PHYSICAL REVIEW A **85**, 022106 (2012)

Relation between wave-particle duality and quantum uncertainty

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(Received 8 July 2011; published 6 February 2012)

We present a study of the relation between wave-particle duality and quantum uncertainty in a two-path interferometer and derive equalities and inequalities involving the visibility (representing wave-like behavior), the predictability (representing particle-like behavior), and their variances. We experimentally demonstrate that, for a single photon in a Mach-Zehnder interferometer, these quantities are related via an equation that connects both duality and uncertainty. This relation holds for the single-photon source prepared either in a pure state or a mixed state.

DOI: 10.1103/PhysRevA.85.022106 PACS number(s): 03.65.Ta, 42.50.Ar, 42.50.Lc, 42.50.Xa

I. INTRODUCTION

The principle of complementarity, which emphasizes equally real but mutually exclusive properties of quantum systems, is fundamentally important in the quantum theory. It states that, for two conjugate variables, the observation of one precludes the observation of the other. A consequence of the complementarity principle is wave-particle duality. The Young's double-slit experiment provides a good example of illustrating wave-particle duality [1,2]. The concept of a quantum eraser and the consequent experiments shed light on duality [3–7] and demonstrate that the observation of wave-like and particle-like behaviors in, for example, a Young's double-slit experiment can even be made after the registration of the quantum [8,9]. Recently, an experimental test of duality relation [1,2,10,11] in an interferometer has been reported [12], and the quantified wave information and particle information can be varied to demonstrate the exchange procedure between wave-like and particle-like behaviors [13]. There has been heated debate whether the appearance or disappearance of fringes in a double-slit experiment is due only to duality or can be interpreted in terms of the Heisenberg uncertainty relation. The relationship between the two principles has been discussed in Feynman's light microscope [14], Scully's quantum eraser [15], and Englert's interferometer [16], but the discussion continues [17–21].

Historically, one of the issues has been that, unlike uncertainty relations, complementarity as espoused by Bohr was not formulated in terms of a formal mathematical relation. Recently, it has, however, been shown that the predictability P (particle nature) and the visibility of the fringes V (wave nature) are connected to each other through the inequality $P^2 + V^2 \leqslant 1$ [16]. This inequality represents a mathematical statement of complementarity but it does not involve uncertainties. It is natural to ask whether there is a relationship of P and V with the uncertainties and whether this inequality can be converted to an equality by adding terms involving uncerainties. In this paper, we answer these questions positively. We show explicitly how the visibility and predictability and their fluctuations are related to each other. In particular, we show that, for the single-photon case, an equality is obtained that involves not only the visibility and predictability but also their fluctuations. We also demonstrate that this equality holds experimentally in a Mach-Zehnder interferometer [22].

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II. THEORY

The particle-like and wave-like behaviors of an electron or a photon in a Mach-Zehnder interferometer (see Fig. 1) is usually measured by the which-path information, i.e., the predictability (P) and the visibility (V) of the interference pattern [1,2,10,11,16,23], respectively. These are given by

$$P = |p_1 - p_2|, \tag{1a}$$

$$V = p_{\max} - p_{\min}, \tag{1b}$$

where $p_{1(2)}$ is the information knowing the probability of passing through path 1 (2), and $p_{\max(\min)}$ is the probability of the maximum (minimum) hits on the screen. The duality relation is given by

$$P^2 + V^2 \leqslant 1, \tag{2}$$

where the equal sign holds for pure states.

Quantum-mathematical descriptions of the visibility and predictability in an interferometer for a single photon are given by the Pauli operators [24–26],

$$\hat{P} = \hat{\sigma}_z, \tag{3a}$$

$$\hat{V} = \cos\phi\hat{\sigma}_x + \sin\phi\hat{\sigma}_y, \tag{3b}$$

where the phase parameter ϕ should be appropriately chosen to maximize the expectation value of the operator, (3b). The visibility and predictability are then the modules of the two expectation values, i.e., $P = |\langle\hat{P}\rangle|$ and $V = |\langle\hat{V}\rangle_{\max}|$. We note that the two operators $\hat{P}$ and $\hat{V}$ are connected to each other through a unitary transformation, $U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -e^{i\phi} \\ e^{-i\phi} & 1 \end{pmatrix}$, which represents the combined action of the phase shifter (PZT) and the 50:50 beam splitter (BS) (see Fig. 1).

We choose $\{|1_1 0_2\rangle, |0_1 1_2\rangle\}$ as the basis for the single-photon system, where the subscripts indicate the two paths, and the two states represent one photon in path 1 or 2. A general state for a single photon in such a basis can be described by a 2×2 density matrix $\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$. Based on the quantum description of the visibility and predictability in Eq. (3), we can derive their expectation values [24,25],

$$P = |\langle \hat{P} \rangle| = |\rho_{11} - \rho_{22}|, \tag{4a}$$

$$V = |\langle \hat{V} \rangle_{\max}| = 2|\rho_{12}|, \tag{4b}$$

and their quantum uncertainties,

$$(\Delta P)^2 = \langle \hat{P}^2 \rangle - (\langle \hat{P} \rangle)^2 = 4\rho_{11}\rho_{22}, \tag{5a}$$

$$(\Delta V)^2 = \langle \hat{V}^2 \rangle - (\langle \hat{V} \rangle)^2 = 1 - 4|\rho_{12}|^2. \tag{5b}$$

In obtaining Eq. (4b) we have maximized the expectation value of the operator, (3b), $\langle \hat{V} \rangle = 2\text{Re}(\rho_{21} e^{-i\phi}) = 2|\rho_{21}| \cos(\theta - \phi)$, with θ being the argument of ρ_{21}, by adjusting $\phi = \theta$.

From Eqs. (4) and (5) we can establish a number of inequalities and equalities. For example, we obtain

$$P^2 + (\Delta P)^2 = 1, \tag{6a}$$

$$V^2 + (\Delta V)^2 = 1. \tag{6b}$$

In order to understand the physical meaning of these equations, we note that the duality relation $P^2 + V^2 \leqslant 1$ can now be rewritten in the form

$$P^2 \leqslant (\Delta V)^2, \tag{7a}$$

$$V^2 \leqslant (\Delta P)^2. \tag{7b}$$

These inequalities play the role of uncertainty relations for the visibility and the predictability, as the maximum visibility ($V = 1$) would require the maximum uncertainty in the predictability ($\Delta P = 1$), and vice versa. We also note that the four quantities in Eqs. (4) and (5) satisfy the equality,

$$P^2 + V^2 + (\Delta P)^2 + (\Delta V)^2 = 2. \tag{8}$$

This establishes a relation between the duality and the quantum uncertainty for the single-photon system. Equation (8) is a general equation for the duality. This relation, however, is restricted in the sense that the choice of P and V determines uniquely the values of the uncertainties ΔV and ΔP, respectively, via Eqs. (6).

These relations between the visibility and predictability and their variances can be tested in an experiment. We present results that satisfy relation (8). However, from the data, we can see that, for both pure and mixed states, all the inequalities and equalities mentioned above are satisfied.

III. EXPERIMENTS

In our experiment, the measurement is performed on an ensemble of single photons passing through the interferometer and recording the photon counting. For the measurement of visibility, we have to appropriately adjust the PZT to a particular value so that the counting difference between the two detectors D_1 and D_2 is maximized within a fixed time interval, e.g., 1 s. The probabilities in Eq. (1b) are thus derived through $p_{\max(\min)} = N_{\max(\min)}/(N_{\max} + N_{\min})$, with $N_{\max(\min)}$ being the photon counting recorded by D_1 and D_2, respectively. For the measurement of predictability, which is independent of the phase difference between the two paths, we need to remove BS in Fig. 1 and determine the probabilities in Eq. (1b) through $p_{1(2)} = N_{1(2)}/(N_1 + N_2)$, with $N_{1(2)}$ being the photon counting in detector $D_{1(2)}$. With the measured $p_{\max}$ (or p_1) and $p_{\min}$ (or p_2), we determine the visibility and predictability through relations (1b) and (1a), and their variances through

$$(\Delta V)^2 = (\Delta p_{\max})^2 + (\Delta p_{\min})^2 - 2\Delta(p_{\max} p_{\min}), \tag{9a}$$

$$(\Delta P)^2 = (\Delta p_1)^2 + (\Delta p_2)^2 - 2\Delta(p_1 p_2). \tag{9b}$$

The probabilities, variances, and covariances in Eqs. (1a), (1b), (9a), and (9b) are achievable from the photon counting recorded by detectors D_1 and D_2.

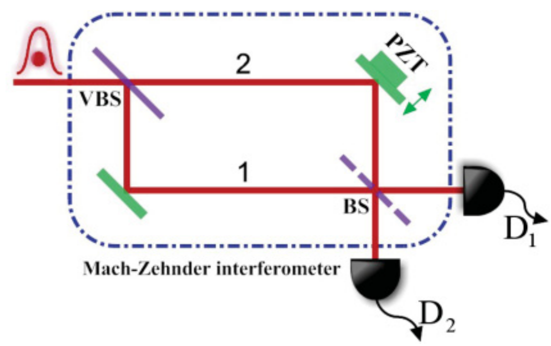

FIG. 1. (Color online) Experimental setup: polarized single photons produced from parametric down-conversion are sent to a Mach-Zehnder interferometer, where VBS is a beam splitter with adjustable reflectivity (R), PZT is a phase shifter, BS is a 50:50 beam splitter, and D_1 and D_2 are two single-photon detectors.

We use the second harmonic generation from a Ti:sapphire laser at 850 nm with pulse width 180 fs and repetition rate 76 M to pump a $4 \times 4 \times 0.6$ mm β-barium borate (BBO) crystal generating 850-nm orthogonally polarized photon pairs through type II parametric down-conversion. One of the two photons is chosen as the single-photon source to feed the interferometer (see Fig. 1). The quality of the single-photon source is tested through the coincidence counts of detectors D_1 and D_2 [12,13], with the coincidence counts no more than 3 among 2100 photons recorded within a time of 0.36 s. The random dark and background counts are about 100 per second. The bin width of each single-photon detector is set at 4 ns. In the experiment, the variable BS (VBS) is composed of a half-wave plate followed by a polarization BS, and the second BS, at the output, is composed of two polarization BSs with a half-wave plate in the middle.

The VBS with adjustable reflectivity R turns the single photon into the superposition,

$$|\Phi\rangle = \sqrt{1-R}|1_1 0_2\rangle + \sqrt{R}e^{i\theta}|0_1 1_2\rangle, \tag{10}$$

where θ is the phase difference between the two paths, induced by the VBS and also the length difference of the two paths. To maximize the visibility based on the above state, the phase shift ϕ, controlled by the PZT, should be set at $\phi = \theta$. In this case, the visibility operator in (3b) becomes $\hat{V} = \begin{pmatrix} 0 & e^{-i\theta} \\ e^{i\theta} & 0 \end{pmatrix}$. For measurement of the predictability P, we remove the BS and count the difference between the two detectors. This yields the measurement of $\hat{\sigma}_z$.

The experimental setup (see Fig. 1) used here by us is a little different from the one used in Ref. [13]: The locations of the 50:50 BS and the VBS are exchanged in the two experiments. In Ref. [13], the particle information is measured after the interference of the photons inside the interferometer, and it is classified as *a posteriori* distinguishability. In this situation, the single photons are fed into the two paths of the interferometer with equal probability, and the path distinguishability is measured through an unbalanced output BS. Given the balanced photon flux along the two paths of the interferometer, the unbalanced output BS changes the photon's path distinguishability by varying its reflectivity. In our experiment, we demonstrate the duality of the single photons in another way. The initial state of the single photon is variable here, which leads to an unbalanced and controllable photon flux into the two paths of the interferometer, and the particle information is directly determined by the initial state of the single photon. This type of particle information, associated with an unbalanced photon flux inside the interferometer, i.e., predictability, is called *a priori* distinguishablity in Ref. [13]. In fact, many theoretical works on duality are based on a variable initial state for the single photon, with the which-way information referred to as the predictability, as in Refs. [26] and [27]. In the Appendix, we show explicitly that the two experimental schemes are equivalent for demonstrating the photon's duality as well as the corresponding measurement uncertainties.

IV. RESULTS AND DISCUSSION

The theoretical values of visibility and predictability [13,16], $V = 2\sqrt{R(1-R)}$ and $P = |1-2R|$, and their variances versus R are obtained from Eqs. (5) and (10); see the curves in Fig. 2. The points with error bars are the experimental results for the squared visibility [dotted (blue) line], the predictability [dashed (burgundy) line], and their sum [solid (red) line] in Fig. 2(a), and for the two variances and their sum (using the same colors and the same types of line) in Fig. 2(b). The total sum of the four quantities in Eq. (8) is plotted in Fig. 2(c), which confirms that Eq. (8) is satisfied. The sum of the four quantities is 2.0 ± 0.11. Note that the results for $R = 0.5$ to 1.0 are the same as those for $R = 0.5$ to 0.0. In the experiment, the highest visibility is 0.91, which is lower than the theoretical value of 1. This discrepancy is caused due to the unbalanced probability of the photons entering the two paths in the interferometer, decoherence of the flying photons, imperfect overlap of the two interfering paths at the output, and unequal detection efficiencies of the detectors.

Next we examine Eq. (8) for a single photon in a complete mixed state (no off-diagonal elements):

$$\rho = (1-R)|1_1 0_2\rangle\langle 1_1 0_2| + R|0_1 1_2\rangle\langle 0_1 1_2|. \tag{11}$$

This state is realized by completely randomizing the phase difference between the two paths in the interferometer. Using the same method, we measured the squared visibility, the predictability, and their variances (see Fig. 3). As expected, almost-vanishing visibility is observed for any R, while the predictability presents the same behavior as in the pure-state

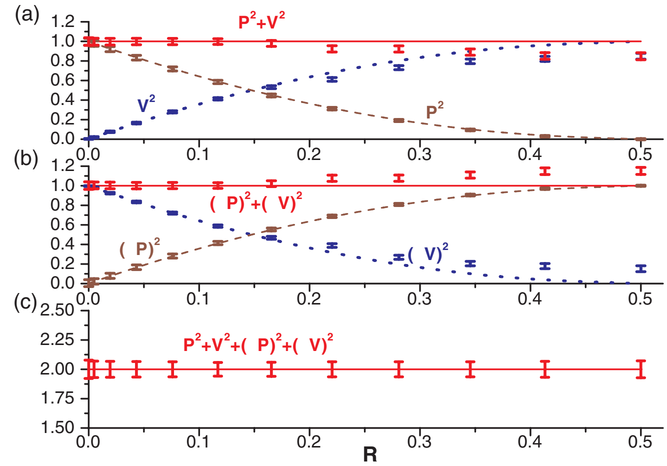

FIG. 2. (Color online) (a) Squared visibility [dotted (blue) line], predictability [dashed (burgundy) line), and their sum [solid (red) line] for the state of Eq. (10). (b) Their variances, using the same colors and the same types of line. (c) Total sum of the four quantities. The x axis is the reflectivity R of the VBS.

case. The missing particle information (for increasing R) does not turn to the wave knowledge of visibility [see Fig. 3(a)]; it causes an increase in uncertainty [see Fig. 3(b) and keeps the sum of the four quantities equal to 2.0 ± 0.1.

If we prepare the system in a state between the pure state, Eq. (10), and the completely mixed one, Eq. (11) (off-diagonal elements not equal to 0), we observe a similar result, with the total sum being 2.0 ± 0.11. We have thus shown that the complementary relation between the duality and the uncertainty, Eq. (8), holds for both pure and mixed states.

V. CONCLUSIONS

We note, in conclusion, that although the derivation of the duality, Eq. (2), does not require Heisenberg's uncertainty relation, this does not mean that the duality has to be independent of the quantum uncertainty. In this paper we

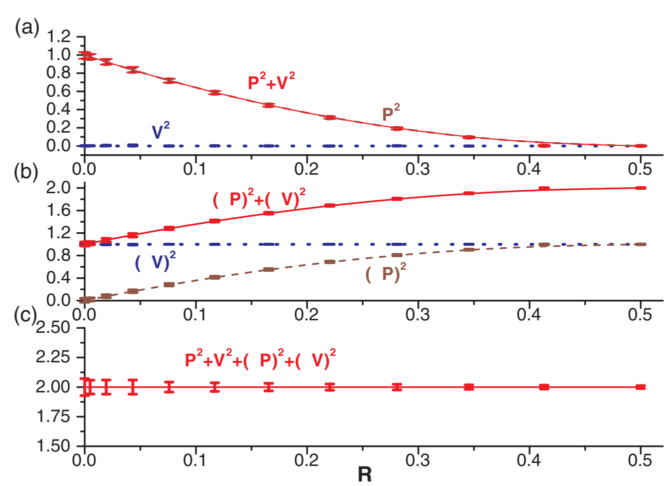

FIG. 3. (Color online) The same as Fig. 2, for the mixed state of Eq. (11).

have elucidated the relationship between the two. We note that, in the case of mixed states, the duality relation presents only an incomplete description of the exclusive relation between wave-like and particle-like behaviors. Our results indicate the intricate relationship between duality and quantum uncertainty. In particular, Eq. (8), which holds for both pure and mixed states, tells us that the missing duality information in fact causes an increase in total uncertainty in the same system. In other words, although neither the duality relation, Eq. (2), nor the quantum uncertainty (represented by the sum of the two variances) can be considered a direct consequence of the other, the duality and quantum uncertainty are intrinsically connected to each other. By feeding the Mach-Zehnder interferometer with single photons and detecting them at the single-photon level, we have experimentally verified this relation for pure or mixed states of a single-photon.

ACKNOWLEDGMENTS

This research is supported by the National Basic Research Program of China (Grants No. 2011CB922203 and No. 2010CB923102) and the National Natural Science Foundation of China (Grants No. 11174026, No. 10774096, No. 60978008, No. 11174118, and No. 10804042). The research of M.S.Z. is supported by a grant from the Qatar National Research Fund under NPRP Project No. 4-520-1-083.

APPENIDX: AN EQUIVALENCE PROOF FOR THE TWO TYPES OF EXPERIMENTAL SETUPS

A schematic of the two types of experimental setups used in the present work and in the work in Ref. [13] is shown in Fig. 4. The main difference is that the locations of the variable VBS and the 50:50 BS are exchanged in these two experimental setups. In the following, we prove that the two experimental schemes are equivalent to each other for demonstrating a photon's duality.

The action of a BS with reflectivity R can be represented by a unitary transformation [28], $U_R = \begin{pmatrix} \sqrt{R} & \sqrt{1-R} \\ -\sqrt{1-R} & \sqrt{R} \end{pmatrix}$, which connects the annihilation operators, $a_{\text{in}1(2)}$, of the input beams of the BS and the creation operators, $a_{\text{out}1(2)}$, of the output beams of the BS via

$$\begin{pmatrix} a_{\text{out}1} \\ a_{\text{out}2} \end{pmatrix} = U_R \begin{pmatrix} a_{\text{in}1} \\ a_{\text{in}2} \end{pmatrix}. \tag{A1}$$

In the following, we describe the action of the 50:50 BS by the unitary transformation $U_{\frac{1}{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ and the action of the variable BS by the unitary transformation U_R, mentioned above.

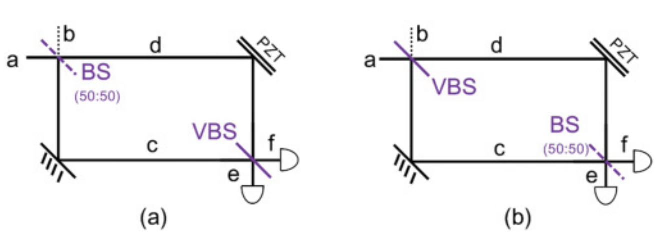

FIG. 4. (Color online) Schematic of the experimental setup used (a) in the work in Ref. [13] and (b) in this work.

In the first experimental setup [see Fig. 4(a)], the combination of the 50:50 BS, the PZT, represented by the unitary transformation $S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$, and the VBS connects the two annihilation operators for the photons in the input beams, a and b, and the annihilation operators for the photons in the output beams, e and f, by the following:

$$\begin{aligned}\begin{pmatrix} e \\ f \end{pmatrix} &= U_R S U_{\frac{1}{2}} \begin{pmatrix} a \\ b \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{R} - \sqrt{1-R}e^{i\phi} & \sqrt{R} + \sqrt{1-R}e^{i\phi} \\ -\sqrt{1-R} - \sqrt{R}e^{i\phi} & -\sqrt{1-R} + \sqrt{R}e^{i\phi} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}.\end{aligned} \tag{A2}$$

The visibility is then measured through the operator $\hat{V}_1 = \frac{e^\dagger e - f^\dagger f}{\langle e^\dagger e + f^\dagger f \rangle}$, with the subscript 1 indicating the first experimental scheme in Fig. 4(a). In terms of the creation and annihilation operators $a^\dagger$, $b^\dagger$, a, and b, this can be described as

$$\begin{aligned}\hat{V}_1 = &\frac{1}{\langle a^\dagger a + b^\dagger b \rangle}\{2\sqrt{R(1-R)}\cos\phi(b^\dagger b - a^\dagger a) \\ &+ (2R-1)(a^\dagger b + b^\dagger a) \\ &- 2i\sqrt{R(1-R)}\sin\phi(b^\dagger a - a^\dagger b)\}.\end{aligned} \tag{A3}$$

Similarly, the combination of the VBS, the PZT, and the 50:50 BS in the second experimental setup in Fig. 4(b) connects the four annihilation operators, a, b, e, and f, for the photons in the input and the output beams by

$$\begin{aligned}\begin{pmatrix} e \\ f \end{pmatrix} &= U_{\frac{1}{2}} S U_R \begin{pmatrix} a \\ b \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{R} - \sqrt{1-R}e^{i\phi} & \sqrt{1-R} + \sqrt{R}e^{i\phi} \\ -\sqrt{R} - \sqrt{1-R}e^{i\phi} & -\sqrt{1-R} + \sqrt{R}e^{i\phi} \end{pmatrix} \begin{pmatrix} e \\ f \end{pmatrix}.\end{aligned} \tag{A4}$$

The operator $\hat{V}_2 = \frac{e^\dagger e - f^\dagger f}{\langle e^\dagger e + f^\dagger f \rangle}$ for the measurement of visibility in this scheme is then equivalent to

$$\begin{aligned}\hat{V}_2 = &\frac{1}{\langle a^\dagger a + b^\dagger b \rangle}\{2\sqrt{R(1-R)}\cos\phi(b^\dagger b - a^\dagger a) \\ &+ 2R\cos\phi(a^\dagger b + b^\dagger a) - (a^\dagger b e^{-i\phi} + b^\dagger a e^{i\phi})\}.\end{aligned} \tag{A5}$$

Given the same input state $|\psi_0\rangle = |1_a 0_b\rangle$ for both experimental setups in Fig. 4, it can be shown that the interference patterns in both types of interferometers exhibit the same visibility, i.e.,

$$V = |\langle \psi_0 | \hat{V}_{1,2} | \psi_0 \rangle_{\max}| = 2\sqrt{R(1-R)}. \tag{A6}$$

The quantum fluctuation of measurements of the visibility can be evaluated as

$$(\Delta V)^2 = \langle \psi_0 | \hat{V}_{1,2}^2 | \psi_0 \rangle - |\hat{V}_{1,2}|^2 = (1-2R)^2. \tag{A7}$$

To measure the which-way information, i.e., distinguishability in Ref. [13] and predictability in the present work, one of the two paths inside the interferometer, e.g., path c, should be blocked in the first experimental scheme [see Fig. 4(a)] or, equivalently, the 50:50 BS should be removed in the second experimental scheme [see Fig. 4(b)]. By ignoring the photons blocked in the first experiment, which are not included in any photon counting in the experiment, the annihilation operator a for the input photons is connected to the annihilation operators

e and f for the output photons by

$$\begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} \sqrt{1-R}e^{i\phi} & \sqrt{R} \\ \sqrt{R}e^{i\phi} & -\sqrt{1-R} \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix}. \quad \text{(A8)}$$

A similar relation can be established in the second experimental scheme, which is

$$\begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} -\sqrt{1-R}e^{i\phi} & \sqrt{R}e^{i\phi} \\ \sqrt{R} & \sqrt{1-R} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \quad \text{(A9)}$$

Accordingly, the operator $\frac{e^\dagger e - f^\dagger f}{\langle e^\dagger e + f^\dagger f\rangle}$ for evaluating the particle information in the interferometer can be rewritten as

$$\hat{D} = \frac{(1-2R)(a^\dagger a - c^\dagger c) + 2\sqrt{R(1-R)}(e^{-i\phi}a^\dagger c + e^{i\phi}c^\dagger a)}{\langle a^\dagger a + c^\dagger c\rangle} \quad \text{(A10a)}$$

for the distinguishability in the first experimental scheme in Fig. 4(a) and as

$$\hat{P} = \frac{(1-2R)(a^\dagger a - b^\dagger b) - 2\sqrt{R(1-R)}(a^\dagger b + b^\dagger a)}{\langle a^\dagger a + b^\dagger b\rangle} \quad \text{(A10b)}$$

for the predictability in the second experimental scheme in Fig. 4(b).

Imposing the above two operators on the initial state $|\psi_0\rangle = |1_a 0_b\rangle$ (or $|\psi_0'\rangle = |1_a 0_c\rangle$), we get the expectation value of the distinguishability in the first experimental scheme to be

$$D = |\langle\psi_0'|\hat{D}|\psi_0'\rangle| = |1-2R| \quad \text{(A11a)}$$

and the predictability in the second experimental scheme to be

$$P = |\langle\psi_0|\hat{P}|\psi_0\rangle| = |1-2R|. \quad \text{(A11b)}$$

The uncertainties of measuring the distinguishability and predictability are

$$(\Delta\hat{D})^2 = \langle\psi_0'|\hat{D}^2|\psi_0'\rangle - D^2 = 4R(1-R) \quad \text{(A12a)}$$

and

$$(\Delta\hat{P})^2 = \langle\psi_0|\hat{P}^2|\psi_0\rangle - P^2 = 4R(1-R), \quad \text{(A12b)}$$

respectively. Since all the results of measuring the particle information and the wave information are identical for the two types of interferometers, we believe they are equivalent for demonstrating the photon's duality.

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