,\omega\tau\bar{\alpha}_1),\\
H_2 &= 2\sqrt{\gamma_3\rho_3}[\gamma^2+2\gamma\rho+\rho^2-\bar{\alpha}_3^2\chi^2-\omega^2\tau^2+\mathrm{i}\,\omega\tau(2\gamma+2\rho)].
\end{aligned}$$

Substituting $\delta^2X_i^{\mathrm{in}}=\delta^2Y_i^{\mathrm{in}}=1$ into Eqs. (21) and (22), we have[28,29]

$$T_X=\frac{\mathrm{SNR}[X_3^{\mathrm{out}}]}{\mathrm{SNR}[X_1^{\mathrm{in}}]}=\frac{A_1^2}{A_1^2+B_1^2+C_1^2+D_1^2+G_1^2+H_1^2}, \quad (23)$$
$$T_Y=\frac{\mathrm{SNR}[Y_3^{\mathrm{out}}]}{\mathrm{SNR}[Y_1^{\mathrm{in}}]}=\frac{A_2^2}{A_2^2+B_2^2+C_2^2+D_2^2+G_2^2+H_2^2}. \quad (24)$$

3. Results and discussion

The dependences of signal transfer coefficient $T_{X(Y)}$ and conversion efficiency η on pump amplitude (β_2) under different values of input signal amplitude (β_1) are shown in Fig. 2.

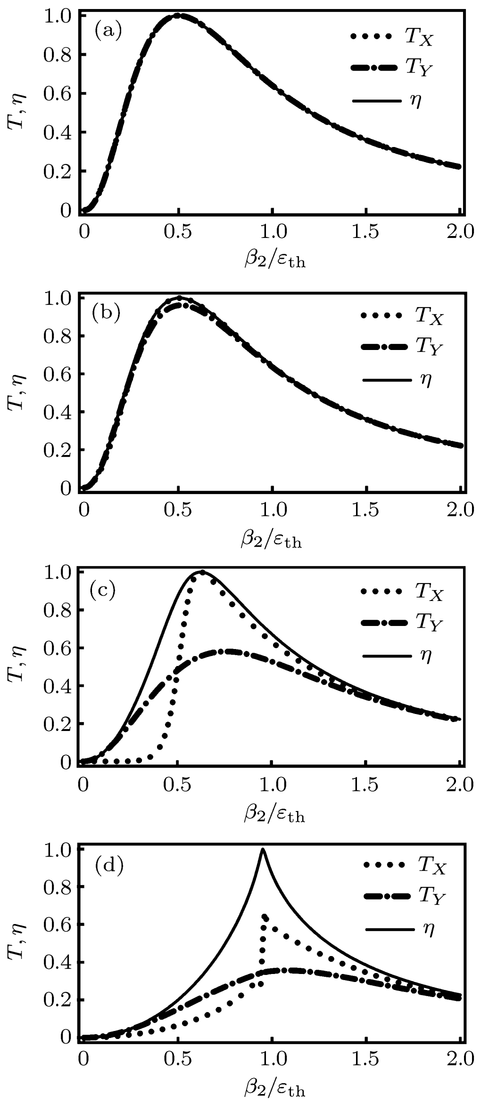

Fig. 2. Signal transfer coefficients T_X, T_Y, and conversion efficiency η versus normalized input pump mode amplitude ($\beta_2/\varepsilon_{\rm th}$) under different values of input signal mode amplitude β_1: (a) $0.01\varepsilon_{\rm th}$, (b) $0.1\varepsilon_{\rm th}$, (c) $0.5\varepsilon_{\rm th}$, (d) $0.95\varepsilon_{\rm th}$ with $\gamma = \gamma_3 = 0.1$, $\rho = \rho_3 = 0$, $\chi = 0.001$, and $\varOmega = \omega\tau/\gamma = 0$.

When the input signal amplitude β_1 is far below the threshold, e.g. $\beta_1 = 0.01\varepsilon_{\rm th}$ in Fig. 2(a), both the signal transfer coefficients T_X, $T_{\rm Y}$, and conversion efficiency η increase monotonically with the increase of the pump amplitude β_2 in the beginning, and they reach the optimum value 1 simultaneously at $\beta_2 = 0.5\varepsilon_{\rm th}$ as shown in Fig. 2(a), which means that almost ideal noise-free frequency up-conversion from $\hat{a}_1^{\rm in}$ to $\hat{a}_3^{\rm out}$ is realized. If the pump amplitude β_2 further increase, the T_X, T_Y, and η will decrease, which shows that for the SHG operating below the threshold, an appropriate pump amplitude is needed for frequency conversion. Note that the signal transfer coefficient of quadrature amplitude (phase) component $T_{X(Y)}$ is always equal to conversion efficiency η under the condition of Fig. 2(a). When the input signal amplitude β_1 increases to $0.1\varepsilon_{\rm th}$ even $0.5\varepsilon_{\rm th}$ (Figs. 2(b) and 2(c)), both T_X and η can approach to 1 almost at the same time, but the $T_{\rm Y}$ drops below 1. Figure 2(d) shows that the T_X as well as T_Y begins to decrease when the signal mode operates near but below the threshold. In this case, the quadrature component squeezing of the harmonic mode will influence the signal transfer coefficient. Accordingly, the similarity between subharmonic and harmonic modes in phase space will decrease. This result accords with the conclusion obtained in Ref. [30], in which the closer to the threshold value the excited subharmonic intensity, the higher the squeezing of the harmonic mode is.[30,31]

When the pump amplitude β_2 is set to be $0.5\varepsilon_{\rm th}$, to what extent does the signal influence the signal transfer coefficient and conversion efficiency? Figure 3 shows that when the signal amplitude β_1 is weak enough ($\leq 0.1\varepsilon_{\rm th}$), the three parameters keep the same and are equal to 1. Once the signal amplitude is larger than $0.1\varepsilon_{\rm th}$, the parameter $T_{\rm Y}$ decreases monotonically, however, the transfer coefficient of quadrature amplitude component T_X and conversion efficiency η keep constant in the beginning and then decrease rapidly in this period. So a weaker signal field is suited for the frequency up-conversion of quantum state.

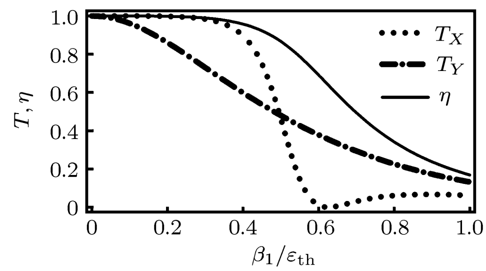

Fig. 3. Signal transfer coefficients T_X, T_Y, and conversion efficiency η versus β_1 with $\beta_2 = 0.5\varepsilon_{\rm th}$, $\gamma = \gamma_3 = 0.1$, $\rho = \rho_3 = 0$, $\chi = 0.001$, and $\varOmega = 0$.

Figure 4 shows that the signal transfer coefficient of quadrature components T_X and $T_{\rm Y}$ versus normalized analysing frequency $\varOmega$ when the SHG is operated below the threshold with $\beta_1 = 0.01\varepsilon_{\rm th}$ and $\beta_2 = 0.5\varepsilon_{\rm th}$. Obviously, at zero frequency ($\varOmega = 0$), the maximum signal transfer coefficient is obtained.

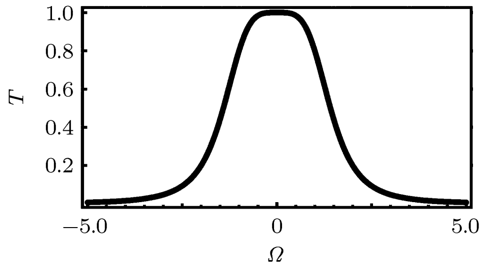

Fig. 4. Signal transfer coefficients T_X, T_Y versus normalized analysing frequency Ω with $\beta_1 = 0.01\varepsilon_{\rm th}$, $\beta_2 = 0.5\varepsilon_{\rm th}$, $\gamma = \gamma_3 = 0.1$, $\rho = \rho_3 = 0$, and $\chi = 0.001$.

From the above analyses, it is evident that the perfect frequency up-conversion of quantum state in an optical cavity can be easily obtained when the cavity is operated far below the threshold with a weak signal and strong pump of proper amplitude. Note that in this discussion, the extra intracavity loss is set to be $\rho_i = 0$ if the unavoidable loss is included, the signal transfer coefficient for quadrature components and conversion efficiency for photon number will decrease monotonically as shown in Fig. 5. So in order to achieve a high performance frequency up-conversion of quantum state, the extra intracavity loss should be far less than the transmission loss from the coupling mirror $\rho_i \ll \gamma_i$.

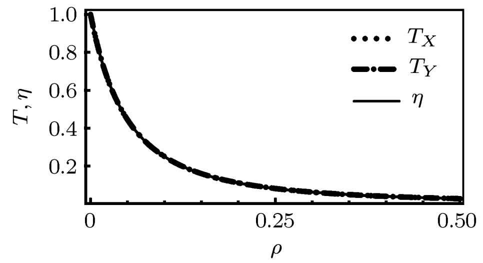

Fig. 5. Signal transfer coefficients T_X, T_Y, and conversion efficiency η versus extra cavity loss $\rho = \rho_3$ with $\beta_1 = 0.01\varepsilon_{\rm th}$, $\beta_2 = 0.5\varepsilon_{\rm th}$, $\gamma = \gamma_3 = 0.1$, $\chi = 0.001$, and $\Omega = 0$.

4. Conclusion

The theoretical discussion shows that the noise-free frequency up-conversion of quantum state with asymmetrically pumped SHG in cavity can be realized when the cavity operates far below the threshold with a weaker signal and proper pump amplitude. The dependences of the signal transfer coefficient and the conversion efficiency on the parameter of the cavity are discussed. The quantum state conversion between different frequencies may have applications in quantum network and quantum computation processes. The experimental realization of this scheme should be achieved with the successful SHG technology.

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