Robust preparation of atomic W states without any excitations

Rong-Can Yang, Tian-Cai Zhang

State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan 030006, PR China
School of Physics and Opto-Electronics Technology, Fujian Normal University, Fuzhou 350007, PR China

ARTICLE INFO

Article history:
Received 29 September 2010
Received in revised form 13 December 2010
Accepted 19 February 2011
Available online 9 March 2011

Keywords:
W state
Cavity QED
Virtual excitation
Casimir effect

ABSTRACT

A scheme is proposed for the preparation of atomic W states with virtual excitations. In the protocol, one of two transitions for A-type atoms is resonant with cavity mode, and the other transition is driven by a weak laser pulse with an appropriate detuning, reducing the influence of Casimir effect on the system vacuum. We also discuss the influence of the atom and photon decays on the W-state preparation, which shows that the system may be robust and feasible in the strong-coupling range.

Quantum entanglement, first proposed by Einstein et al. [1] and known today as a key feature of quantum mechanics, plays an important role in test of the quantum nonlocality [2–4] and the implementation of quantum communications [5,6]. So far, most of the mysteries regarding two-qubits entanglement have been revealed. By contrast, entanglement among three or more qubits still remains as a challenge due to its richer and more complex structures [7]. It is well known that entanglement can be divided into several classes which cannot be transformed to each other under stochastic local operations and classical communications [8,9]. For example, there are two classes for three-qubits entanglement (GHZ-type and W-type states) [8] and nine ones for four-qubits [10]. Recently, much attention has been focused on W states with the form

\[ |W\rangle = \frac{1}{\sqrt{10}} \left( |100\rangle + |010\rangle + \cdots + |001\rangle \right). \]  

The reason is that these states have the persistent property to protect entanglement between the remaining qubits against the particle loss and are immune to global dephasing and bit-flip noise [11]. Thus the preparation of W states becomes a popular topic in quantum information science [12–20].

Cavity quantum electrodynamics (CQED), known as an effective system to study the interaction between light and atoms (or ions, atomic ensembles, etc.) in a confined space, is thought to be a potential candidate for the demonstration of quantum state engineering [21–25]. Based on CQED, there have been many proposals suggested for the generation of W states [26–31]. However, in those schemes, the excitation is usually used to store (carry) quantum information, making the system sensitive to the decay. In order to solve this difficulty, Zheng demonstrated a method in which the excitations are virtual [32], which is then generally used for state preparation [33–36] and gate implementation [37], etc. In these works, the Rabi frequency of classical field focused on the atoms is usually not smaller than the atom-cavity rate in the large detuning. In this case, laser power should be large enough to maintain the system evolution, especially in the strong-coupling range \( |g^2| > \kappa, \gamma \). However, the larger power of the laser injected into the cavity is, the higher temperature the cavity mirrors gets and the more obvious change of the effect length of the cavity is. As a result, the vacuum within the cavity will be strongly distorted due to Casimir effect first predicted by Casimir [38] and then extensively studied with cavity QED [39–41]. In this paper, we show an alternate method to prepare W states in a cavity without any excitations, where only weak laser pulse is needed to be focused on the atoms, reducing the effect of the laser pulse on the system’s vacuum in the cavity. The main idea is that one atomic transition for A-type atoms is driven by a weak laser pulse with a suitable detuning and the other atomic transition resonant with cavity field, so that the higher levels of atoms and photons can be virtually excited.

Now, let us begin our proposal. Suppose that there are \( n \) A-type atoms trapped in a single-mode cavity. Each atom has one excited level \( |e\rangle \) and two ground levels \( |g\rangle \) and \( |s\rangle \). A classical laser pulse is focused on atoms to drive the \( |g\rangle \rightarrow |e\rangle \) transition with a detuning \( \Delta \).
while the \(|s\rangle \to |e\rangle\) transition is resonant with cavity mode. The illustration is shown in Fig. 1. In the rotating-wave approximation, the interaction Hamiltonian for the whole system, in the interacting picture, is given by

\[
H_{\text{int}} = -\Delta \sum_{j=1}^{n} \hat{g}_j^\dagger \hat{g}_j + \sum_{j=1}^{n} \Omega_j |e\rangle_j \langle g | + \sum_{j=1}^{n} \hat{g}_j^\dagger \hat{g}_j |s\rangle a + H.c.,
\]

where the subscript \(j\) represents \(j\)-th atom, \(a (a^\dagger)\) is the annihilation (creation) operator for cavity field and \(\hat{\Omega}(\hat{g})\) indicates the Rabi frequency of laser pulses (atom–cavity coupling coefficient). \(H.c\). stands for Hermitian conjugate. In this Hamiltonian, the first term describes the detuning of ground state \(|g\rangle\), the second one shows the interaction between classical field and the \(|g\rangle \to |e\rangle\) transition, while the third term depicts the resonant interaction between the \(|s\rangle \to |e\rangle\) transition and cavity mode.

For simplicity, we consider a symmetrical situation \(\Omega_2 = \Omega_3 = \cdots = \Omega_n = \Omega\), \(g_2 = g_3 = \cdots = g_n = g\) and suppose that \(\Omega_1\), \(\Omega\), \(g_1\) and \(g\) are real. In the closing subspace

\[
|\xi_1\rangle = |ss\rangle|123\cdots n\rangle 0, \quad |\xi_2\rangle = |ss\rangle|123\cdots n\rangle 1, \\
|\xi_3\rangle = |s\rangle \Omega^\dagger \frac{1}{\sqrt{\Delta-1}} (|\Delta-ss\rangle|234\cdots n\rangle + |ss\rangle|234\cdots n\rangle - |ss\rangle|234\cdots n\rangle 0), \\
|\xi_4\rangle = |s\rangle \Omega^\dagger \frac{1}{\sqrt{\Delta-1}} (|\Delta-ss\rangle|234\cdots n\rangle + |ss\rangle|234\cdots n\rangle + |ss\rangle|234\cdots n\rangle 0),
\]

the interaction Hamiltonian of Eq. (2) reduces to

\[
H_{\text{int}} = -\Delta \langle \xi_1 | H_1 | \xi_1 \rangle - \Delta \langle \xi_2 | H_1 | \xi_2 \rangle + \Omega_1 \langle \xi_1 | H_1 | \xi_2 \rangle + g_1 \langle \xi_2 | H_1 | \xi_3 \rangle + \sqrt{\Delta-1} g_1 \langle \xi_2 | H_1 | \xi_4 \rangle + \Omega \langle \xi_4 | H_1 | \xi_3 \rangle.
\]

Introduce three orthogonal vectors

\[
|\eta_1\rangle = \frac{1}{N_1} (\sqrt{\Delta-1} g_1 |\xi_2\rangle - g_1 |\xi_3\rangle), \\
|\eta_2\rangle = \frac{1}{\sqrt{2N_1}} (g_1 |\xi_2\rangle + N_1 |\xi_3\rangle + \sqrt{\Delta-1} g_1 |\xi_4\rangle), \\
|\eta_3\rangle = \frac{1}{\sqrt{2N_1}} (g_1 |\xi_2\rangle - N_1 |\xi_3\rangle + \sqrt{\Delta-1} g_1 |\xi_4\rangle),
\]

with \(N_1 = \frac{1}{\sqrt{g_1^2 + (\Delta-1)^2}}\). The three vectors correspond to three eigenvectors of \(g_1 |\xi_2\rangle + N_1 |\xi_3\rangle + \sqrt{\Delta-1} g_1 |\xi_4\rangle\) with eigenvalues \(0, N_1, -N_1\), respectively. The Hamiltonian described in Eq. (4), in the rotating frame \(|\xi_1\rangle, |\xi_2\rangle, |\xi_3\rangle, |\xi_4\rangle, |\eta_1\rangle, |\eta_2\rangle\), becomes

\[
H = H_0 + V,
\]

where

\[
H_0 = -\Delta \langle \xi_1 | H_1 | \xi_1 \rangle - \Delta \langle \xi_2 | H_1 | \xi_2 \rangle + N_1 \langle \eta_1 | H_1 | \eta_1 \rangle - N_1 \langle \eta_2 | H_1 | \eta_2 \rangle.
\]

\[
V = \frac{1}{\sqrt{\Delta N_1}} \langle \xi_3 | \hat{g}_1^\dagger (\langle \eta_1 | H_1 | \eta_1 \rangle + \sqrt{\Delta-1} g_1 |\eta_1\rangle) + \langle \eta_2 | H_1 | \eta_2 \rangle + \sqrt{\Delta-1} g_1 |\eta_2\rangle) + H.c.\).
\]

In order to solve the system evolution under the Hamiltonian (6), we return to the interaction picture with respect to \(H_0\). Then, we have

\[
H_t = e^{iH_0 t} H e^{-iH_0 t} = \frac{1}{\sqrt{2N_1}} \frac{1}{\sqrt{\Delta N_1}} \langle \eta_1 | H_1 | \eta_1 \rangle + \langle \eta_2 | H_1 | \eta_2 \rangle |\eta_1\rangle + \sqrt{\Delta-1} g_1 |\eta_2\rangle |\eta_1\rangle + H.c.
\]

For the sake of simplicity, we consider large detuning conditions

\[
\frac{\Omega_1 g_1}{\sqrt{2N_1}} \sqrt{\Delta-1} g_1 \leq \Delta, N_1,
\]

meaning that there are no energies exchanged between the \(|\xi_1\rangle, |\xi_2\rangle, |\xi_3\rangle\) and \(|\eta_1\rangle, |\eta_2\rangle, |\eta_3\rangle\). The effective Hamiltonian for the interaction Hamiltonian described in Eq. (8) is then given by the expression [42]

\[
H_{\text{eff}} = -iH_1(t) \int H_1(t') dt'
\]

\[
\frac{1}{2N_1} \left( \frac{1}{\Delta + N_1} + \frac{1}{\Delta - N_1} \right) \langle \Omega_1 g_1 | H_1 | \xi_1 \rangle + \sqrt{\Delta-1} g_1 |\eta_1\rangle)
\]

\[
\frac{1}{\sqrt{\Delta}} \left( \sqrt{\Delta-1} g_1 |\eta_1\rangle + \Omega g_1 |\eta_2\rangle + \Omega g_1 |\xi_1\rangle \right) \left( \frac{1}{\Delta + N_1} |\eta_1\rangle + \frac{1}{\Delta - N_1} |\eta_2\rangle \right)
\]

\[
\frac{1}{\sqrt{\Delta}} \left( \sqrt{\Delta-1} g_1 |\eta_1\rangle + \Omega g_1 |\eta_2\rangle + \Omega g_1 |\xi_1\rangle \right) \left( \frac{1}{\Delta + N_1} |\eta_1\rangle + \frac{1}{\Delta - N_1} |\eta_2\rangle \right)
\]

where the high-frequency oscillating terms have been neglected.

Now, let us return to consider the initial system state. If the system is initially prepared in the state \(|\xi_1\rangle\), i.e. the first atom is initially cooled to the ground state \(|g\rangle\), all of the other \(n-1\) atoms in
the ground state \( |s\rangle \) and the cavity mode in the vacuum state \( |0\rangle \). In this situation, the effective Hamiltonian is simplified as

\[
H_t = \frac{1}{2Nt} \left( \frac{1}{\Delta + N_1} + \frac{1}{\Delta - N_1} \right) \left( \Omega_1 |s_1\rangle \langle s_1| + \sqrt{n-1} \Omega_g |s_1\rangle \langle s| \right) 
\times \left( \Omega_1 |s_1\rangle \langle s_1| + \sqrt{n-1} \Omega_g |s_1\rangle \langle s| \right) + \frac{1}{Nt} \left( \sqrt{n-1} \Omega_1 g |s_1\rangle \langle g| - \Omega g |s_1\rangle \langle g| \right)
\]

\[
\times \left( \sqrt{n-1} \Omega_1 g |s_1\rangle \langle g| - \Omega g |s_1\rangle \langle g| \right) + \frac{1}{Nt} \left( \sqrt{n-1} \Omega_1 g |s_1\rangle \langle g| - \Omega g |s_1\rangle \langle g| \right)
\]

\[
= x|\langle s_1| + z|\langle s_1| + y\left(|\langle s_1| + |s_1\rangle\langle s_1|\right)
\]

\[(11)\]

with

\[
x = \frac{\Omega_1^2 \left( \Delta^2 - (n-1)g^2 \right)}{\Delta \left( \Delta^2 - N_1^2 \right)} , \quad y = \frac{\sqrt{n-1} \Omega_1 \Omega g g}{\Delta \left( \Delta^2 - N_1^2 \right)} , \quad z = \frac{\Omega_1^2 \left( \Delta^2 - g^2 \right)}{\Delta \left( \Delta^2 - N_1^2 \right)} .
\]

\[(12)\]

It is easy to verify the initial system state, under the effective Hamiltonian (11) and the free Hamiltonian (7a), evolves to

\[
|S(t)\rangle = \exp(-iH_t t) \exp(-i\mathcal{H}_i t) |\langle s_1| \rangle
\]

\[
= \left( \begin{array}{c} \frac{e^{\frac{et}{2}}}{\sqrt{2}} + i \frac{x e^{-\frac{et}{2}}}{\sqrt{2}} \frac{e^{\frac{et}{2}}}{\sqrt{2}} \frac{e^{\frac{et}{2}}}{\sqrt{2}} \frac{e^{2y t}}{\sqrt{2}} \end{array} \right) - i \frac{2y t}{e} \left( \begin{array}{c} \frac{e^{\frac{et}{2}}}{\sqrt{2}} + i \frac{x e^{-\frac{et}{2}}}{\sqrt{2}} \frac{e^{\frac{et}{2}}}{\sqrt{2}} \frac{e^{2y t}}{\sqrt{2}} \end{array} \right).
\]

\[(13)\]

with \( e = \sqrt{(x - \frac{1}{2})^2 + 4y^2} \). Here we have eliminated the common phase \( e^{i(A - x^2/2)/2} \). It should be noted that the system state is W-type [43], where we have inserted Eq. (3) into (13). However, if we want to prepare all the atoms in the W states described in Eq. (1), we should consider two more conditions.

At first, the Rabi frequencies, atom–cavity coupling coefficients and the interaction time \( t \) should fulfill

\[
\sqrt{n-1} \cos \frac{et}{2} + i \frac{x - \frac{1}{2}}{e} \sin \frac{et}{2} \frac{e^{2y t}}{\sqrt{2}} = \frac{2y t}{e} \frac{e^{\frac{et}{2}}}{\sqrt{2}}
\]

\[(14)\]

With a simple calculation, we obtain

\[
t = \tau = \frac{2}{e} \arcsin \left( \frac{\sqrt{n-1}}{n-1} \frac{e}{2} \right) + \frac{4m \pi}{e}, \quad m = 0, 1, 2, \ldots
\]

\[(15)\]

and

\[
\sqrt{n-1} \frac{e}{2} \frac{e}{2} \leq 1.
\]

\[(16)\]

Submitting the expression (14) into (13), we obtain the system state expressed by

\[
|S(t)\rangle = e^{i\theta_1} \frac{1}{\sqrt{n}} \left( |s_1\rangle + e^{i(\theta_2 - \theta_1 - \pi/2)} \sqrt{n-1} |s_\rangle \right)
\]

\[(17)\]

with

\[
e^{i\theta_1} = \frac{\cos \frac{et}{2} + i \frac{x - \frac{1}{2}}{e} \sin \frac{et}{2} \frac{e^{2y t}}{\sqrt{2}}}{\cos \frac{et}{2} + i \frac{x - \frac{1}{2}}{e} \sin \frac{et}{2} \frac{e^{2y t}}{\sqrt{2}}},
\]

\[
e^{i\theta_2} = \frac{\sin \frac{et}{2}}{\sin \frac{et}{2}}
\]

\[(18)\]

Secondly, a single-qubit operation

\[
|g\rangle \rightarrow |g\rangle, \quad |s\rangle \rightarrow e^{-i(\theta_2 - \theta_1 - \pi/2)} |s\rangle
\]

\[(19)\]
of $\gamma g(\kappa g)$ and the decreasing rates induced by photon decay and atom decay are more or less the same. If we consider both, then we obtain $F=92.5\% (74.3\%)$ when $\kappa=\gamma=0.1g(0.5g)$, meaning that the W-state preparation is robust. The paragraph is ended with an example for the experimental parameters $(g,\gamma,\kappa)=(23.9,2.6,2.6)$ MHz obtained in our lab [45], leading to the required time and the fidelity of the generated W states to be about $t\approx5.68\mu s$ and $F\approx92.0\%$, respectively.

We now demonstrate how many atoms can be entangled in W states described in Eq. (1) based on the present method. The key factor for the W-state preparation is that the chosen parameters should fulfill the conditions described in Eqs. (9) and (16). For simplicity, we also choose $\Delta=g_1=g$ and $\Omega=\Omega_1=0.1g$. According to Eq. (16), we obtain

$$n-1\leq\frac{n}{2}\sqrt{\frac{n}{2}}.$$ 

This means that there are no more than four atoms that can be entangled as W states. Luckily, if we choose $\Omega_1, \Omega, g_1, g$ and $\Delta$ appropriately to make $x=z$ (such as the choice of $g_1=\sqrt{n-1}g$ and $\Delta=g$), then $\sqrt{n-1}\leq\frac{n}{2}$ is always less than 1, meaning that we can prepare W states without the restriction of the atomic number in theory.

In summary, we have proposed a scheme for the generation of W states without any excitations. In this proposal, one of the atomic transitions is resonant with cavity mode and the other atomic transition is driven by the weak laser pulses with a suitable detuning, leading the excited states of atoms and cavity to be only virtually excited in the whole procedure. At the same time, the effect of the laser pulse on the system’s vacuum can be reduced. We also analyze the effect of atom and photon decays on the prepared-W state fidelity, which shows that the scheme is robust in the strong-coupling range and may be feasible based on current technologies.

Acknowledgements

This work is supported in part by the NSFC (Grants No. 10974125, 60821004, 60878059), the State Basic Key Research Program of China, in part by the Educational Committee of Fujian Province (JA09041) and FJNU (2008100220).

References