# Squeezing-enhanced rotating-angle measurement beyond the quantum limit

Kui Liu, Chunxiao Cai, Juan Li, Long Ma, Hengxin Sun, and Jiangrui Gao

Citation: Appl. Phys. Lett. **113**, 261103 (2018); doi: 10.1063/1.5066028
View online: https://doi.org/10.1063/1.5066028
View Table of Contents: http://aip.scitation.org/toc/apl/113/26
Published by the American Institute of Physics

APPLIED PHYSICS LETTERS **113**, 261103 (2018)

# Squeezing-enhanced rotating-angle measurement beyond the quantum limit

Kui Liu, Chunxiao Cai, Juan Li, Long Ma, Hengxin Sun, and Jiangrui Gao[a)]

*State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan, Shanxi 030006, China*

(Received 11 October 2018; accepted 4 December 2018; published online 28 December 2018)

Aided by quantum sources, quantum metrology helps to enhance measurement precision. Here, we introduce a method to enhance the measurement of a rotation angle. As a proof of principle, assisted by a quantum state called the squeezed orbital-angular-position (OAP) state and balance homodyne detection, we demonstrate in experiments 3 dB-enhanced measurements of a rotation-angle beyond the shot noise limit. A precision of up to 17.7 nrad/$\sqrt{\text{Hz}}$ is obtained. Furthermore, we discuss means to further improve the measurement with a high-order precision OAP squeezed state. The method holds promise for future practical applications, such as in high-sensitive Sagnac interferometry.  https://doi.org/10.1063/1.5066028

Improving measurement precision is paramount in developing science and technology. In the classical domain, if $N$ is the number of measurements, the precision of a measurement is limited by the statistical scaling of errors of order $\sqrt{N}$, which corresponds to the standard quantum limit or shot noise limit (SNL) in quantum theory. Quantum metrology[1,2] is one field where new quantum technologies may emerge and offer genuine quantum advantages aided by nonclassical states. It could offer enhanced precision of measurements beyond the SNL and even beat the ultimate precision the "Heisenberg limit," or often referred to as the "Heisenberg bound", which scales as $N^{-1}$.

The orbital angular momentum (OAM) of light is associated with the transverse spatial distribution of the optical beam, described, for example, by the combination of Laguerre-Gaussian (LG) modes.[3] It has attracted wide attention and covered many applications, including optical manipulation and trapping,[4] super-resolution imaging,[5] high-precision measurement,[6] optical spin-orbital coupling, and the optical topological effect.[7] Because of the inherent higher dimensional spaces associated with OAM states, high-dimensional entanglement is achievable for quantum information processing and building high-capacity quantum communication networks.[4,8] Moreover, based on the "mechanical Faraday effect" for OAM states, rotation-angle can be measured with high precision. Recently, D̓Ambrosio *et al.*[9] realized ultra-sensitive angular measurements beyond the SNL using the NOON state ($N=2$) with OAM, and Magaña-Loaiza *et al.*[10] realized angular-rotation measurements employing weak measurements. However, it is experimentally difficult to generate NOON states with a high number of photons[11] as these states are fragile in the presence of losses,[1] and the weak measurement proposal is unable to go beyond the SNL.[12] Squeezing is a true quantum effect first observed 30 years ago in atomic vapour with the optical four-wave mixing process.[13] Squeezing has been expanded to other mesoscopic systems, such as spins of atoms,[14] and mechanical oscillators.[15,16] The squeezed state is viewed as a nice quantum state for metrology and more practical for true applications in metrology, such as in the advanced LIGO (Laser Interferometer Gravitation Wave Observatory) detectors.[17] Other attributes of the squeezing state have also been studied and found to be promising for the corresponding measurements of quantities in quantum enhanced metrology,[12,18] such as spin squeezing in magnetic field measurements,[19] spatial squeezing for displacement measurements,[20] and temporal-mode squeezing in clock synchronization.[21] Recently, optical squeezing levels have risen to 15.3 dB,[22] which is more attractive and promises improvements in measurement precision. To date, no report on angular rotation measurements based on squeezing has been published.

In this letter, we report a method for the angular rotation measurement based on squeezed light and balanced homodyne detection (BHD). We first give the commutation relation between the orbital angular position (OAP) and OAM and also demonstrate the SNL with classical light. Based on the commutation relation, we define the associated squeezed state, referred to as the squeezed "OAP" state, with which we can enhance the precision of rotation measurements in the experiment, by $3.00 \pm 0.03$ dB squeezing of the probe light to detect small rotation. We demonstrate a precision beyond the SNL and up to 17.7 nrad/$\sqrt{\text{Hz}}$ using an optimal detection system based on BHD, which reaches the Cramer-Rao bound limit.[9] The sub-shot noise rotation measurement has potential applications in high-precision sensing and monitoring of rotation vibrations,[23] for example, in combination with a rotational optomechanics system[24] to realize a high precision quantum gyroscope.

Generally, for a rotation-angle measurement system, the probe field is a spatial multimode field, which in a quantum mechanical description is written as follows:[25]

$$E_p^+(\mathbf{r},t) = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 cT}}\sum_{p=0}^{\infty}\sum_{l=-\infty}^{\infty}\hat{a}_{p,l}(t)u_{p,l}^{\sin}(\mathbf{r}), \tag{1}$$

a)Electronic mail: jrgao@sxu.edu.cn

 

where $\omega$ is the field frequency, $T$ the integration time, $u^{\sin}_{p,l}(\mathbf{r})$ the transverse beam amplitude function of the *sinusoidal* LG modes ($LG^{\sin}$),[26] $p$ the radial mode index, and $l$ the azimuthal mode index. The sinusoidal LG mode is an alternative form of LG modes with a sinusoidal amplitude dependence on the azimuthal angle [Fig. 1(a)]. The sinusoidal LG modes with $l = \pm 1$ and $p = 0$ are also the first-order Hermite-Gaussian (HG) modes. The corresponding annihilation operators, $\hat{a}_{p,l} = \hat{X}_{p,l} + i\hat{Y}_{p,l}$, can be written in a linear approximation in the form $\hat{a}_{p,l} = \langle \hat{a}_{p,l}\rangle + \delta\hat{a}_{p,l}$, where $\langle \hat{a}_{p,l}\rangle$ denotes the coherent amplitude and $\delta\hat{a}_{p,l}$ corresponds to the small quantum fluctuations.

If the probe beam has only a bright $LG^{\sin}_{0,n}$ mode ($l = n$, $p = 0$), then $\langle \hat{a}_{p,l}\rangle = \sqrt{N}$ (here, $N$ is the number of photons of the $LG^{\sin}_{0,n}$ mode) with all other modes giving $\langle \hat{a}_{p\neq 0, l\neq n}\rangle = 0$, the probe field may be rewritten as

$$E^{+}_{p} = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 cT}}\left\{\sqrt{N}u^{\sin}_{0,n}(\mathbf{r}) + \sum_{p=0}^{\infty}\sum_{l=-\infty}^{\infty}\delta\hat{a}_{p,l}(t)u^{\sin}_{p,l}(\mathbf{r})\right\}. \tag{2}$$

When the probe beam is rotated by a small angle $\theta$ about its direction of propagation $z(\theta \ll 1)$ [Fig. 1(b)], the rotated field $E^{+}(\theta)$ may be expanded in a Taylor series as follows:

$$E^{+}_{p}(\theta) = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 cT}}\sqrt{N}\left[u^{\sin}_{0,n}(\mathbf{r}) + n\theta u^{\sin}_{0,-n}(\mathbf{r})\right] + \sum_{p=0}^{\infty}\sum_{l=-\infty}^{\infty}\delta\hat{a}_{p,l}(t)u^{\sin}_{p,l}(\mathbf{r}). \tag{3}$$

Equation (3) states that the rotation-angle of the $LG^{\sin}_{0,n}$ mode beam is transferred to the amplitude of the $LG^{\sin}_{0,-n}$ mode, and the rotation-angle $\theta$ can be extracted by measuring the $LG^{\sin}_{0,-n}$ mode component of the field. Given the above equations, we introduce an OAP operator given by

$$\hat{\theta} = \frac{1}{n\sqrt{N}}\hat{X}_{0,-n}, \tag{4}$$

where $\theta = \langle\hat{\theta}\rangle$, $\hat{X}_{0,-n}$ is the amplitude quadrature of the $LG^{\sin}_{0,-n}$-mode component of the field, and the fluctuation of OAP is $\Delta\hat{\theta} = \frac{1}{n\sqrt{N}}\Delta\hat{X}_{0,-n}$.

Moreover, in accordance with the definition of a continuous-variable OAM state,[27–29] $\hat{O}$ denoting the OAM along the z axis is given by

$$\hat{O} = |l|(\hat{a}^{\dagger}_{LG_0^{+l}}\hat{a}_{LG_0^{+l}} - \hat{a}^{\dagger}_{LG_0^{-l}}\hat{a}_{LG_0^{-l}}) = 2n\sqrt{N}\hat{Y}_{0,-n}, \tag{5}$$

where $\hat{a}_{LG_0^{l}}$ is the $l$-order *helical* LG mode annihilation operator. The fluctuation of OAM is $\Delta\hat{O} = 2n\sqrt{N}\hat{Y}_{0,-n}$. Therefore, the OAM and OAP are conjugate observables and satisfy the following commutation relation[30,31] $[\hat{\theta}, \hat{O}] = i$ and related uncertainty $\Delta\hat{\theta} * \Delta\hat{O} \geq 1$. From the above equations, we find that when the probe beam is a coherent state ($\Delta\hat{X}_{0,-n} = 1/2$ and $\Delta\hat{Y}_{0,-n} = 1/2$), then it corresponds to the SNL of both OAM and OAP, $\Delta\hat{\theta}_{SNL} = \frac{1}{2n\sqrt{N}}$ and $\Delta\hat{O}_{SNL} = 2n\sqrt{N}$. If the vacuum $LG^{\sin}_{0,-n}$ mode component in the probe field is squeezed in amplitude or phase quadrature ($\Delta\hat{X}_{0,-n} < 1/2$ or $\Delta\hat{Y}_{0,-n} < 1/2$), then the fluctuation of OAP or OAM is below the SNL ($\Delta\hat{\theta} < \frac{1}{2n\sqrt{N}}$ or $\Delta\hat{O} < 2n\sqrt{N}$). This is analogous to the definition of the quadrature-squeezed state. We define the probe beam, which is a combination of a bright $LG^{\sin}_{0,n}$ mode coherent state and a squeezed vacuum $LG^{\sin}_{0,-n}$ mode state, as an n-order OAP- or OAM-squeezed state [refer Fig. 1(c)]. In metrology, using a OAP-squeezed state as the probe beam, we can obtain a precise measurement of a rotation-angle beyond the SNL, $\Delta\hat{\theta} = \frac{1}{2n\sqrt{N}}e^{-r}$, where $r$ is the factor associated squeezing.

To achieve the best possible measurement precision corresponding to the Cramer-Rao bound limit, we take the BHD method to demodulate the value of the angular displacement. In the BHD system, we generally consider the local oscillator field not to be in a pure $LG^{\sin}_{0,-n}$ mode but in a spatial multimode field $u_L(\mathbf{r})$ (here, $u_L(\mathbf{r}) = \sum_{p=0}^{\infty}\sum_{l=-\infty}^{\infty}\Gamma_{p,l}u^{\sin}_{p,l}(\mathbf{r})$ and $\sum_n \Gamma_n^2 = 1$). The local oscillator field is then written as

$$E^{+}_{L} = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 cT}}\left(\sqrt{N_L}u_L(\mathbf{r}) + \sum_{p=0}^{\infty}\sum_{l=-\infty}^{\infty}\delta\hat{a}^{L}_{p,l}(t)u^{\sin}_{p,l}(\mathbf{r})\right)e^{i\varphi_L}, \tag{6}$$

where $\varphi_L$ is the phase between the local oscillator field and the probe beam. Then, the output signal of the BHD system is given by

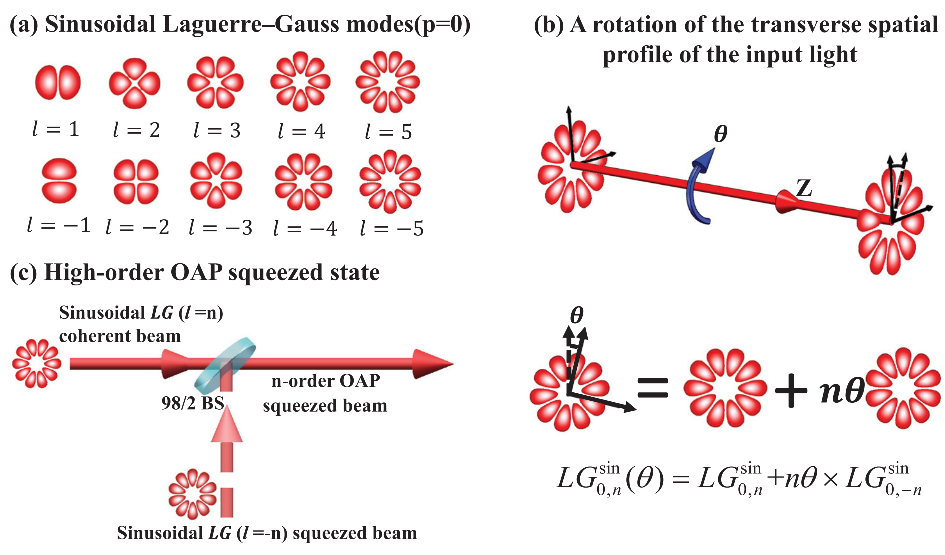


FIG. 1. (a) Spatial intensity distribution of a sinusoidal Laguerre-Gaussian mode for $p = 0$. (b) A beam is rotated by a small angle $\theta$ about its direction of propagation $z$ and the rotation-angle information is carried by its orthogonal mode component. (c) Schematic for the generation of the OAP squeezed state.

$$\hat{i}_- \propto \int dr(E_L^\dagger E_p(\theta) - E_p^\dagger(\theta)E_L)$$
$$= 2\sqrt{N_L N}(\Gamma_{0,n} + n\Gamma_{0,-n}\theta(\Omega))\cos(\varphi_L)$$
$$+2\sqrt{N_L}\sum_{p=0}^{\infty}\sum_{l=-\infty}^{\infty}\Gamma_{p,l}\delta\hat{X}_{p,l}^{\varphi_L}(\Omega). \quad (7)$$

The signal noise ratio (SNR), which equals the intensity of the rotation signal [the term in Eq. (7) depending on $\theta$] divided by the detection noise [the last term in Eq. (7)], may be deduced as follows:

$$SNR = \left|\frac{n\sqrt{N}\Gamma_{0,-n}\theta(\Omega)\cos(\varphi_L)}{\sqrt{\sum_{p=0}^{\infty}\sum_{l=-\infty}^{\infty}\Gamma_{p,l}^2\delta^2\hat{X}_{p,l}^{\varphi_L}(\Omega)}}\right|, \quad (8)$$

where $\Omega$ is the modulation frequency of the rotation signal. When $\varphi_L = m\pi(m=0,\ 1,\ 2\ldots)$, we obtain maximum $SNR$ corresponding to the measurement of the amplitude quadrature. When the probe beam is a coherent state, then $\delta\hat{X}_{p,l}^{\varphi_L}(\Omega) = 1/2$, and the $SNR = 2n\sqrt{N}\Gamma_{0,-n}\theta(\Omega)$. When the probe beam is an OAP-squeezed state, then $\delta\hat{X}_{p,l\neq 01}(\Omega) = 1/2$ and $\delta\hat{X}_{0,-n}(\Omega) = e^{-r}/2$. Hence, $S/N = \frac{2n\sqrt{N}\Gamma_{0,-n}\theta(\Omega)}{\sqrt{1-\Gamma_{0,-n}^2(1-e^{-2r})}}$. If the local field is a perfect $LG_{0,-n}^{\sin}$ mode, the measurement precision attains its optimal theoretical value. As for the impurity of the mode, detection loss modifies SNR giving a final form

$$SNR = \frac{2n\sqrt{N}\Gamma_{0,-n}\sqrt{\eta_{det}}\theta(\Omega)}{\sqrt{1-\Gamma_{0,-n}^2\eta_{det}(1-e^{-2r})}}, \quad (9)$$

where $\eta_{det}$ is the detection efficiency, which accounts for the propagation loss and detector efficiency. When $S/N = 1$, we obtain the minimum precision of the measurement

$$\theta_{min} = \frac{\sqrt{1-\Gamma_{0,-n}^2\eta_{det}(1-e^{-2r})}}{2n\sqrt{N}\Gamma_{0,-n}\sqrt{\eta_{det}}}. \quad (10)$$

In the experiment, the photon number $N(N = P\Delta t/\hbar\omega)$ is determined from both the optical power of the probe beam $P$ and the integration time of detection system $\Delta t(\Delta t \approx 1/RBW$; here, RBW is the resolution bandwidth of the spectrum analyzer).

Here, we demonstrate the proof of principle experiment with a first-order OAP squeezed state. In the experimental setup for the rotation-angle measurement (Fig. 2), a 2 $\mu$W squeezed beam in the $LG_{0,-1}^{\sin}$ mode at 1080 nm was generated using an optical parametric amplifier (OPA). The squeezed beam was coupled with a 100 $\mu$W bright coherent beam in the $LG_{0,1}^{\sin}$ mode at a 98/2 beam splitter (the coupling efficiency was 99.0 ± 0.5%) to generate an OAP-squeezed beam [Fig. 2(a)].[32] Next, the OAP-squeezed beam as the probe beam is passed through a beam rotation device which simulates an object rotating with rotational frequency $\Omega$ [Fig. 2(b)]. The probe light receives a small rotation about its propagation direction. Finally, the probe light carrying the rotational information of the rotation device is interrogated by the BHD using a 4 mW local oscillator (LO) beam in the $LG_{0,-1}^{\sin}$ mode incident on a spectrum analyzer to read the measurement results [Fig. 2(c)]. The beam rotation device is a rotated Dove Prime. The rotation of the Dove Prism is achieved using a PZT (Piezo-electric Transducer)-actuator based on the lever principle with the center of the Dove Prism as fulcrum [Fig. 2(d)].

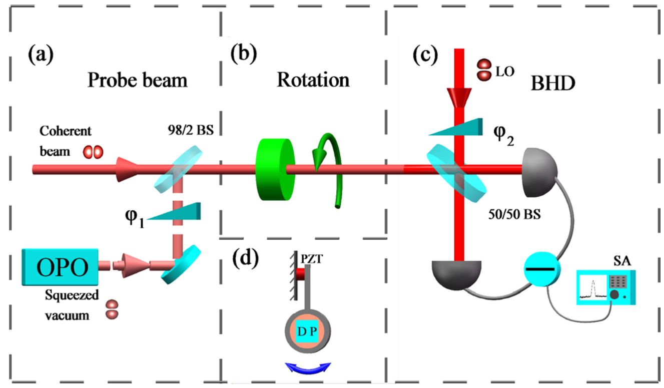


FIG. 2. Schematic of the setup to measure rotation-angles. (a) Generation of the probe beam. (b) Probe beam passing through the rotation device. (c) Rotated probe light is measured by the balance homodyne detection (BHD). (d) Schematic of the rotation device. OPO: optical parametric oscillator; BS: beam splitter; LO: local oscillator beam; PD: photodiode; SA: spectrum analyzer.

First, we analyzed the squeezing spectrum of the $LG_{0,-1}^{\sin}$-mode squeezed state from 100 kHz to 900 kHz (Fig. 3) showing squeezing to be 3.10 ± 0.03 dB. Here, the total detection efficiency is 0.74 ± 0.04, where the transmitting efficiency is 0.91 ± 0.02, the measuring efficiency of the photodiode (ETX500) is 0.90 ± 0.01, and the spatial overlap efficiency between the squeezed beam and the local oscillator beam on the homodyne detector is 0.90 ± 0.02. Hence, the inferred squeezing in the $LG_{0,-1}^{\sin}$ mode is 5.12 ± 0.68 dB. We demonstrate the rotation-angle measurement of a rotating device revolving at a frequency of $\Omega$ = 600 kHz. The measurements were recorded and displayed (Fig. 4) using a spectrum analyser with a resolution bandwidth (RBW) of 300 kHz and a video bandwidth (VBW) of 100 Hz at analyzed frequencies from 400 to 800 kHz. Trace (i) shows the shot-noise-level, which is measured by blocking the probe light. In the experiment, the power of the LO is 4 mW, which is much higher than the power of the probe light (100 $\mu$W), guaranteeing that the SNL is accurate; Trace (ii) is achieved when the probe light is coherent; in the experiment, only the

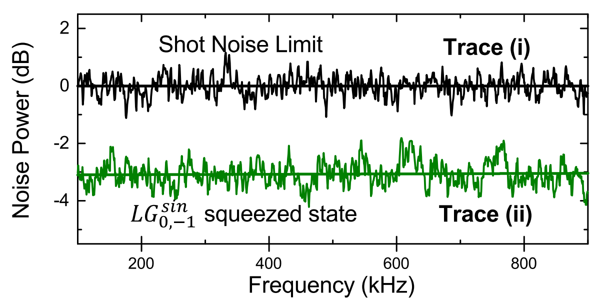


FIG. 3. Squeezing spectra of the $LG_{0,-1}^{\sin}$ mode. Trace (i) quantum noise limit; Trace (ii) the noise of the $LG_{0,-1}^{\sin}$ mode normalized to the quantum noise limit. The spectrum shows a broadband squeezing level of 3.10 ± 0.03 dB relative to the shot-noise-level.

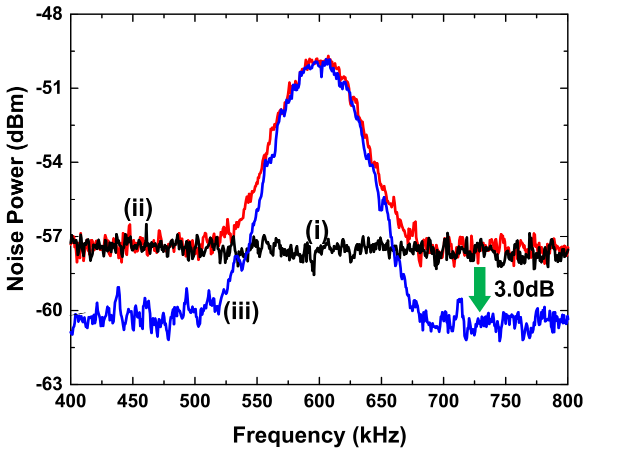


FIG. 4. Results of rotation-angle measurements at a central frequency of 600 kHz. RBW = 300 kHz and VBW = 100 Hz. Trace (i) shows the shot-noise-level without angular modulation. Trace (ii) shows the shot-noise-level of the rotation-angular measurements with the rotation modulated at 600 kHz when an OAP probe beam (not squeezed) is used. Trace (iii) shows the same signal obtained using OAP-squeezed light. The signal is centered about the peak at 600 kHz corresponding to the modulated frequency.

$LG^{\sin}_{0,-1}$-mode squeezed light is blocked when measured. Trace (iii) corresponds to the measurement result with the probe light in the OAP-squeezed state. Traces (ii) and (iii) are obtained when the phase of the LO is locked to ensure that it is in phase with the probe light. From Fig. 4, we see a distinct peak centered at a frequency of 600 kHz corresponding to the frequency of the measured rotation signal. With the probe light in the OAP-squeezed state, the measurement noise is below the SNL $3.00 \pm 0.03$ dB, which is a slightly lower than the squeezing of the $LG^{\sin}_{0,-1}$-mode mainly because of losses at the 98/2 beam splitter [Fig. 2(a)].

We also give the measurement of the rotational signal, the amplitude of which slowly increased over time, at 600 kHz with a resolution bandwidth (RBW) of 68 kHz and a video bandwidth (VBW) of 68 Hz. As seen in Fig. 5(a), Trace (ii) corresponds to the measurement performed with a coherent beam and represents the optimal precision that can be achieved with classical light. With no modulation signal, it corresponds to the SNL for background noise with a coherent beam. Trace (iii) corresponds to the measurement performed with an OAP-squeezed beam. With no modulation signal, it corresponds to the level of squeezing of the state and represents the background noise of the squeezed beam. Figure 5(b) shows the SNR obtained by processing the data from (a) to normalize the respective noise levels for the coherent beam [Trace (v)] and squeezed beam [Trace (iv)]. The horizontal axis corresponds to the rotation-angle which determines the theoretical value based on Eq. (9). In ramping the rotation-angle, the SNR obtained using the squeezed beam increases more rapidly than for the coherent beam. For a given SNR, the squeezed measurement yields a smaller rotation-angle than does the coherent case. A SNR equalling one (solid black line) corresponds to the best precision in measurement with a 68% confidence level. In our experiment, we have a power of 100 $\mu$W, a detection efficiency of 0.74, RBW = 68 kHz, and VBW = 68 Hz; the best precision in measurements with a coherent beam yield a $\theta^{coh}_{min}$ of 6.50 urad, whereas for the squeezed beam it is $\theta^{sq}_{min} = 4.60$ urad, which corresponds to sensitivity values of $24.9\,\text{nrad}/\sqrt{\text{Hz}}$ and $17.7\,\text{nrad}/\sqrt{\text{Hz}}$. Thus, an improvement by a factor of 1.4 has been achieved over the quantum noise limited coherent state.

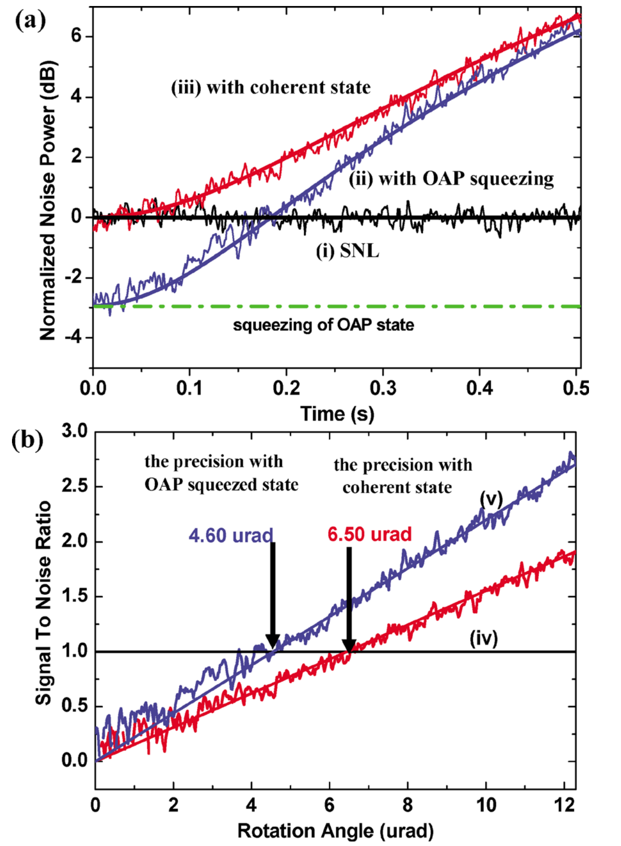


FIG. 5. Rotation angle measurement. (a) Measurement of the rotation signal ramped in time at 600 kHz. RBW = 68 kHz and VBW = 68 Hz, with (i) coherent and (ii) squeezed beams; (b) Data taken from (a) and processed to show the signal-to-noise ratio versus increasing rotation-angle. Traces (iv) and (v) show the signal-to-noise ratio for the coherent and squeezing states, respectively.

In conclusion, as a proof-of-principle experiment, we have demonstrated a new method to measure rotation-angle using an OAP-squeezed state that gives a precision beyond the SNL. Our proposal employs the BHD system and exploits its high measurement efficiency over a photon number counter. Moreover, weak signals may be measured using a sufficiently intense local oscillator. Furthermore, as a probe beam, we take squeezed light, which has the advantage that the squeezed mode is more robust that the NOON state and more practical in regard to future applications. Whether using lower frequency squeezing or the heterodyne detection technique, the method could garner a broader range of applications. Moreover, if we apply advanced techniques to obtain better squeezing, for example, above 10 dB, and use high-order OAP-squeezing states, it will be promising in improving vastly the sensitivity of the Sagnac interferometer[23,33,34] and detection accuracy in rotational motion of nano-materials and atoms and hence to deepen our understanding of light-matter interactions[16,24,35,36] and build a high precision quantum gyroscope. Furthermore, the proposal is promising to extension to the hyper-squeezed state with more than one degree of freedom to realize quantum metrology to estimate simultaneously more than one physical parameter.

This work was supported by the Key Project of the Ministry of Science and Technology of China (2016YFA0301404), National Natural Science Foundation of China (NSFC) (91536222, 11674205), and Program for OIT of Shanxi and Shanxi "1331 Project".

[1]V. Giovannetti, S. Lloyd, and L. Maccone, "Advances in quantum metrology," Nat. Photonics **5**, 222–229 (2011).
[2]V. Giovannetti, S. Lloyd, and L. Maccone, "Quantum metrology," Phys. Rev. Lett. **96**, 010401 (2006).
[3]A. M. Yao and M. J. Padgett, "Orbital angular momentum: Origins, behavior and applications," Adv. Opt. Photonics **3**, 161–204 (2011).
[4]M. Terriza, G. J. P. Torres, and L. Torner, "Twisted photons," Nat. Phys. **3**, 305–310 (2007).
[5]F. Tamburini, G. Anzolin, G. Umbriaco, A. Bianchini, and C. Barbieri, "Overcoming the Rayleigh criterion limit with optical vortices," Phys. Rev. Lett. **97**, 163903 (2006).
[6]M. P. Lavery, F. C. Speirits, S. M. Barnett, and M. J. Padgett, "Detection of a spinning object using light's orbital angular momentum," Science **341**, 537–540 (2013).
[7]K. Y. Bliokh and F. Nori, "Transverse and longitudinal angular momenta of light," Phys. Rep. **592**, 1–38 (2015).
[8]J. W. Pan, Z. B. Chen, C. Y. Lu, H. Weinfurter, A. Zeilinger, and M. Zukowski, "Multi-photon entanglement and interferometry," Rev. Mod. Phys. **84**, 777 (2012).
[9]V. D'Ambrosio, N. Spagnolo, L. Del Re, S. Slussarenko, Y. Li, L. C. Kwek, L. Marrucci, S. P. Walborn, L. Aolita, and F. Sciarrino, "Photonic polarization gears for ultra-sensitive angular measurements," Nat. Commun. **4**, 2432 (2013).
[10]O. S. Magaña-Loaiza, M. Mirhosseini, B. Rodenburg, and R. W. Boyd, "Amplification of angular rotations using weak measurements," Phys. Rev. Lett. **112**(20), 200401 (2014).
[11]I. Afek, O. Ambar, and Y. Silberberg, "High-NOON states by mixing quantum and classical light," Science **328**, 879–881 (2010).
[12]L. Zhang, A. Datta, and I. A. Walmsley, "Precision metrology using weak measurements," Phys. Rev. Lett. **114**, 210801 (2015).
[13]R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, "Observation of squeezed states generated by four-wave mixing in an optical cavity," Phys. Rev. Lett. **55**, 2409–2412 (1985).
[14]K. Hammerer, A. S. Søorensen, and E. S. Polzik, "Quantum interface between light and atomic ensembles," Rev. Mod. Phys. **82**, 1041–1093 (2010).
[15]E. E. Wollman, C. U. Lei, A. J. Weinstein, J. Suh, A. Kronwald, F. Marquardt, A. A. Clerk, and K. C. Schwab, "Quantum squeezing of motion in a mechanical resonator," Science **349**(6251), 952–955 (2015).
[16]J.-M. Pirkkalainen, E. Damskägg, M. Brandt, F. Massel, and M. A. Sillanpää, "Squeezing of quantum noise of motion in a micromechanical resonator," Phys. Rev. Lett. **115**(24), 243601 (2015).
[17]J. Aasi, J. Abadie, B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, C. Affeldt, O. D. Aguiar, P. Ajith, B. Allen, E. Amador Ceron, D. Amariutei, S. B. Anderson, W. G. Anderson, K. Arai, M. C. Araya, C. Arceneaux, S. Ast, S. M. Aston, D. Atkinson, P. Aufmuth *et al*., "Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light," Nat. Photonics **7**, 613–619 (2013).
[18]U. L. Andersen, T. Gehring, C. Marquardt, and G. Leuchs, "30 years of squeezed light generation," Phys. Scr. **91**, 053001 (2015).
[19]F. Wolfgramm, A. Cere, F. A. Beduini, A. Predojević, M. Koschorreck, and M. W. Mitchell, "Squeezed-light optical magnetometry," Phys. Rev. Lett. **105**, 053601 (2010).
[20]N. Treps, N. Grosse, W. P. Bowen, C. Fabre, H.-A. Bachor, and P. K. Lam, "A quantum laser pointer," Science **301**, 940–943 (2003).
[21]B. Lamine, C. Fabre, and N. Treps, "Quantum improvement of time transfer between remote clocks," Phys. Rev. Lett. **101**, 123601 (2008).
[22]H. Vahlbruch, M. Mehmet, K. Danzmann, and R. Schnabel, "Detection of 15 dB squeezed states of light and their application for the absolute calibration of photoelectric quantum efficiency," Phys. Rev. Lett. **117**, 110801 (2016).
[23]S. Xiao, L. Zhang, D. Wei, F. Liu, Y. Zhang, and M. Xiao, "Orbital angular momentum-enhanced measurement of rotation vibration using a Sagnac interferometer," Opt. Express **26**, 1997–2005 (2018).
[24]M. Bhattacharya, "Rotational cavity optomechanics," J. Opt. Soc. Am. B **32**, B55–B60 (2015).
[25]V. Delaubert, N. Treps, M. Lassen, C. C. Harb, C. Fabre, P. K. Lam, and H.-A. Bachor, "TEM10 homodyne detection as an optimal small-displacement and tilt-measurement scheme," Phys. Rev. A **74**, 053823 (2006).
[26]P. Fulda, K. Kokeyama, S. Chelkowski, and A. Freise, "Experimental demonstration of higher-order Laguerre-Gauss mode interferometry," Phys. Rev. D **82**, 012002 (2010).
[27]M. T. L. Hsu, W. P. Bowen, and P. K. Lam, "Spatial-state Stokes-operator squeezing and entanglement for optical beams," Phys. Rev. A **79**, 043825 (2009).
[28]M. Lassen, G. Leuchs, and U. L. Andersen, "Continuous variable entanglement and squeezing of orbital angular momentum states," Phys. Rev. Lett. **102**, 163602 (2009).
[29]K. Liu, J. Guo, C. Cai, S. Guo, and J. Gao, "Experimental generation of continuous-variable hyperentanglement in an optical parametric oscillator," Phys. Rev. Lett. **113**, 170501 (2014).
[30]S. Franke-Arnold, S. Barnett, E. Yao, J. Leach, J. Courtial, and M. Padgett, "Uncertainty principle for angular position and angular momentum," New J. Phys. **6**, 103 (2004).
[31]J. Leach, B. Jack, J. Romero, A. K. Jha, A. M. Yao, S. Franke-Arnold, D. G. Ireland, R. W. Boyd, S. M. Barnett, and M. J. Padgett, "Quantum correlations in optical angle-orbital angular momentum variables," Science **329**, 662–665 (2010).
[32]J. Guo, C. Cai, L. Ma, K. Liu, H. Sun, and J. Gao, "Higher order mode entanglement in a type ii optical parametric oscillator," Opt. Express **25**, 4985–4993 (2017).
[33]T. L. Gustavson, P. Bouyer, and M. A. Kasevich, "Precision rotation measurements with an atom interferometer gyroscope," Phys. Rev. Lett. **78**(11), 2046–2049 (1997).
[34]M. Granata, C. Buy, R. Ward, and M. Barsuglia, "Higher-order Laguerre-Gauss mode generation and interferometry for gravitational wave detectors," Phys. Rev. Lett. **105**, 231102 (2010).
[35]S. Franke-Arnold, G. Gibson, R. W. Boyd, and M. J. Padgett, "Rotary photon drag enhanced by a slow-light medium," Science **333**(6038), 65–68 (2011).
[36]O. Korech, U. Steinitz, R. J. Gordon, I. S. Averbukh, and Y. Prior, "Observing molecular spinning via the rotational Doppler effect," Nat. Photonics **7**(9), 711–714 (2013).