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Fidelity with quadrature component variances for continuous-variable quantum teleportation

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Abstract

The efficiency of continuous-variable quantum teleportation is typically quantified by fidelity especially in experiment. From the original definition of fidelity in two pure states, we give an expansion of fidelity which is related to the variances of output and input states. With a coherent or a squeezed input state, it is convenient to quantify the quantum teleportation experiment, since the variances of quadrature components of the input and output states are measurable. Furthermore, the fidelity was discussed when the quantum channel lies in the phase- and amplitude-noisy environment, which is unavoidable in experiment, and this showed that the effect of phase noise on teleportation is more sensitive than that of the amplitude-noisy environment. The classical fidelity limit of squeezed state quantum teleportation is also obtained when the entanglement resources do not exist.

(Some figures may appear in colour only in the online journal)

Quantum teleportation is one of the most important manifestations of quantum mechanics [1, 2] and plays a central role in quantum information science, which can transmit unknown quantum states between distant users without sending quantum states directly. Quantum teleportation was originally proposed and realized in a discrete variable domain [3, 4]. Recently, it was extended to the teleportation of continuous variables [5]. Braunstein and Kimble [6] gave a protocol of continuous-variable quantum teleportation for a coherent state and soon experimentally demonstrated it [7]. Since then, many groups have realized the continuous-variable quantum teleportation for the vacuum state, coherent states [8–11] and even a squeezed state [12]. Recently, Mišta Jr *et al* [13] investigated how to preserve the negative Wigner function of the single photon state in continuous-variable quantum teleportation. In 2011, Lee *et al* realized the continuous-variable quantum teleportation of the Schrödinger cat state [14].

The fidelity, which is the overlap between the input and output states, is used to quantify the quality of the quantum teleportation. A fidelity expression for the coherent-state quantum teleportation with the quantum channel (EPR state) was given, and experimentally measured [7]. For nonclassical state quantum teleportation, such as a squeezed state, an expression for the fidelity has been proposed [15–18], which depends on the squeezing parameter of the input state and the correlation parameter of the EPR state of the quantum channel. However, it is not very useful in experiment because the squeezing parameter of the input state and the correlation parameter of the EPR state are not observed directly; experimentally, the measurable quantity is the variance. If the input state is a coherent state and the EPR state is a two-mode squeezed vacuum (Wigner functions of both are Gaussian), the fidelity can be expressed with the variances of the two quadrature components of the output state [19].

The fidelity for a pure input state [5], $|\alpha_{in}\rangle = \frac{1}{2}|x_{in} + iy_{in}\rangle$, with x_{in} (y_{in}) being the expectation value of the amplitude (phase) quadrature, is defined by Schumacher $F = \langle\alpha_{in}|\rho_{out}|\alpha_{in}\rangle$, where ρ_{out} is the density operator of the output state, and it can equivalently be expressed by the overlap of their Wigner functions:

$$F = \pi \int d^2\alpha W_{in}(\alpha) W_{out}(\alpha), \tag{1}$$

where $W_{in}(\alpha)$ and $W_{out}(\alpha)$ are the Wigner functions of the input and output states, respectively. If the input state is a coherent state and the EPR state is a two-mode squeezed vacuum (Wigner functions of both are Gaussian), the fidelity can be expressed as [19]

$$F = \frac{2}{\sqrt{(1+\langle\delta^2\hat{X}_{out}\rangle)(1+\langle\delta^2\hat{Y}_{out}\rangle)}} \times \exp\left[-\frac{(x_{out}-gx_{in})^2}{2(1+\langle\delta^2\hat{X}_{out}\rangle)} - \frac{(y_{out}-gy_{in})^2}{2(1+\langle\delta^2\hat{Y}_{out}\rangle)}\right], \tag{2}$$

where $\langle\delta^2\hat{X}_{out}\rangle$ and $\langle\delta^2\hat{Y}_{out}\rangle$ are the quadrature component variances of the output state, x_{in} and y_{in} (x_{out} and y_{out}) are the two quadrature component displacements of the input state (the output state) in the phase space and g is the normalized gain. The two quadrature variances, $\langle\delta^2\hat{X}_{out}\rangle$ and $\langle\delta^2\hat{Y}_{out}\rangle$, are measurable quantities. In a quantum teleportation (or quantum clone) process, the displacement x_{in} (y_{in}) of an input state can be easily reconstructed at the output station by setting the gains of classical channels to unity, i.e. $x_{out} = x_{in}$; $y_{out} = y_{in}$, so that the fidelity is peaked:

$$F = \frac{2}{\sqrt{(1+\langle\delta^2\hat{X}_{out}\rangle)(1+\langle\delta^2\hat{Y}_{out}\rangle)}}. \tag{3}$$

Furusawa *et al* [7] used this expression to investigate the fidelity of continuous-variable quantum teleportation for a coherent input state. The experimental results for the coherent states fit well with the theoretical calculation expressed in equation (3). However, if the input state is a nonclassical state, such as a squeezed state, calculating the fidelity is still a mystery [12, 13].

In this paper, we will derive an expression with the quadrature components' variances of the input and output states for the fidelity of continuous variables quantum teleportation with squeezed states as the input. Furthermore, we will discuss the influence of the noisy environment of the quantum channel on the fidelity.

In fact, equation (1) defines quantitatively the similarity of the two states. Based on equation (1), we will consider the similarity (fidelity) between the two squeezed states.

For a squeezed state with a squeezing parameter s, the annihilation operator $\hat{\alpha} = \hat{X} + i\hat{Y}$ can be expressed in terms of its mean value $\langle\hat{\alpha}\rangle$ plus two fluctuating quadrature components, for example,

$$\hat{\alpha} = \langle\hat{\alpha}\rangle + \delta\hat{X} + i\delta\hat{Y}, \tag{4}$$

where $\delta\hat{X} = e^{s}\hat{X}^{(0)}$ and $\delta\hat{Y} = e^{-s}\hat{Y}^{(0)}$ are the quadrature-phase amplitude operators with the canonical commutation relation $[\delta\hat{X}, \delta\hat{Y}] = i/2$ and $\hat{X}^{(0)}$ and $\hat{Y}^{(0)}$ are the quadrature-phase amplitude operators of a vacuum mode. This state with $s = 0$ corresponds to the coherent state. The Wigner function of the squeezed (coherent) state is Gaussian, considering the two squeezed states, $\hat{\alpha}_1$ and $\hat{\alpha}_2$, (their Wigner functions both are Gaussian):

$$W_1(x, y) = \frac{2}{\pi\sqrt{\langle\delta^2\hat{X}_1\rangle\langle\delta^2\hat{Y}_1\rangle}} \times \exp\left[-\frac{2(x-x_1)^2}{\langle\delta^2\hat{X}_1\rangle} - \frac{2(y-y_1)^2}{\langle\delta^2\hat{Y}_1\rangle}\right], \tag{5}$$

$$W_2(x, y) = \frac{2}{\pi\sqrt{\langle\delta^2\hat{X}_2\rangle\langle\delta^2\hat{Y}_2\rangle}} \times \exp\left[-\frac{2(x-x_2)^2}{\langle\delta^2\hat{X}_2\rangle} - \frac{2(y-y_2)^2}{\langle\delta^2\hat{Y}_2\rangle}\right], \tag{6}$$

respectively.

The similarity (fidelity) between $\hat{\alpha}_1$ and $\hat{\alpha}_2$ can be calculated by using equation (2):

$$\begin{aligned} F &= \pi \int dx\, dy W_{in}(x, y) W_{out}(x, y) \\ &= \int dx\, dy \frac{4}{\pi\sqrt{\langle\delta^2\hat{X}_{in}\rangle\langle\delta^2\hat{Y}_{in}\rangle}\sqrt{\langle\delta^2\hat{X}_{out}\rangle\langle\delta^2\hat{Y}_{out}\rangle}} \\ &\quad\times \exp\left[-\frac{2(x-x_{in})^2}{\langle\delta^2\hat{X}_{in}\rangle} - \frac{2(y-y_{in})^2}{\langle\delta^2\hat{Y}_{in}\rangle}\right] \\ &\quad\times \exp\left[-\frac{2(x-x_{out})^2}{\langle\delta^2\hat{X}_{out}\rangle} - \frac{2(y-y_{out})^2}{\langle\delta^2\hat{Y}_{out}\rangle}\right] \\ &= \frac{2}{\sqrt{\left(\langle\delta^2\hat{X}_{in}\rangle + \langle\delta^2\hat{X}_{out}\rangle\right)\left(\langle\delta^2\hat{Y}_{in}\rangle + \langle\delta^2\hat{Y}_{out}\rangle\right)}} \\ &\quad\times \exp\left[-\frac{(x_{out}-x_{in})^2}{2\left(1+\langle\delta^2\hat{X}_{out}\rangle\right)} - \frac{(y_{out}-y_{in})^2}{2\left(1+\langle\delta^2\hat{Y}_{out}\rangle\right)}\right]. \end{aligned} \tag{7}$$

It can be seen that the fidelity depends not only on the variances of two states but also on the difference of the two mean values. Note that the variances of quadrature-phase amplitudes in equation (7) are normalized by 1/4 with respect to the definition in equation (4). In the case of the same mean value of $\hat{\alpha}_1$ and $\hat{\alpha}_2$, i.e. $x_2 = x_1, y_2 = y_1$, respectively, the fidelity is at maximum:

$$F\left(\hat{\alpha}_1, \hat{\alpha}_2\right) = \frac{2}{\sqrt{(\langle\delta^2\hat{X}_1\rangle + \langle\delta^2\hat{X}_2\rangle)(\langle\delta^2\hat{Y}_1\rangle + \langle\delta^2\hat{Y}_2\rangle)}}. \tag{8}$$

When $\hat{\alpha}_1$ is a minimum uncertainty state, i.e.$\langle\delta^2\hat{X}_1\rangle * \langle\delta^2\hat{Y}_1\rangle = 1$, the fidelity becomes

$$F = \frac{2}{\sqrt{\left(1+\frac{\langle\delta^2\hat{X}_2\rangle}{\langle\delta^2\hat{X}_1\rangle}\right)\left(1+\frac{\langle\delta^2\hat{Y}_2\rangle}{\langle\delta^2\hat{Y}_1\rangle}\right)}}. \tag{9}$$

For a pure coherent state $\hat{\alpha}_1$, due to $\langle\delta^2\hat{X}_1\rangle = \langle\delta^2\hat{Y}_1\rangle = 1$, equation (9) becomes equation (3), which is widely used to qualify the coherent quantum teleportation and the quantum clone [9, 10, 20, 21]. Note that here the similarity (fidelity) is related to the quadrature variances of the (input and output) states, which are experimentally measurable.

Considering a squeezed state quantum teleportation, Alice and Bob share an EPR state in the quantum teleportation. Usually, this EPR state is a two-mode squeezed vacuum, which consists of an initially amplitude-quadrature squeezed-vacuum state and an initially phase-quadrature squeezed-vacuum state on a 50:50 beam splitter. In the Heisenberg representation, we have [23]

$$\begin{aligned} \hat{X}_1 &= \left(e^{+r}\hat{X}_1^{(0)} + e^{-r}\hat{X}_2^{(0)}\right)/\sqrt{2},\\ \hat{Y}_1 &= \left(e^{-r}\hat{Y}_1^{(0)} + e^{+r}\hat{Y}_2^{(0)}\right)/\sqrt{2},\\ \hat{X}_2 &= \left(e^{+r}\hat{X}_1^{(0)} - e^{-r}\hat{X}_2^{(0)}\right)/\sqrt{2},\\ \hat{Y}_2 &= \left(e^{-r}\hat{Y}_1^{(0)} - e^{+r}\hat{Y}_2^{(0)}\right)/\sqrt{2}. \end{aligned} \tag{10}$$

The superscript (0) denotes initial vacuum modes and r is the EPR correlation parameter. With a unity gain in the quantum teleportation, as seen in [7], the mean value of the input state is equal to that of the output state. So we can only consider the fluctuating components of the quadrature operators. For simplicity, we rewrite equation (4) as

$$\hat{\alpha}_{\mathrm{in}} = \delta\hat{X}_{\mathrm{in}} + \delta\hat{Y}_{\mathrm{in}} = e^{+s}\hat{X}^{(0)} + e^{-s}\hat{Y}^{(0)}. \tag{11}$$

Alice mixes a part of the two-mode squeezed-vacuum state, named mode 1, with the input state on a balanced beam splitter (BS) and measures two commuting quadratures by means of two balanced homodyne detectors (BHD). Then she sends the measurement results to Bob who subsequently applies a unitary transformation to his part of the shared two-mode squeezed-vacuum state, named mode 2. For an ideal quantum teleportation (meaning no losses), Bob recovers the original minimum uncertainty state as

$$\begin{aligned} \hat{\alpha}_{\mathrm{out}} &= \hat{X}_{\mathrm{out}} + i\hat{Y}_{\mathrm{out}},\\ \hat{X}_{\mathrm{out}} &= \hat{X}_{\mathrm{in}} + (\hat{X}_1 - \hat{X}_2),\\ \hat{Y}_{\mathrm{out}} &= \hat{Y}_{\mathrm{in}} + (\hat{Y}_2 + \hat{Y}_2), \end{aligned} \tag{12}$$

and the variances of the output state are related to the variances of the operators $\hat{X}_1 - \hat{X}_2$, $\hat{Y}_1 + \hat{Y}_2$ by

$$\begin{aligned} \langle\delta^2\hat{X}_{\mathrm{out}}\rangle &= \langle\delta^2\hat{X}_{\mathrm{in}}\rangle + 2\langle\delta^2(\hat{X}_1 - \hat{X}_2)\rangle,\\ \langle\delta^2\hat{Y}_{\mathrm{out}}\rangle &= \langle\delta^2\hat{Y}_{\mathrm{in}}\rangle + 2\langle\delta^2(\hat{Y}_1 + \hat{Y}_2)\rangle. \end{aligned} \tag{13}$$

Here, $\langle\delta^2(\hat{X}_1 - \hat{X}_2)\rangle = \langle\delta^2(\hat{Y}_1 + \hat{Y}_2)\rangle = e^{-2r}$. Equation (13) is substituted for equation (8); therefore, the fidelity becomes

$$F(\hat{\alpha}_{\mathrm{in}}, \hat{\alpha}_{\mathrm{out}}) = \frac{1}{\sqrt{(\langle\delta^2\hat{X}_{\mathrm{in}}\rangle + e^{-2r})(\langle\delta^2\hat{Y}_{\mathrm{in}}\rangle + e^{-2r})}}, \tag{14}$$

which is consistent with the conclusion given by Bowen [8]. Note that the fidelity expression of equation (14) is in agreement with the theoretical fidelity expressed by the squeezing parameter of the input state and the correlation parameter of the EPR state [15–18]. Furthermore, for an input squeezed state, we have $\langle\delta^2\hat{X}_{\mathrm{in}}\rangle = e^{-2s}$ and $\langle\delta^2\hat{Y}_{\mathrm{in}}\rangle = e^{2s}$. Our fidelity expressed with variances, as seen in equations (7)–(9), is also consistent with the previous theoretical results of the squeezing parameters in the continuous-variable quantum teleportation [15–18].

In real experimental situations, the two-mode squeezed-vacuum state might be influenced by the noise of its thermal environment, which is not included in equation (14). Therefore, we need to consider the influence of the thermal noise on the fidelity of the quantum teleportation. There are two kinds of noise in the continuous variable quantum teleportation: phase damping and amplitude damping. Here we will consider the phase-damping and amplitude-damping models separately.

First, we will consider the phase-damping model. Assuming that the thermal environment gives the same effect on each mode of the two-mode squeezed-vacuum state, the evolution of the system's density operator in the interaction picture is described by the master equation [22]:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\Gamma}{2}\,(L_1 + L_2)\,\rho, \tag{15}$$

where Γ is the overall damping rate and

$$L_i\rho = 2\hat{a}_i^{+}\hat{a}_i\rho\hat{a}_i^{+}\hat{a}_i - (\hat{a}_i^{+}\hat{a}_i)^2\rho - \rho(\hat{a}_i^{+}\hat{a}_i)^2. \tag{16}$$

The density operator of the initial two-mode squeezed-vacuum state is $\rho = \frac{1}{\cosh^2 r}\sum_{n_1,n_2=0}^{\infty}(\tanh r)^{n_1+n_2}\,|n_1, n_1\rangle\,\langle n_2, n_2|$ in the Fock state basis [23]. The solution of equation (15) is calculated as

$$\rho(t) = \frac{1}{\cosh^2 r}\sum_{n_1,n_2=0}^{\infty}(\tanh r)^{n_1+n_2} \times \exp\left(-\Gamma t\,|n_1 - n_2|^2\right)|n_1, n_1\rangle\langle n_2, n_2|. \tag{17}$$

Consequently, the covariance matrix of $\rho(t)$ is obtained as

$$C(\hat{X}_1, \hat{Y}_1, \hat{X}_2, \hat{Y}_2) = \frac{1}{2}\begin{pmatrix} \cosh 2r & 0 & e^{-\Gamma t}\sinh 2r & 0\\ 0 & \cosh 2r & 0 & -e^{-\Gamma t}\sinh 2r\\ e^{-\Gamma t}\sinh 2r & 0 & \cosh 2r & 0\\ 0 & -e^{-\Gamma t}\sinh 2r & 0 & \cosh 2r \end{pmatrix}. \tag{18}$$

From equation (18), the variances of the operators $\hat{X}_1 - \hat{X}_2$, $\hat{Y}_1 + \hat{Y}_2$ are easily found to be

$$\begin{aligned} \langle\delta^2(\hat{X}_1 - \hat{X}_2)\rangle &= \tfrac{e^{2r}}{2}(1 - T) + \tfrac{e^{-2r}}{2}(1 + T), \text{ and}\\ \langle\delta^2(\hat{Y}_1 + \hat{Y}_2)\rangle &= \tfrac{e^{2r}}{2}(1 - T) + \tfrac{e^{-2r}}{2}(1 + T), \end{aligned} \tag{19}$$

respectively, where $T = \exp(-\Gamma t)$ is the transmission coefficient of the noisy quantum channel. The output state variances of the quantum teleportation now become

$$\begin{aligned} \langle\delta^2\hat{X}_{\mathrm{out}}\rangle &= \langle\delta^2\hat{X}_{\mathrm{in}}\rangle + e^{2r}(1 - T) + e^{-2r}(1 + T),\\ \langle\delta^2\hat{Y}_{\mathrm{out}}\rangle &= \langle\delta^2\hat{Y}_{\mathrm{in}}\rangle + e^{2r}(1 - T) + e^{-2r}(1 + T), \end{aligned} \tag{20}$$

By substituting equation (20) for equation (9), we obtain the fidelity

$$F_1 = \frac{1}{\sqrt{\left(1 + \frac{e^{2r}(1-T)+e^{-2r}(1+T)}{2\langle\delta^2\hat{X}_{\mathrm{in}}\rangle}\right)\left(1 + \frac{\left[e^{2r}(1-T)+e^{-2r}(1+T)\right]\langle\delta^2\hat{X}_{\mathrm{in}}\rangle}{2}\right)}}. \tag{21}$$

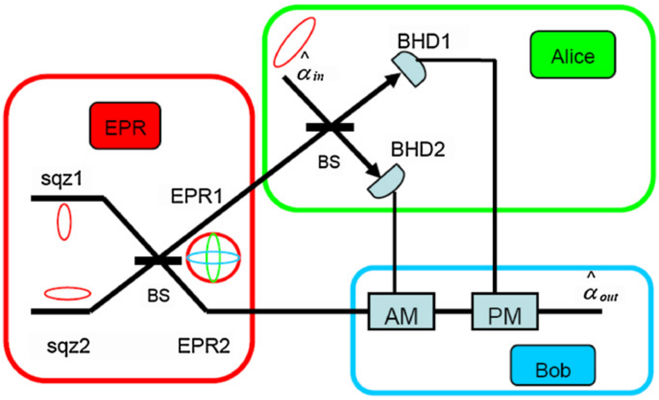

Figure 1. Schematic setup of the continuous-variable quantum teleportation, which consists of a source of a two-mode squeezed-vacuum state, a BS, two BHD, along with amplitude modulation and phase modulation.

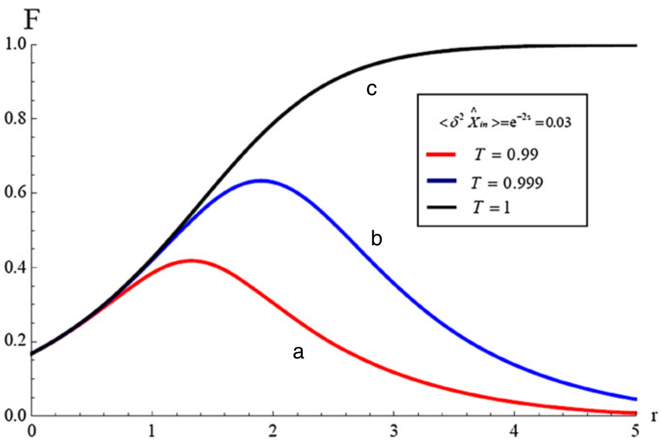

Figure 2. The influence of the phase-damping model on the fidelity for different EPR correlation parameters r of the noisy quantum channel. The input state with the squeezing variance $\langle\delta^2\hat{X}_{\rm in}\rangle = 0.03$. The lines *a, b* and *c* are corresponding to the different quantum channel transmission coefficients $T = 0.99$, 0.999 and 1, respectively.

It can be found from equation (21) that, if we have a perfect EPR ($e^{-2r} = 0$ and $T = 1$), for any input state, we always have a perfect quantum teleportation with unit fidelity. We plot the fidelity, equation (21), versus r in figure 2 with the input state of the squeezing variance $\langle\delta^2\hat{X}_{\rm in}\rangle = 0.03$. Lines *a, b* and *c* correspond to the different quantum channel transmission coefficients $T = 0.99$, 0.999 and 1 (different losses), respectively. For the same input state, the smaller the transmission coefficient T, the worse the fidelity will be. The fidelity decreases to a fixed value, $F_1 = \left[\sqrt{\left(1+\frac{1}{\langle\delta^2\hat{X}_{\rm in}\rangle}\right)\left(1+\langle\delta^2\hat{X}_{\rm in}\rangle\right)}\right]^{-1}$, when the correlation parameter r decreases to 0. Furthermore, the fidelity decreases to 0.5 when the correlation parameter r decreases to 0 and the input state is the coherent state. This is the classical fidelity limit of coherent state quantum teleportation. The fidelity has a maximum value when $e^{2r}(1-T) = e^{-2r}(1+T)$. Then the fidelity decreases exponentially as the correlation parameter r increases and $T \neq 1$, since the anti-squeezed noise, e^{2r}, is induced in the quantum channel, which means that the two-mode squeezed-vacuum state is quickly destroyed in the phase-damping model.

We can obtain an inequation from equation (21) as

$$F_1 \leqslant \frac{1}{\sqrt{\left(1+\frac{\sqrt{1-T^2}}{\langle\delta^2\hat{X}_{\rm in}\rangle}\right)\left(1+\sqrt{1-T^2}\langle\delta^2\hat{X}_{\rm in}\rangle\right)}}. \quad (22)$$

Equation (22) is an upper limit of fidelity, which only depends on the T and variances of the input state. Note that the upper limit of fidelity has nothing to do with the correlation parameter r of the two-mode squeezed-vacuum state. It is shown that the smaller the squeezing variances of the input state $\langle\delta^2\hat{X}_{\rm in}\rangle$, the worse the fidelity with the same transmission coefficient T.

Next, we will consider the amplitude-damping model. The master equation for the density matrix is the same as equation (15), but

$$\begin{aligned} L_i\rho &= (\bar{n}+1)(2\hat{a}_i\rho\hat{a}_i^+ - \hat{a}_i^+\hat{a}_i\rho - \rho\hat{a}_i^+\hat{a}_i) \\ &\quad + \bar{n}(2\hat{a}_i^+\rho\hat{a}_i - \hat{a}_i\hat{a}_i^+\rho - \rho\hat{a}_i\hat{a}_i^+), \end{aligned} \quad (23)$$

where $\bar{n}$ is the average photon number of the thermal environment [23]. We can also calculate the covariance matrix of $\rho(t)$ as

$$\begin{aligned} &C(\hat{X}_1, \hat{Y}_1, \hat{X}_2, \hat{Y}_2) \\ &= \frac{e^{-\Gamma t}}{2}\begin{pmatrix} \cosh 2r & 0 & \sinh 2r & 0 \\ 0 & \cosh 2r & 0 & -\sinh 2r \\ \sinh 2r & 0 & \cosh 2r & 0 \\ 0 & -\sinh 2r & 0 & \cosh 2r \end{pmatrix} \\ &+(1-e^{-\Gamma t})\begin{pmatrix} \bar{n}+\frac{1}{2} & 0 & 0 & 0 \\ 0 & \bar{n}+\frac{1}{2} & 0 & 0 \\ 0 & 0 & \bar{n}+\frac{1}{2} & 0 \\ 0 & 0 & 0 & \bar{n}+\frac{1}{2} \end{pmatrix}. \end{aligned} \quad (24)$$

From the covariance matrix in equation (24), the variances of the operators $\hat{X}_1 - \hat{X}_2$, $\hat{Y}_1 + \hat{Y}_2$ are found to be

$$\begin{aligned} \langle\delta^2(\hat{X}_1 - \hat{X}_2)\rangle &= (1+2\bar{n})(1-T) + Te^{-2r}, \\ \langle\delta^2(\hat{Y}_1 + \hat{Y}_2)\rangle &= (1+2\bar{n})(1-T) + Te^{-2r}. \end{aligned} \quad (25)$$

In experimental situations, the thermal environment is considered as the vacuum state [6, 7, 9, 10], which means $\bar{n} = 0$; therefore, the parameter e^{-2r} which appeared in equation (13) is replaced with $1 - T(1 - e^{-2r})$ [24, 25], so the output state variances now become

$$\begin{aligned} \langle\delta^2\hat{X}_{\rm out}\rangle &= \langle\delta^2\hat{X}_{\rm in}\rangle + 2\left[1 - T\left(1 - e^{-2r}\right)\right], \\ \langle\delta^2\hat{Y}_{\rm out}\rangle &= \langle\delta^2\hat{Y}_{\rm in}\rangle + 2\left[1 - T\left(1 - e^{-2r}\right)\right], \end{aligned} \quad (26)$$

where the quantities $\langle\delta^2\hat{X}_{\rm out}\rangle(\langle\delta^2\hat{Y}_{\rm out}\rangle)$ are experimentally measurable. The second term includes the extra noise from the imperfect transport, detecting and so on.

By substituting equation (26) for equation (9), the fidelity is obtained as

$$F = \frac{1}{\sqrt{\left(1+\frac{1-T[1-\exp(-2r)]}{\langle\delta^2\hat{X}_{\rm in}\rangle}\right)[1+\langle\delta^2\hat{X}_{\rm in}\rangle[1-T(1-\exp(-2r))]]}}. \quad (27)$$

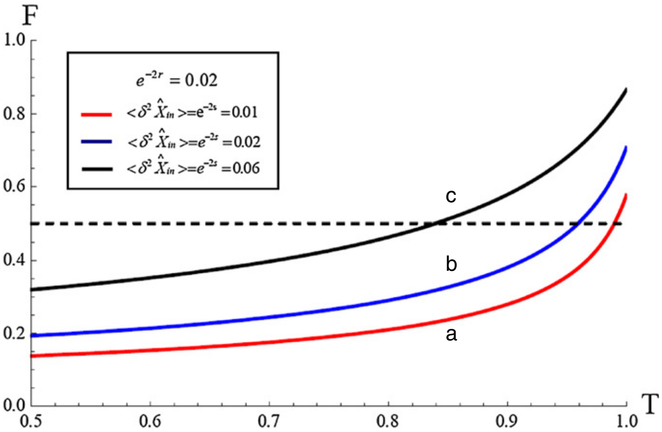

Figure 3. The influence of the amplitude-damping model on the fidelity, where r is the EPR correlation parameter and T is the transmission coefficient of the noisy quantum channel. The lines *a*, *b* and *c* correspond to the squeezing variance of the input state $\langle\delta^2\hat{X}_{\text{in}}\rangle = 0.01,\ 0.02$ and 0.06, respectively.

It can be found from equation (27) that, if we have a perfect EPR ($\exp(-2r) = 0$ and $T = 1$), for any input state, we always have a perfect quantum teleportation whose fidelity is 1. Therefore, we plot the fidelity, equation (27), versus T in figure 3 with the EPR quantum channel of $\exp(-2r) = 0.02$. Lines *a, b* and *c* correspond to the squeezing variances of the input state $\langle\delta^2\hat{X}_{\text{in}}\rangle = 0.01, 0.02$ and 0.06, respectively. For the same quantum channel, the smaller the squeezing variances of the input state $\langle\delta^2\hat{X}_{\text{in}}\rangle$, the worse the fidelity will be. To perform the quantum teleportation with high fidelity, the 'noise' of each mode of the quantum channel needs to be large enough (which means strong entanglement) to 'hide' the quantum fluctuations of the input state. For the transmission coefficient $T = 0$, it means that the entanglement state is lost completely, and it is a complete classical teleportation, where the fidelity is never larger than 0.5. When $\langle\delta^2\hat{X}_{\text{in}}\rangle < 1$, the fidelity is always below 0.5. The fidelity will be very sensitive to the losses ($T = \exp(-\Gamma t)$), if the transmission coefficient T approaches to 1.

Comparing equations (20) and (26) in the two damping models with the same input state and the same transmission coefficient T in the noisy quantum channel, we find that the variances of the output state in the phase-damping model are always larger than the variances of the output state in the amplitude-damping model. This means the fidelity in the phase-damping model is always smaller than the fidelity in the amplitude-damping model when T is the same for both ($T = \exp(-\Gamma t)$, same Γ).

In classical teleportation, the entanglement state between Alice and Bob is replaced by a vacuum state. So we obtain the fidelity limit of the classical teleportation for the squeezed state input from equation (8):

$$F = \frac{1}{\sqrt{2 + \mathrm{e}^{2s} + \mathrm{e}^{-2s}}}. \tag{28}$$

For any classical teleportation, the fidelity for the squeezed input state is always smaller than 0.5. When the squeezing parameter of the input state is $s = 0$ (i.e. coherent state), we obtain the classical fidelity limit of 0.5 for the coherent state. The classical fidelity limit of the squeezed state decreases exponentially as the squeezing parameter s increases, which is due to the fragile nature of squeezing.

In conclusion, we have presented a general expression of the fidelity for any Gaussian input state teleportation directly related to the experimental measurable variances of the output and the input state. Our result shows that this expression can accurately quantify the quantum teleportation for both the coherent and the squeezed input states. Furthermore, the fidelity was discussed when the quantum channel lies in the phase- and amplitude-noisy environment, which is unavoidable in experiment, and it was showed that the effect of the noisy phase on teleportation is more sensitive than that of the noisy amplitude, so in experiment, the faithful teleportation must prevent the environment from the phase damping. The classical fidelity limit of the squeezed state is also obtained when the entanglement state does not exist in quantum teleportation.

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References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)

[2] Braunstein S L and Pati A K 2002 *Quantum Information Theory with Continuous Variables* (Dordrecht: Kluwer)

[3] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895–9

[4] Bouwmeester D, Pan J W, Mattle K, Elbl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575–9

[5] Schumacher B 1996 *Phys. Rev.* A **54** 2614–28

[6] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869–72

[7] Furusawa A, Srensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706–9

[8] Bowen W P, Treps N, Buchler B C, Schnabel R, Ralph T C and Bachor H A 2003 *Phys. Rev.* A **67** 032302

[9] Zhang T C, Goh K W, Chou C W, Lodahl P and Kimble H J 2003 *Phys. Rev.* A **67** 033802

[10] Zhai Z H, Li Y M, Wang S K, Guo J, Zhang T C and Gao J R 2005 *Acta Phys. Sin.* **54** 2710

[11] Milburn G J and Braunstein S L 1999 *Phys. Rev.* A **60** 937–42

[12] Takei N *et al* 2005 *Phys. Rev.* A **72** 042304

[13] Mišta L Jr, Filip R and Furusawa A 2010 *Phys. Rev.* A **82** 012322

[14] Lee N, Benichi H, Takeno Y, Takeda S, Webb J, Huntington E and Furusawa A 2011 *Science* **333** 330–3

[15] Lee J, Kim M S and Jeong H 2000 *Phys. Rev.* A **62** 032305

[16] Ban M 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** 224–30

[17] Zhang J X, Xie C D, Li F, Zhu S Y and Zubairy M S 2000 *Europhys. Lett.* **56** 478–84

[18] Zhang J X, Xie C D and Peng K C 2005 *Chin. Phys. Lett.* **22** 3005

[19] Grangier P and Grosshans F 2001 arXiv:quant-ph/0010107
[20] Andersen U L, Josse V and Leuchs G 2005 *Phys. Rev. Lett.* **94** 240503
[21] Zhang J, Xie C D and Peng K C 2005 *Phys. Rev. Lett.* **95** 170501
[22] Hiroshima T 2001 *Phys. Rev.* A **63** 022305
[23] Walls D F and Milburn G J 1994 *Quantum Optics* (Berlin: Springer-Verlag)
[24] Ban M, Sasaki M and Takeoka M 2002 *J. Phys. A: Math. Gen.* **35** L401–5
[25] Takeoka M, Ban M and Sasaki M 2002 *J. Opt. B: Quantum Semiclass. Opt.* **4** 114–22