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Generation of Enhanced Three-Mode Continuously Variable Entanglement *

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The generation of enhanced three-mode continuously variable (CV) entanglement via difference-frequency amplification in an optical cavity above the threshold is investigated. The quantum entanglement characteristics among the pump, signal, and idler beams are demonstrated by applying a sufficient inseparability criterion for CV entanglement proposed by van Loock and Furusawa. Bright three-mode CV entanglement with different frequencies can be generated in this simple system when the optical cavity operates above its threshold, and the best three-mode CV entanglement can be obtained when the pump threshold parameter is modulated at about $\sigma = 1.3$.

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Quantum entanglement attracts a good deal of interest since it is the central resource in applications such as quantum communication and computation. Multipartite CV entanglement beams with different frequencies are necessary to connect different physical systems at the nodes of quantum networks[1] since multicolor entanglement beams are easy to separate in the application.[2] It was predicted that two-color CV entanglement can be produced by a nondegenerate optical parametric oscillator[3] and it has been demonstrated experimentally below and above the oscillator threshold.[4,5] Then Villar *et al.* predicated that three-color CV entanglement can be generated using an optical parametric oscillator operating above the threshold[6] and this scheme has been verified by recent experiments.[2] In fact, when one pump photon is destroyed, exactly two photons are created, i.e., when N pump photons are destroyed, $2N$ longer-wavelength photons are created. Thus the intensities of three beams, pump, signal and idler are correlated when the losses and uncertainties are minimized.[7] There are numerous schemes to produce multicolor CV entanglement, such as cascaded nonlinearities inside an optical cavity[8−11] and four-wave mixing.[12]

Usually, the parametric process is weak. Therefore, the difference-frequency amplification is easier to realize experimentally in comparison with the parametric process. In the difference-frequency amplification process, a pump photon and an idler photon produce a signal photon, and the signal and idler beams are amplified by each other via interaction with the pump. If one puts the difference-frequency amplification inside an optical cavity, the signal and idler beams are further amplified for the energy transferred from pump to signal and idler beams duo to the energy conservation. Thus the intensities of pump, signal, and idler beams correlate when the losses and uncertainties are minimized, which is similar to the case of an optical parametric oscillator operating above the threshold.[2,6] The three-mode CV entanglement should exist in the difference-frequency amplification process when considering the quantum characteristics of the pump. In this Letter, we investigate whether the enhanced three-mode CV entanglement can indeed be generated in the difference-frequency amplification in an optical cavity operated above the threshold.

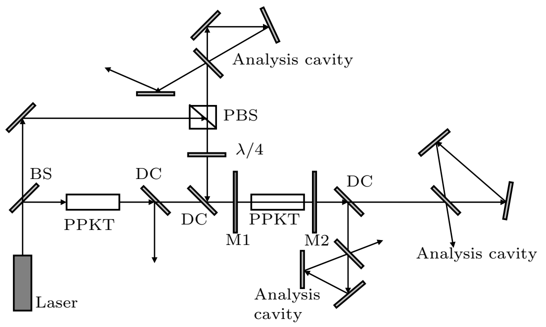

Fig. 1. A schematic of the experiment. BS: 50/50 beam splitter; PPKTP: periodically poled $KTiOPO_4$ crystal used to produce signal and idler beams; DC: dichroic splitter; PBS: polarizing beam splitter; M1 and M2: cavity mirrors.

Our study is different from the optical parametric oscillator above the threshold reported in Refs. [2, 6], which has only one pump entering the optical cavity and the nonlinear process is the parametric down-conversion. Here we consider two beams, a pump with frequency ω_0 and a weak idler with frequency ω_1 entering an optical cavity. The signal beam with frequency ω_2 is generated by the difference-frequency process between the pump and idler beams (i.e. $\omega_0 = \omega_1 + \omega_2$). A schematic of the experiment can be seen in Fig. 1. Our experimental scheme is similar to that used in Ref. [2]. However, there is a weak idler beam seed entering the cavity which is different from the case in Ref. [2]. It

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is well known that the seed can not only enhance the stability of the cavity, but also increase the nonlinear conversion efficiency. Therefore, it is easier to be experimentally realized than the scheme in Ref. [2] and better enhanced three-mode entanglement can be obtained in our scheme by the difference-frequency generation process.

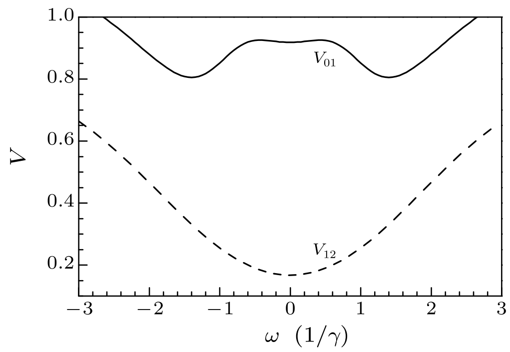

Fig. 2. The minimized values of V_{01} and V_{12} versus normalized analyzing frequency $\omega = \nu/\gamma$ with $\gamma = 0.015$, $\gamma_0 = 0.025$, and $\sigma = 2$.

The Hamiltonian of the free modes can be written as[13]

$$H_f = \hbar\Delta_0\gamma_0 a_0^\dagger a_0 + \hbar\Delta_1\gamma a_1^\dagger a_1 + \hbar\Delta_2\gamma a_2^\dagger a_2. \tag{1}$$

Here we introduce the detuning parameters $\Delta_0 = \frac{\omega_0-\varpi_0}{\gamma_0}$, $\Delta_1 = \frac{\omega_1-\varpi_1}{\gamma}$, and $\Delta_2 = \frac{\omega_2-\varpi_2}{\gamma}$; ϖ_0, ϖ_1, and ϖ_2 are the quasi-resonant frequencies of the three modes, respectively; γ_0 is the damping rate of ω_0. In this study we use the same damping rate for these modes (as $\gamma_1 = \gamma_2 = \gamma$) for simplicity.

The interaction Hamiltonian is $H_I = i\hbar g(a_1^\dagger a_2^\dagger a_0 - a_1 a_2 a_0^\dagger)$, where g is the dimensionless coupling constant. Considering that the system is driven by an external pump[13] $H_{ext} = i\hbar\varepsilon(a_0^\dagger - a_0)$, where ε is taken to be real and positive for definiteness, and the losses of the three modes are given by $\Lambda_i\rho = \gamma_i(2a_i\rho a_i^\dagger - a_i^\dagger a_i\rho - \rho a_i^\dagger a_i)$.

The master equation for the density operator ρ of this three-mode system in the interaction picture reads[14]

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_f + H_I + H_{ext}, \rho] + (\Lambda_0 + \Lambda_1 + \Lambda_2)\rho. \tag{2}$$

When all fluctuations and correlations are neglected, the equations for the mean values of the three modes are[13–15]

$$\begin{aligned}
\frac{d}{dt}\alpha_1 &= -\gamma(1+i\Delta_1)\alpha_1 + g\alpha_2^*\alpha_0,\\
\frac{d}{dt}\alpha_2 &= -\gamma(1+i\Delta_2)\alpha_2 + g\alpha_1^*\alpha_0,\\
\frac{d}{dt}\alpha_0 &= \varepsilon - \gamma_0(1+i\Delta_0)\alpha_0 - g\alpha_1\alpha_2,
\end{aligned} \tag{3}$$

where α_i^* is the complex conjugate of α_i. After introducing the normalized quantities $A_i = \frac{g\alpha_i}{\sqrt{\gamma_0\gamma}} (i = 1, 2)$, $A_0 = \frac{g\alpha_0}{\gamma}$, and $E = \frac{g\varepsilon}{\gamma\gamma_0}$, Eq. (3) can be rewritten as

$$\begin{aligned}
\frac{1}{\gamma}\frac{d}{dt}A_1 &= -(1+i\Delta_1)A_1 + A_2^*A_0,\\
\frac{1}{\gamma}\frac{d}{dt}A_2 &= -(1+i\Delta_2)A_2 + A_1^*A_0,\\
\frac{1}{\gamma_0}\frac{d}{dt}A_0 &= E - (1+i\Delta_0)A_0 - A_1A_2.
\end{aligned} \tag{4}$$

The stationary solutions can be obtained by setting $\frac{dA_0}{dt} = \frac{dA_1}{dt} = \frac{dA_2}{dt} = 0$. There is a trivial solution $A_0 = \frac{E}{1+i\Delta_0}$ when $A_1 = A_2 = 0$. Let $\sigma = \frac{|E|^2}{|A_0|^2}$ be the pump intensity normalized to the pump threshold. Setting the phases of A_1, A_2, and A_0 to be φ_1, φ_2, and φ_0, respectively, then $A_1 = |A|e^{i\varphi_1}$, $A_2 = |A|e^{i\varphi_2}$, and $A_0 = |A_0|e^{i\varphi_0}$. One can obtain $e^{i(\varphi_1+\varphi_2)} = \frac{E}{(1+i\Delta)(1+i\Delta_0)+|A|^2}$. In order to simplify the calculation we assume $\varphi_1 = \varphi_2 = \varphi$. Finally, we obtain the stationary solution as

$$\begin{aligned}
&A_1 = A_2,\\
&e^{i2\varphi} = \frac{E}{(1+i\Delta)(1+i\Delta_0)+|A_1|^2},\\
&e^{i\varphi_0} = \frac{1+i\Delta}{\sqrt{1+\Delta^2}}e^{i2\varphi}.
\end{aligned} \tag{5}$$

In this case, the fluctuations can be obtained by solving the classical equations for the fields linearized around the considered mean values.[13–15] The source terms include the vacuum fluctuations entering through the coupling mirror of the cavity. One can obtain $\frac{d\boldsymbol{A}}{dt} = \boldsymbol{M}\boldsymbol{A} + \boldsymbol{B}\boldsymbol{A}^{in}$, where $\boldsymbol{A} = [\delta A_1, \delta A_1^*, \delta A_2, \delta A_2^*, \delta A_0, \delta A_0^*]^{\mathrm{T}}$, $\boldsymbol{B} = \mathrm{diag}[\sqrt{2\gamma}, \sqrt{2\gamma}, \sqrt{2\gamma}, \sqrt{2\gamma}, \sqrt{2\gamma_0}, \sqrt{2\gamma_0}\ ]$ is the diagonal matrix of the transmission coefficients, and $\boldsymbol{A}^{in}$ is source of the fields; $\boldsymbol{M}$ is the drift matrix:

$$\boldsymbol{M}=\begin{bmatrix}
-\gamma(1+i\Delta) & 0 & 0 & \gamma A_0 & \gamma A_2^* & 0\\
0 & -\gamma(1-i\Delta) & \gamma A_0^* & 0 & 0 & \gamma A_2\\
0 & \gamma A_0 & -\gamma(1+i\Delta) & 0 & \gamma A_1^* & 0\\
\gamma A_0^* & 0 & 0 & -\gamma(1-i\Delta) & 0 & \gamma A_1\\
-\gamma_0 A_2 & 0 & -\gamma_0 A_1 & 0 & -\gamma_0(1+i\Delta_0) & 0\\
0 & -\gamma_0 A_2^* & 0 & -\gamma_0 A_1^* & 0 & -\gamma_0(1-i\Delta_0)
\end{bmatrix}. \tag{6}$$

The modes of the cavity can be solved in the frequency domain by Fourier transformation: $\boldsymbol{A}(\nu) = (i\nu\boldsymbol{I} - \boldsymbol{M})^{-1}\boldsymbol{B}\boldsymbol{A}(\nu)^{\text{in}}$, where ν is the analysis frequency. Applying the equation between the input fields and the output fields at the coupling mirror,[16] i.e. $\boldsymbol{A}(\nu)^{\text{out}} = \boldsymbol{B}\boldsymbol{A}(\nu) - \boldsymbol{A}(\nu)^{\text{in}}$, one can obtain the output fields as $\boldsymbol{A}(\nu)^{\text{out}} = [\boldsymbol{B}(i\nu\boldsymbol{I} - \boldsymbol{M})^{-1}\boldsymbol{B} - \boldsymbol{I}]\boldsymbol{A}(\nu)^{\text{in}}$, where $\boldsymbol{I}$ is the identity matrix.

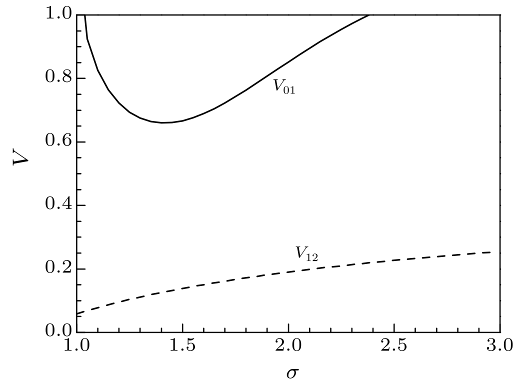

Fig. 3. The minimized values of V_{01} and V_{12} versus normalized pump power σ with $\gamma = 0.015$, $\gamma_0 = 0.025$, and $\omega = 1$.

We define the quadrature amplitude and phase components as $X_i = \frac{\alpha_i+\alpha_i^\dagger}{2}$ and $Y_i = \frac{\alpha_i-\alpha_i^\dagger}{2i}$. The sum of the energy of the signal and idler beams is equal to the energy of the pump beam. Thus the quadrature amplitudes of the signal and idler beams are anti-correlated to that of pump. According to the sufficient inseparability criterion for CV multimode entanglement proposed by van Loock and Furusawa, for this three-mode system, we obtain the inequalities[17]

$$\begin{aligned}
V_{01} &= \langle\delta^2(\frac{X_0+X_1}{\sqrt{2}})\rangle + \langle\delta^2(\frac{Y_1-Y_0}{\sqrt{2}} + g_2Y_2)\rangle \geq 1,\\
V_{12} &= \langle\delta^2(\frac{X_1-X_2}{\sqrt{2}})\rangle + \langle\delta^2(\frac{Y_1+Y_2}{\sqrt{2}} + g_3Y_0)\rangle \geq 1,\\
V_{20} &= \langle\delta^2(\frac{X_2+X_0}{\sqrt{2}})\rangle + \langle\delta^2(\frac{Y_2-Y_0}{\sqrt{2}} + g_1Y_1)\rangle \geq 1,
\end{aligned} \tag{7}$$

where g_i $(i = 1, 2, 3)$ are adjustable factors. The minimized values on the left of the inequalities can be obtained by choosing an appropriate g_i. Violation of any pair of the above inequalities is sufficient for full inseparability of three modes.[17]

Since the signal is exchangeable with the idler, the inequality V_{01} is equal to V_{20}. In the following we only calculate the equalities of V_{01} and V_{12}. In this case, for simplicity, we assume that the three modes are all on resonance in the cavity with $\varDelta = \varDelta_0 = 0$. Figure 2 shows that the minimized values of V_{01} and V_{12} versus normalized analyzing frequency $\omega = \nu/\gamma$ with $\gamma = 0.015$, $\gamma_0 = 0.025$, and $\sigma = 2$. From Fig. 2 one can see that the minimized values of the inequalities are both below 1 in a wide frequency range which is enough to assert that the inequalities V_{01} and V_{12} are violated. This indicates that the signal, idler and pump beams are CV entangled with each other. Moreover, the intensities of signal and idler beams are enhanced by a difference-frequency process and the optical cavity operating above the threshold. A bright three-mode entanglement can be produced in this scheme.

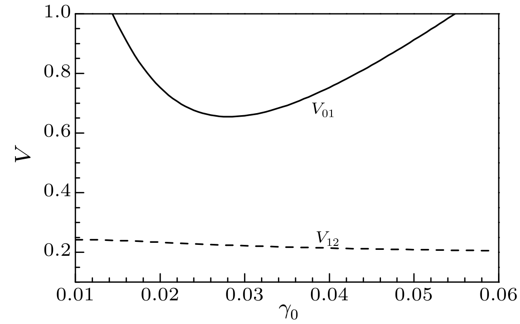

Fig. 4. The minimized values of V_{01} and V_{12} versus the damping constant γ_0 with $\gamma = 0.015$, $\sigma = 1.5$, and $\omega = 1$.

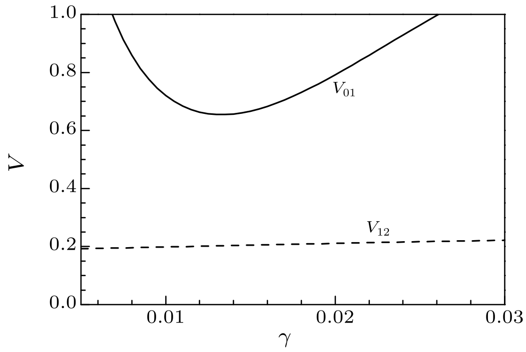

Fig. 5. The minimized values of V_{01} and V_{12} versus the damping constant γ with $\gamma_0 = 0.025$, $\sigma = 1.5$, and $\omega = 1$.

In Fig. 3 we plot the minimized values of V_{01} and V_{12} versus normalized pump power σ with $\gamma = 0.015$, $\gamma_0 = 0.025$, and $\omega = 1$. Figure 3 clearly shows that V_{12} increases with the increasing normalized pump power, but V_{01} decreases initially, then increases. Moreover, there is no quantum correlation between the pump and idler beams, only signal and idler are entangled, when the pump power is small. Since the nonlinear conversion efficiency is low when the pump power is below the threshold, the pump power is much higher than that of the idler and signal beams, and its quantum characteristic does not present. When the pump power is above the threshold, the signal power will increase with the increasing conversion efficiency and the idler power will also increase due to the difference-frequency amplification at the same time. However, the pump power becomes lower for the depletion and its quantum characteristics starts to appear above the oscillation threshold at this time which is similar to the case in the optical parametric oscillator above the threshold in Refs. [2, 6]. With the increase of pump power, one can see that the best three-mode entanglement can be obtained at about $\sigma = 1.3$, where the inequality has a minimum. Then, the value of V_{01} increases with the increasing normalized pump power. With the further increase, the gain begins to saturate, the pump power intensity is larger than that of the signal and idler beams and its quantum characteristics vanishes when the value of V_{01} exceeds 1.

Figure 4 plots the minimized values of V_{01} and V_{12} versus the damping constant γ_0 with $\gamma = 0.015$, $\sigma = 1.5$, and $\omega = 1$. One can see that the best three-mode entanglement can be obtained at about $\gamma_0 = 0.028$. We also plot the minimized values of V_{01} and V_{12} versus the damping constant γ in Fig. 5 with $\gamma_0 = 0.025$, $\sigma = 1.5$, and $\omega = 1$. It is easy to find that the best three-mode entanglement can be achieved at about $\gamma = 0.013$. When the losses are very small, the damping constants are related to the amplitude transmission coefficients t_i by $t_i^2 = 2\gamma_i$.[9] A larger transmission coefficient can make the optical field have more losses in the cavity. By contrast, a smaller transmission coefficient can make the output field become weaker. Therefore, in order to obtain the best entangled beams, one has to choose appropriate transmission coefficients according to the damping constants by theoretical calculation.

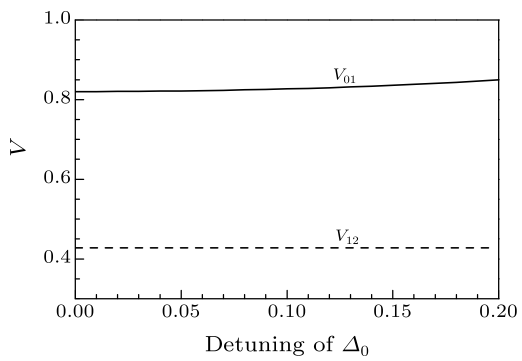

Fig. 6. The minimized values of the inequalities versus the detuning parameter Δ_0.

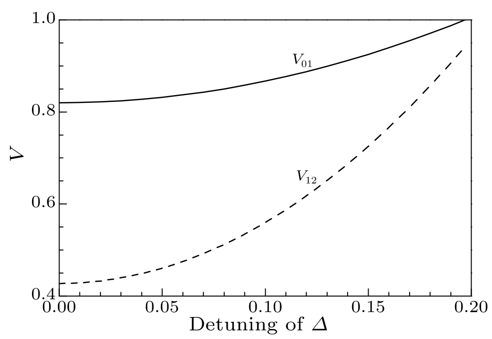

Fig. 7. The minimized values of the inequalities versus the detuning parameter Δ.

In the above discussions we have taken $\Delta = \Delta_0 = 0$ for the triple resonance condition. It is interesting to see what happens when certain frequency detuning exists. In Fig. 6 we plot the minimized values of the inequalities versus the detuning parameter Δ_0 with $\gamma = 0.02$, $\gamma_0 = 0.01$, $\Omega = 0.2, \sigma = 2$, and $\Delta = 0$. One can see that there is little influence on the three-mode entanglement when the pump deviates from resonance. Figure 7 plots the minimized values of the inequalities versus the detuning parameter Δ. It shows that the three-mode entanglement is sensitive to a change of Δ. However, there is little effect on the three-mode entanglement characteristics when frequency detuning is very small. When $\Delta = \Delta_0 = 0$ the degree of three-mode squeezing is at its greatest, as is the degree of entanglement. When the frequencies deviate from the resonance frequency, both the degree of entanglement and the degree of squeezing become lower, which indicates a close relation between the two quantum effects.

In summary, we have proposed a simple scheme to directly produce bright enhanced three-mode CV entanglement by putting difference-frequency amplification inside an optical cavity operating above the threshold. The pump, signal and idler are CV entangled with each other by applying a sufficient inseparability criterion for CV multimode entanglement. Because the entering seed can not only enhance the stability of the cavity, but also increase the nonlinear conversion efficiency, our scheme is easier to realize experimentally than the scheme used in Ref. [2]. The intensities of the entanglement beams are enhanced by the difference-frequency generation process and the enhanced three-mode CV entanglement can be generated when the optical cavity is operating above the threshold. When the pump power is small, one can only obtain signal and idler two-mode CV entanglement. The best bright three-mode entanglement can be obtained at $\sigma = 1.3$. In addition, the three-mode entanglement characteristic is related to the damping constants and the detuning of frequency. The results indicate that the three-mode CV entanglement is rather more sensitive to the detuning of Δ than of Δ_0. The maximum degree of entanglement can be obtained when $\Delta = \Delta_0 = 0$. Theoretical calculation may provide referenced data for experimental studies. This enhanced three-mode CV entanglement can be taken as a resource in the applications of quantum communication and computation networks.

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