Quantum Entanglement Dynamics of Two Atoms in Two Coupled Cavities*

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Abstract *Quantum entanglement dynamics for two atoms trapped in two coupled cavities is investigated. Numerical results show that the present of the two atomic excitations is mainly accounted for the entanglement-sudden-death (ESD) effect with the two cavities initially in the vacuum. The entanglement can also be controlled by the hopping rate and the imbalances between the two atom-cavity coupling rates.*

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1 Introduction

Quantum entanglement is one of the fundamental features that distinguish quantum systems from their classical counterparts and plays an important role in quantum information science,[1] such as quantum teleportation,[2] quantum computing,[3] and so on. However, the real physical system is inevitable to interact with the environment, leading the entanglement to deteriorate. Hence, it is of great importance to study the quantum entanglement dynamics (ED). Recently, Yu and Eberly have shown that two initially entangled particles without interaction between each other can decay to 0 in a finite time, which is much shorter than that of their spontaneous emission.[4] This phenomenon is known as the entanglement sudden death (ESD).[5–6] On the other hand, two initially separated atoms could be entangled in a finite time, which is termed the entanglement sudden birth (ESB).[7] These quantum entanglement features have been explored in diverse scenarios,[8–19] but the nature of ESD and ESB is still not very explicit.[20]

Cavity quantum electrodynamics (CQED), known as an effective system to study the interaction between light and atoms (or ions, atomic ensembles etc.) in a confined space, is thought to be a potential candidate for the demonstration of quantum information processing.[21–22] Based on CQED, there are several proposals suggested for the research of ED via J-C models and the generalized J-C models.[23–28] However, in most of these schemes, the investigated atoms are usually trapped in one cavity or two isolated cavities, respectively.

Recently, much attention has been focused on the coupled-cavities arrays,[29–30] for which some potential technologies have been demonstrated in experiment.[31–32] The system is thought to be suitable for building a large-scale architecture for quantum information processing.[33] In such systems, atoms are usually used for the storage of quantum information and cavity modes act as carrying qubits. For this reason, it is important to study the ED for atoms on the basis of the coupled-cavities arrays. For simplicity, in this paper, we only consider two cavities coupled directly and there is a two-level atom inside each cavity. These two atoms are initially prepared in the superposition of the two-excitation state and the zero-excitation state, i.e. Φ-type Bell states. The numerical results show that the entanglement dynamics is strongly affected by the probability of two-excitation state of the initial state, the hopping rates, and the ratio between two atom-cavity coupling rates. The duration for the disentanglement before recovery can be enhanced via the increase of the two-excitation state probability when the atom-cavity coupling rate and the hopping rate are both real, which is different from that obtained via the two atoms individually interacting with two isolated cavities,[34] where the quantum entanglement is necessary for the ESD when the cavities are initially in the vacuum. In addition, we also show that the entanglement dynamics can be controlled by the hopping rate (including the amplitude and the phase) between two cavities and the ratio between two atom-cavity coupling rates.

2 Model for Two Two-Level Atoms Trapped in Two Coupled Cavities

Let us consider a two-cavity system shown in Fig. 1. There are two identical cavities and each one has a two-

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level atom holding the excited state $|e\rangle$ and the ground state $|g\rangle$.[35] The two cavities are coupled directly and each of them is resonant with the atomic transition. Without loss of generality, we use the subscript k ($k = 1, 2$) to identify each atom and each cavity. In the rotating-wave approximation, the interaction Hamiltonian is expressed as[36]

$$H = \sum_{j=1,2} g_j \sigma_j^+ a_j + \nu a_1^\dagger a_2 + \text{H.c.}\,, \tag{1}$$

where $g\,(\nu)$ and $a\,(a^\dagger)$ represent the atom-cavity coupling rate (the hopping rate between two cavities) and the annihilation (creation) operator for the cavity mode, $\sigma^- = |g\rangle\langle e|$ ($\sigma^+ = |e\rangle\langle g|$) is the raising (lowing) operators for atoms, and H.c. stand for the conjugate terms. For simplicity, we suppose that the atom-cavity coupling rates are both real, while the hopping rate can be chosen to be a complex number. Initially, the atoms are considered to be prepared in the Φ-type Bell state

$$|\Phi(\alpha)\rangle = \cos\alpha|ee\rangle_{12} + \sin\alpha|gg\rangle_{12}\,, \tag{2}$$

and both cavities are in the vacuum state $|00\rangle$. Thus, the state for the whole system is

$$|S\rangle = (\cos\alpha|ee\rangle + \sin\alpha|gg\rangle)|00\rangle\,, \tag{3}$$

where the subscripts have been eliminated.

We define the operator

$$N = \sum_{j=1}^{2} \Bigl(\frac{1}{2}\sigma_{z,j} + a_j^\dagger a_j\Bigr)$$

with $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ being a Pauli operator and one can easily prove that $[N, H] = 0$. Thus, under the action of the interaction Hamiltonian (1), the zero-excitation state $|gg\rangle|00\rangle$ would not change since there is no excitation and the time evolution for the two-excitation state $|ee\rangle|00\rangle$ will be confined in the subspace $\{|ee\rangle|00\rangle, |eg\rangle|01\rangle, |ge\rangle|10\rangle, |eg\rangle|10\rangle, |ge\rangle|01\rangle, |gg\rangle|20\rangle, |gg\rangle|02\rangle, |gg\rangle|11\rangle\}$. This leads to the interaction Hamiltonian

$$H' = \begin{pmatrix} 0 & g_2 & g_1 & 0 & 0 & 0 & 0 & 0 \\ g_2 & 0 & 0 & \nu & 0 & 0 & 0 & g_1 \\ g_1 & 0 & 0 & 0 & \nu & 0 & 0 & g_2 \\ 0 & \nu^* & 0 & 0 & 0 & \sqrt{2}g_1 & 0 & 0 \\ 0 & 0 & \nu^* & 0 & 0 & 0 & \sqrt{2}g_2 & 0 \\ 0 & 0 & 0 & \sqrt{2}g_1 & 0 & 0 & 0 & \sqrt{2}\nu \\ 0 & 0 & 0 & 0 & \sqrt{2}g_2 & 0 & 0 & \sqrt{2}\nu \\ 0 & g_1 & g_2 & 0 & 0 & \sqrt{2}\nu^* & \sqrt{2}\nu^* & 0 \end{pmatrix}. \tag{4}$$

As a result, the state of the whole system state at time t can be given by

$$\begin{aligned} |S(t)\rangle = {} & C_{ee00}(t)|ee\rangle|00\rangle + C_{eg01}(t)|eg\rangle|01\rangle + C_{ge10}(t)|ge\rangle|10\rangle + C_{eg10}(t)|eg\rangle|10\rangle + C_{ge01}(t)|ge\rangle|01\rangle \\ & + C_{gg20}(t)|gg\rangle|20\rangle + C_{gg02}(t)|gg\rangle|02\rangle + C_{gg11}(t)|gg\rangle|11\rangle + \sin\alpha|gg\rangle|00\rangle\,, \end{aligned} \tag{5}$$

where $C_{kk'll'}(t)$ ($k, k' = e, g$ and $l, l' = 0, 1$) is complex and time-dependent. Since the explicit expressions of these parameters are very complicated, here we skip the details and give our results in terms of numerical calculations.

By tracing the two cavity fields, the density matrix for the two atoms, in the basis of $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$, reads

$$\rho = \begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}, \tag{6}$$

with

$$\begin{aligned} & a = |C_{ee00}(t)|^2\,, \quad b = |C_{eg01}(t)|^2 + |C_{eg10}(t)|^2\,, \\ & c = |C_{ge10}(t)|^2 + |C_{ge01}(t)|^2\,, \\ & d = \sin^2\alpha + |C_{gg20}(t)|^2 + |C_{gg02}(t)|^2 + |C_{gg11}(t)|^2\,, \\ & z = C_{eg01}(t)C^*_{ge01}(t) + C_{eg10}(t)C^*_{ge10}(t)\,, \\ & w = \sin\alpha C_{ee00}(t)\,. \end{aligned} \tag{7}$$

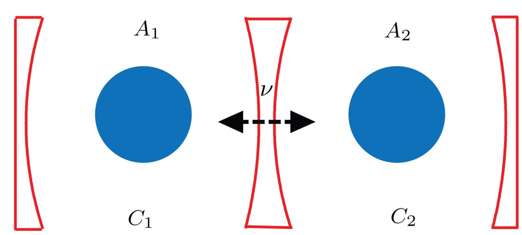

Fig. 1 Sketched setup for two atoms individually trapped in two coupled cavities.

3 Entanglement Dynamics of Two Atoms

In order to investigate the entanglement dynamics of the bipartite system, we use Wootters concurrence to quantify the degree of the quantum entanglement.[37] The concurrence of the density matrix ρ is given by

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}\,, \tag{8}$$

with λ_i ($i = 1, 2, 3, 4$) being the square root of the eigenvalues of the non-Hermitian matrix $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ in decreasing order, where ρ^* is the complex conjugation of ρ in the standard basis and σ_y is the Pauli matrix expressed as $\begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$. According to Ref. [38], we can quickly obtain the concurrence for the X-class density matrix described in Eq. (6),

$$C(\rho) = 2\max(0, C_1, C_2)\,, \tag{9}$$

with

$$\begin{aligned} C_1 &= |z| - \sqrt{ad} = |C_{eg01}C^*_{ge01} + C_{eg10}C^*_{ge10}| \\ &\quad - |C_{ee00}|\sqrt{\sin^2\alpha + |C_{gg20}|^2 + |C_{gg02}|^2 + |C_{gg11}|^2}\,, \\ C_2 &= |w| - \sqrt{bc} = |\sin\alpha\, C_{ee00}| \\ &\quad - \sqrt{(|C_{eg01}|^2 + |C_{eg10}|^2)(|C_{ge10}|^2 + |C_{ge01}|^2)}\,. \end{aligned} \tag{10}$$

For the sake of simplicity, we first assume ν is real and study the entanglement dynamics of the bipartite system in two diverse situations: the symmetrical case where the two atom-cavity coupling rates are equal ($g_1 = g_2 = g$) and the unsymmetrical one ($g_1 \neq g_2$) and then we generalize the hopping rate to be a complex number ($|\nu|\,\mathrm{e}^{\mathrm{i}\theta}$) to investigate how the phase of the hopping rate (θ) affect the entanglement dynamics.

3.1 *Symmetrical Case ($g_1 = g_2 = g$) with ν Being Real*

Firstly, the evolution of the concurrence for various initial atomic states (different parameters α) can be investigated via numerical simulation. For simplicity, we firstly assume the hopping rate is $\nu = g$. The time evolution of the concurrence as the function of the scale time gt/π and α is plotted in Fig. 2, where we have confined α within the range $[0, \pi]$ without loss of generality. The result shows (i) The initial separate state of two atoms ($\cos\alpha = 0, 1$, i.e. $\alpha = 0, \pi/2, \pi$) cannot be entangled at any time; (ii) The maximal entanglement can exist only when the two atoms are initially entangled maximally; (iii) The initial entanglement of the two atoms can fall abruptly to zero and will remain zero for a period of time before it recovers. The larger the probability of the two excitations ($\cos^2\alpha \neq 1$), the longer the state will be in the disentangled separable state.

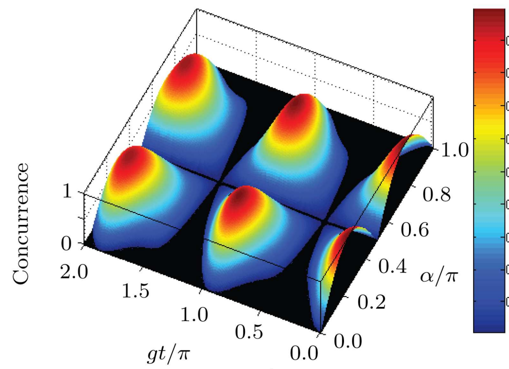

Fig. 2 The time evolution of the concurrence as the function of the scale time gt/π and α, where $g_1 = g_2 = g$ and $\nu = g$.

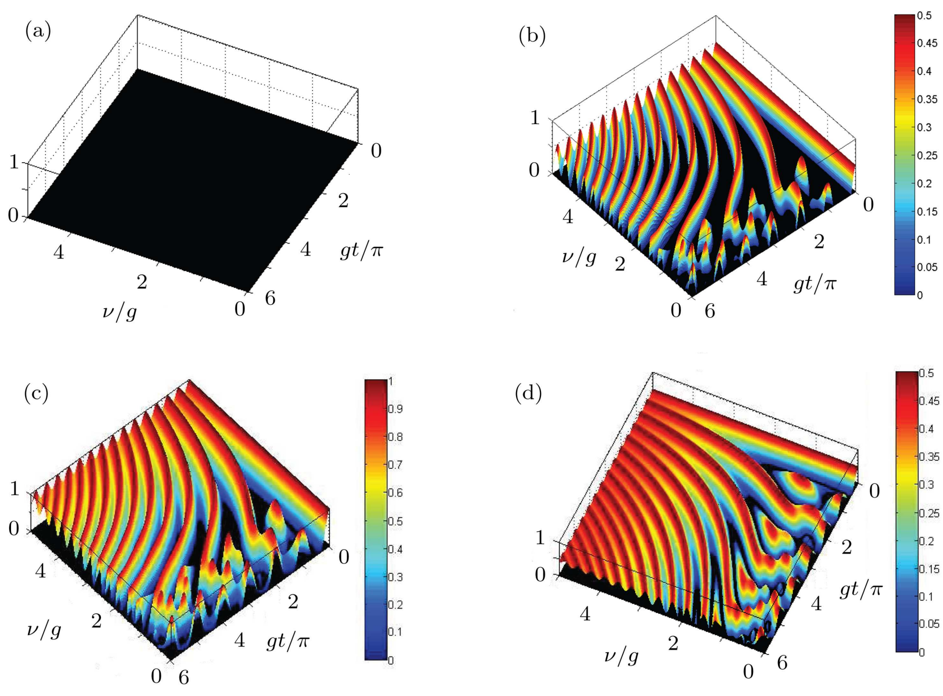

Fig. 3 The concurrence versus the scale time gt/π and the ratio ν/g, where $g_1 = g_2 = g$ and $\alpha =$ (a) 0, (b) $\pi/12$, (c) $\pi/4$, (d) 5/12.

Secondly, the concurrence versus the scale time gt/π and the ratio ν/g is taken into account in Fig. 3. To do so, the hopping rate ν can be controlled experimentally by controlling the length of the cavity in principle, which can be considered as an adjustable parameter. For simplicity, we choose α = (a) 0 (b) $\pi/12$ (c) $\pi/4$, (d) 5/12 again. The numerical results demonstrate (i) the separable states $|ee\rangle$ ($\alpha = 0, \pi$) and $|gg\rangle$ ($\alpha = \pi/2$) can never be entangled no matter what the hopping rate is (Fig. 3(a)); (ii) for the initial entangled states, the result is shown in the other three figures (Figs. 3(b), 3(c), and 3(d)), where each picture is divided into two regions with the boundary $\nu/g = 1$. In the region of $\nu/g < 1$, the ESD effect can always take place for the initial entangled state within $0 < \alpha < \pi/4$, but the dynamics of the ESD is dependent on the initial two-excitation state within $\pi/4 \leq \alpha < \pi/2$. The larger the two-excitation-state probability, the longer time interval the ESD occurs. While for the case of $\nu/g > 1$, the entanglement can suddenly disappear when ν/g is small. The ESD effect is most obvious at about $\nu/g = 1.7$ and this effect will eventually disappear as ν/g increases. For this reason, we can conclude that there exists a decoherence-free state (DFS) when ν/g is large enough. It is actually true because there is no any energy exchange in the case of the large detuning between atom-cavity coupling rate and the hopping rate (we call it Δ), where Δ is closely related to the initial two-excitation state. The larger the initial two-excitation-state probability, the larger hopping rate will be required for satisfying the large-detuning condition.

3.2 *Unsymmetrical Case ($g_1 \neq g_2$) with ν Being Real*

We now consider entanglement dynamics of the system influenced by the imbalances between g_1 and g_2. The concurrence as the function of the scale time gt/π and the ratio g_2/g_1 is depicted in Fig. 4, where we have set $g_1 = \nu = g$ and α = (a) 0 (b) $\pi/12$ (c) $\pi/4$, (d) $5\pi/12$. It is clearly seen that the separable two atoms, initially in the two-excitation state $|ee\rangle$, can be entangled with a delayed time in the case of $g_2 \neq g_1$ ($g_2 \neq 0$), i.e. the ESB effect occurs. While for the initial entangled state, the sudden death effect can take place much more easily in the small ratio g_2/g_1 and large probability of the initial two-excitation state.

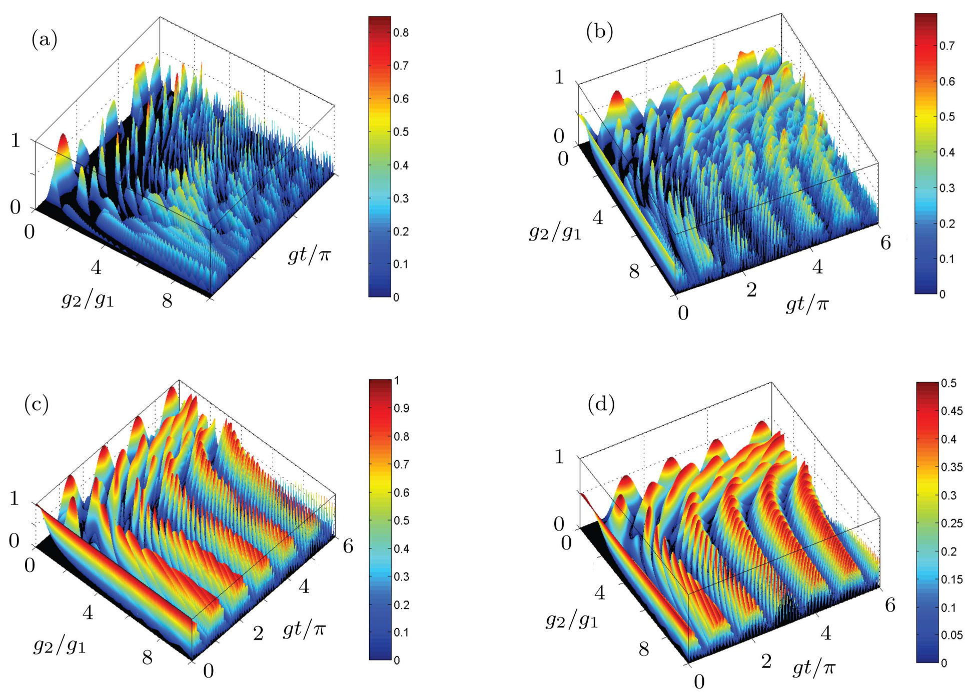

Fig. 4 The concurrence versus the scale time gt/π and the ratio g_2/g_1, where $g_1 = \nu = g$ and α = (a) 0, (b) $\pi/12$, (c) $\pi/4$, (d) $5\pi/12$.

3.3 *ED with ν being Complex*

With the investigation, we make a conclusion that the phase of the hopping rate can also affect the ED, and even destroy the symmetry similar to the case of $g_1 \neq g_2$. For the sake of the simplicity, we only consider the ED with the initial separate atomic state $|ee\rangle$ in the case of ν being complex. We know that there is no any ESB exist when $g_1 = g_2$ and the initial state $|ee\rangle|0\rangle$ as shown in Subsect. 3.1. In the following passage, let us show how this phase makes an influence on the entanglement evolution, which is depicted in Fig. 5 where we have set $g_1 = g_2 = |\nu| = g$. The illustration tells us ESB does not occurs only when $\theta = n\pi$ with n being an integer. In other words, we can prepare the atomic entangled state even when the initial atomic state both in the excited state in the case of the symmetrical system with the adjustment of the hopping rate's phase.

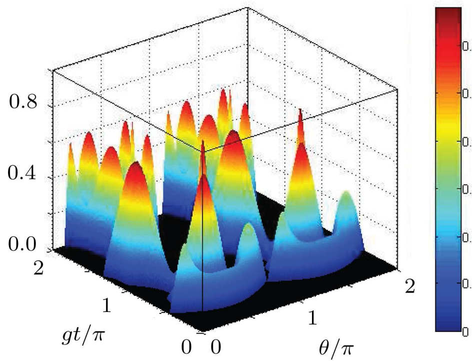

Fig. 5 The concurrence versus the scale time gt/π and phase of the hopping rate θ, where $g_1 = g_2 = |\nu| = g$ and $\alpha = 0$.

4 Discussion

We have studied the entanglement dynamics based on the system with two atoms and two cavities. The two cavities are coupled directly and both are initially in the vacuum states. The entanglement dynamics shows distinctive features for different initial atomic states, the ratios g_2/g_1 and the hopping rates. The ESD and ESB are more obvious when the two atoms are prepared initially in the excited states, but this behavior is also dependent on the symmetry of the two atom-cavity interaction system as well as the hopping rate including its amplitude and phase. If the system is completely symmetry ($g_1 = g_2$) and the phase of hopping rate is set to $\theta = n\pi$ (n is an integer), there will be no any ESB effect occurred for the separable atoms. This result is similar to the phenomenon obtained from the two atoms individually interacting with two isolated vacuums. However, the ESB phenomenon exist in the case of $\theta \neq n\pi$. We also show that the ESD effect does not occur when the hopping rate is large enough. A decoherence-free state may exist in the large-detuning condition, which is determined by the probability of the initial two-excitation state. Although the above-mentioned features of the quantum entanglement could not be well explained quantitatively at this moment, we still see that entanglement dynamics is strongly affected by the initial excitations of the two atoms, the imbalance of the two atom-cavity coupling rates and the hopping rates between two cavities. The proposal presented here is feasible as the development of the modern technology. The two coupled cavities can be achieved either by using the adjacent cavities or optical fiber connected cavities.[30] The two-level state of atoms could be chosen as, for example, the excited state $|6P_{3/2}, F = 4\rangle$ and the ground state $|6S_{1/2}, F = 3\rangle$ or $|6S_{1/2}, F = 4\rangle$ of the cesium atoms (^{133}Cs), respectively.[36]

5 Summary

We have proposed a scheme to study the entanglement dynamics for two atoms interacted with two coupled cavities. Based on the numerical simulation, we conclude that the excitations of the atoms play the important role in the entanglement dynamics. The larger probability of the initial two-excitation state, the more obvious the ESD effect occurs. It shows that the entanglement dynamics is also closely related to the hopping rates including its amplitude and phase and the imbalance of the two atom-cavity systems. The atomic entanglement dynamics has shown distinctive features with different conditions and different initial atomic states, which can help us to understand these interesting quantum behaviors.

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